Combinatorics and Physics

Chapter 0
Introduction
Overview of the course
(part 3)

IIT-Madras
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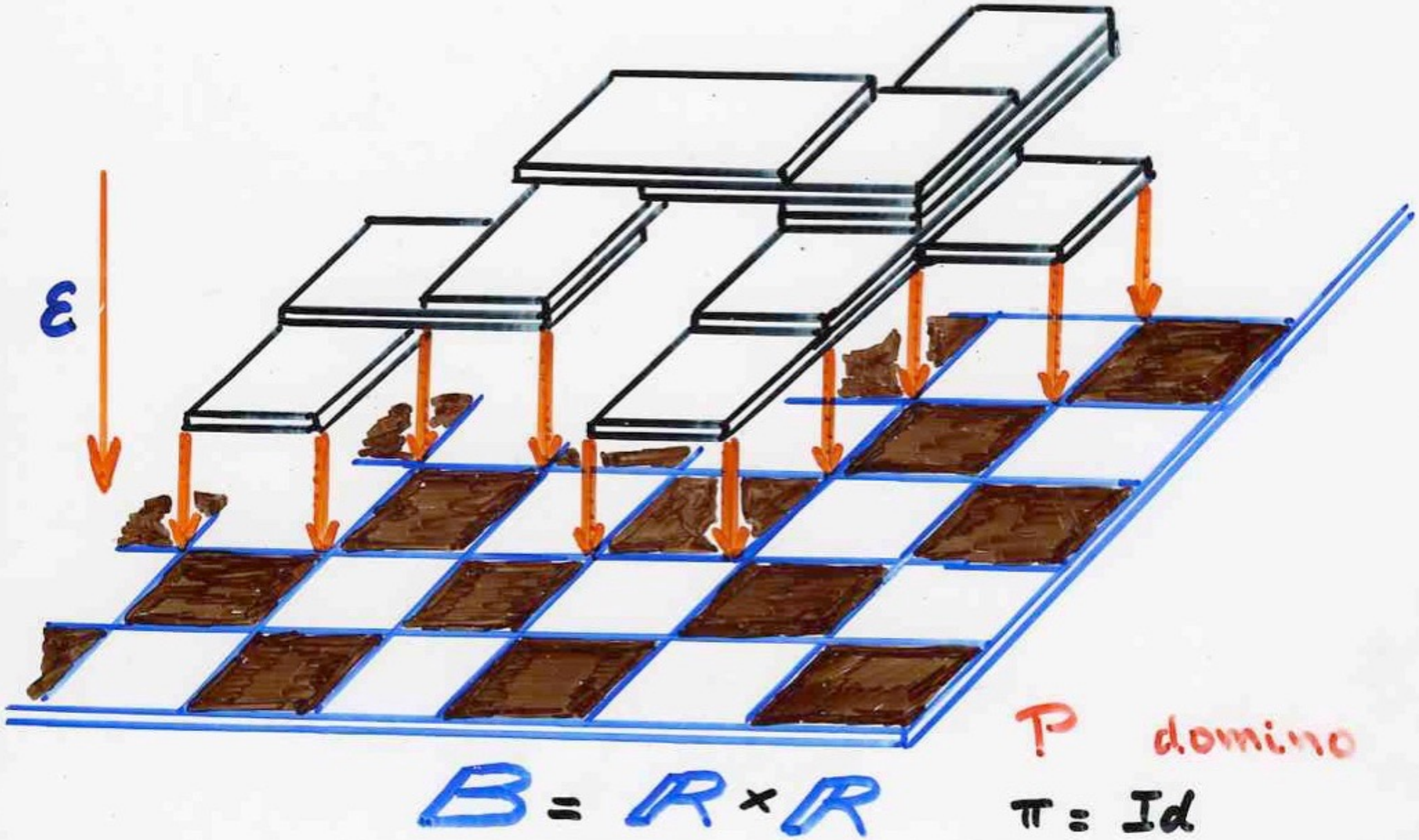
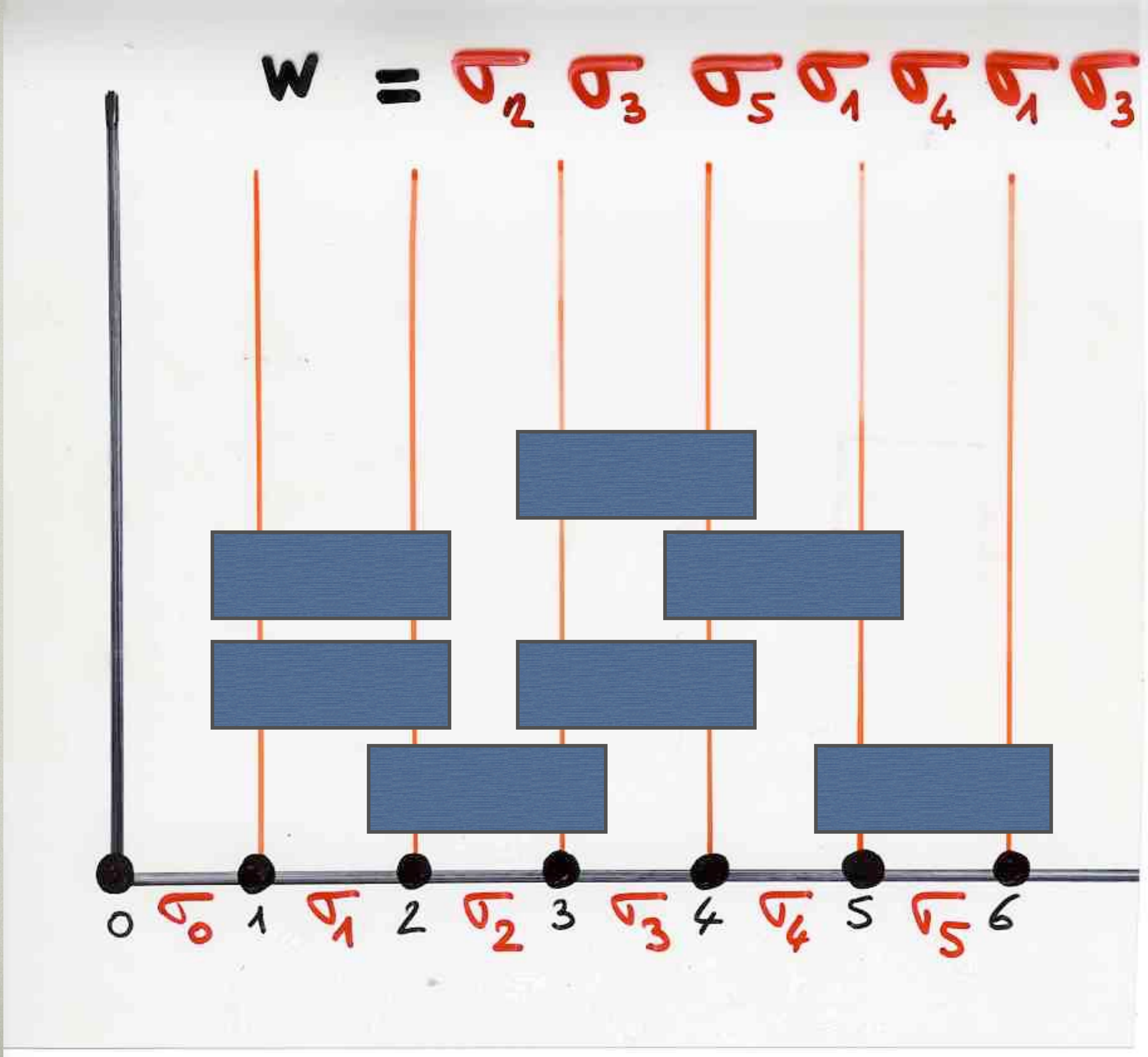
Heaps of pieces
and
commutations



ε
P domino
B = ℝ × ℝ
π = Id


$W = \sigma_2 \sigma_3 \sigma_5 \sigma_1 \sigma_4 \sigma_1 \sigma_3$
$0 \ \sigma_0 \ 1 \ \sigma_1 \ 2 \ \sigma_2 \ 3 \ \sigma_3 \ 4 \ \sigma_4 \ 5 \ \sigma_5 \ 6$

3 basic lemma

- $\left(\text{heaps}\right) = \dfrac{1}{\left(\text{trivial heaps}\right)}$
- $\log\left(\text{heaps}\right) = \text{pyramids}$
- path = heap

interactions with physics

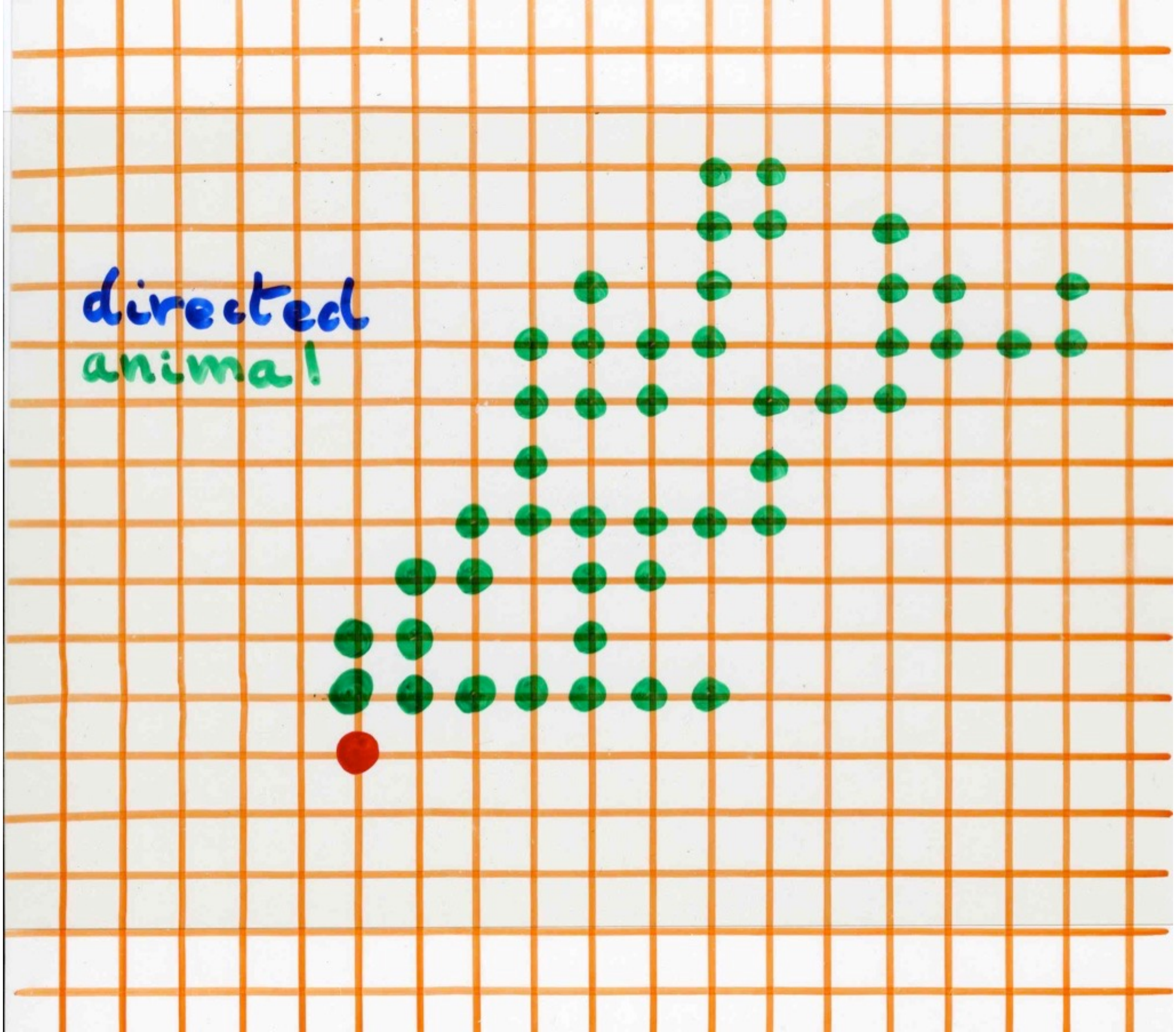
- loop-erased walks

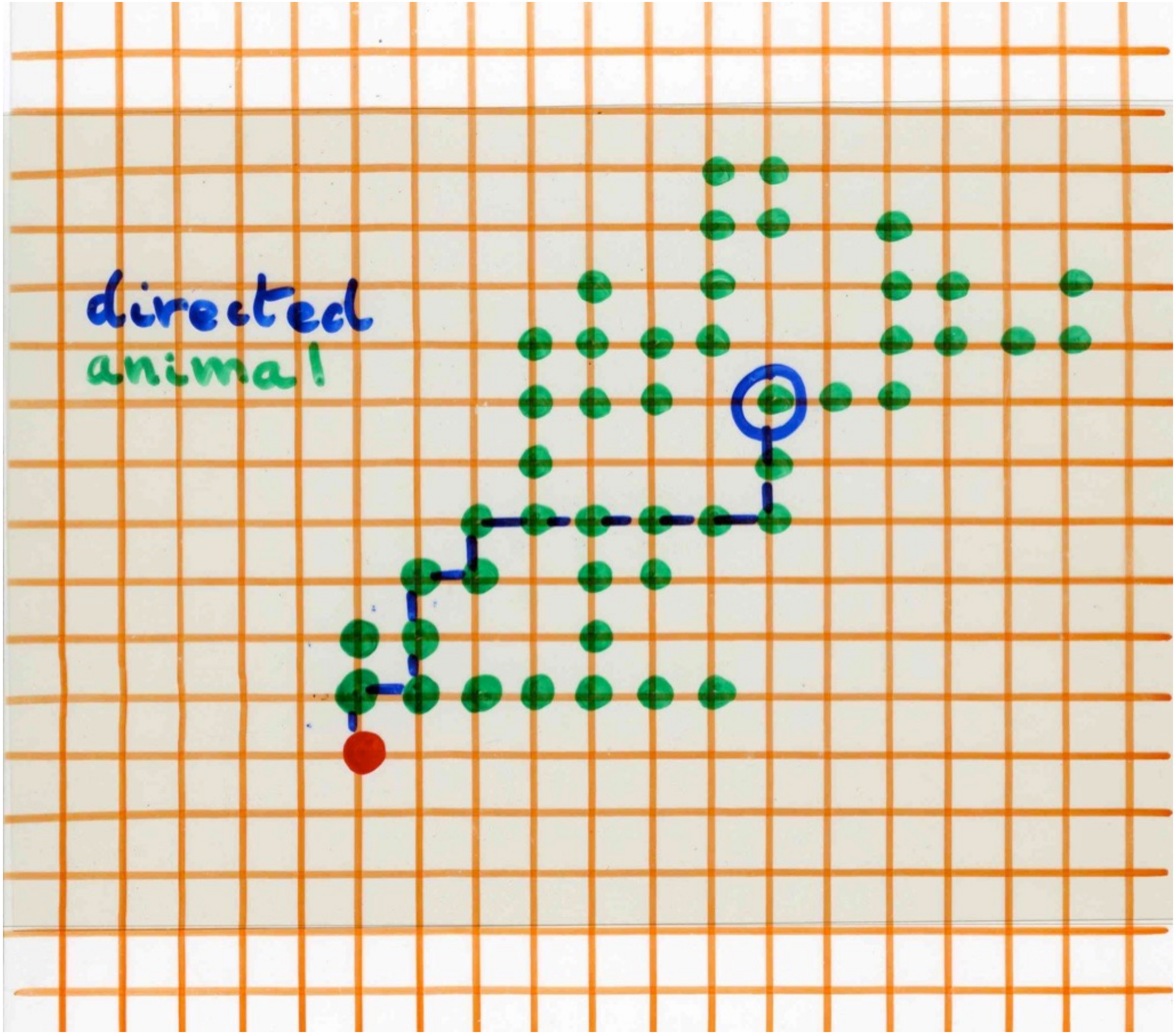
- the directed animal model
  and hard particule gas model

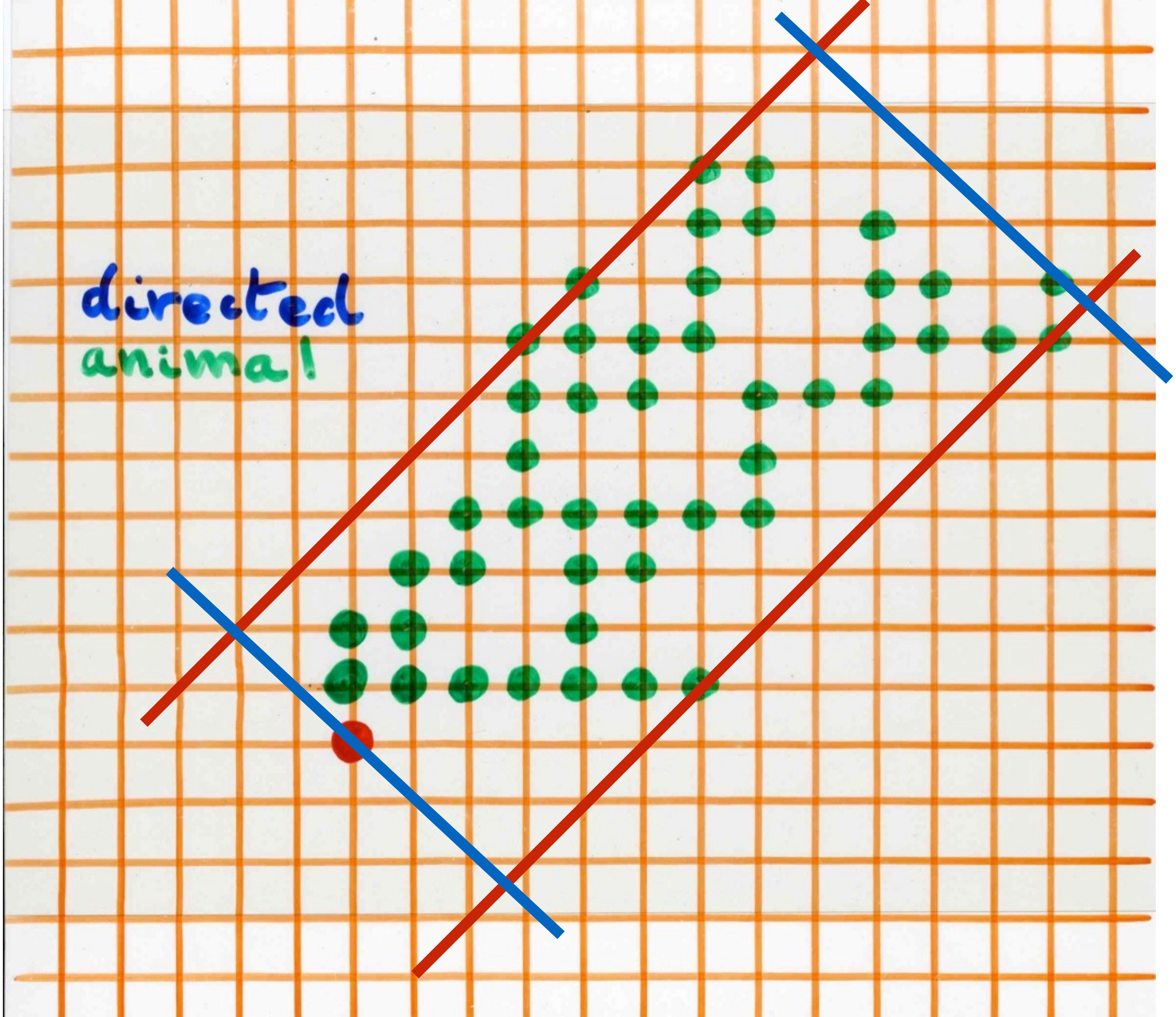
- q-Bessel functions in statistical physics:
  staircase polygons, SOS model, ...

- Lorentzian triangulations in 2D quantum gravity

directed animals


directed
animal


directed
animal


directed
animal

$$a_n \sim \mu^n \, n^{-\theta}$$

number of directed animals n points

$$\ell_n \sim n^{\nu_\perp}$$

average width

$$L_n \sim n^{\nu_\parallel}$$

average length

$$a_n \sim \mu^n n^{-\theta}$$

$\mu = 3 \quad \theta = 1/2$

$\nu_\perp = 1/2$

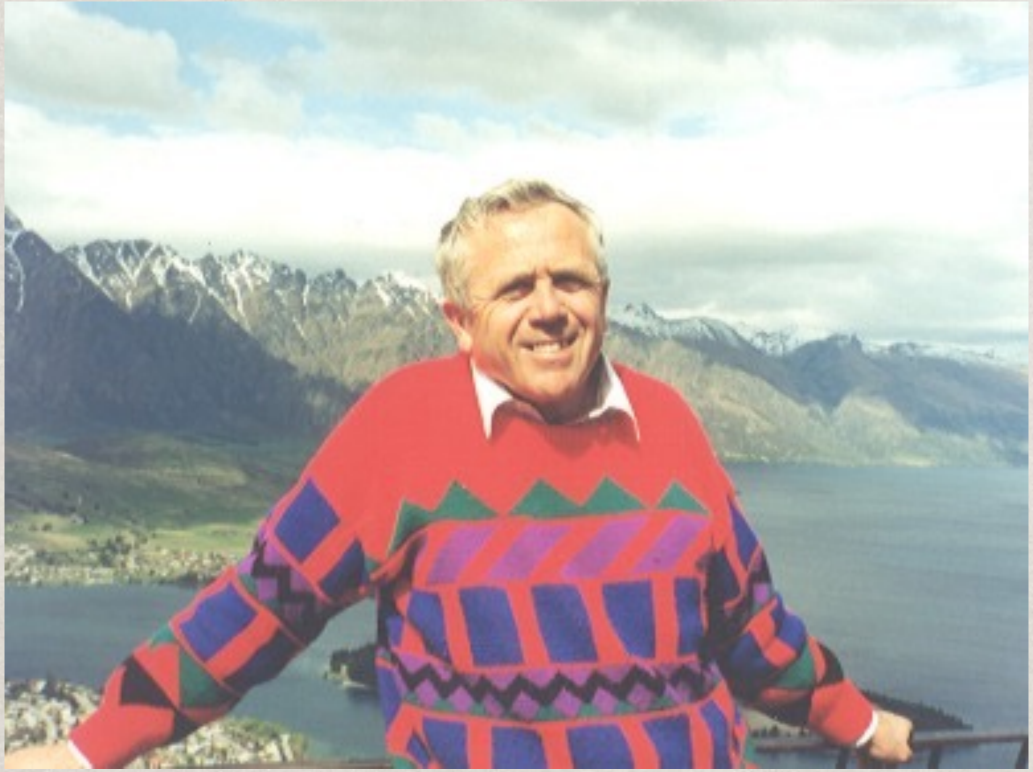
$\nu_\parallel = \cancel{9/11}$?

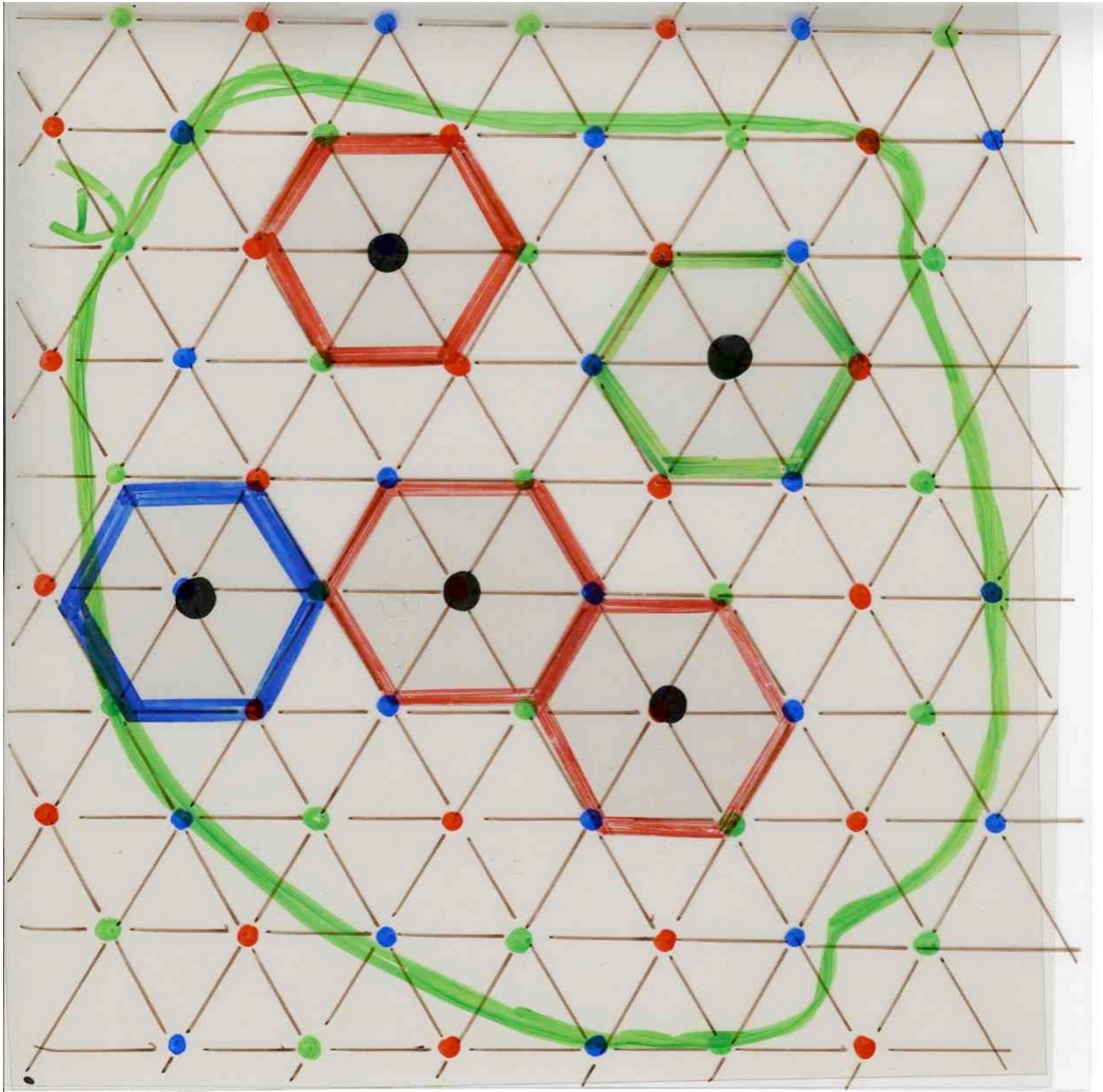
Hard Hexagons

Baxter

1980
1982

gas model
with "hardcore interactions"





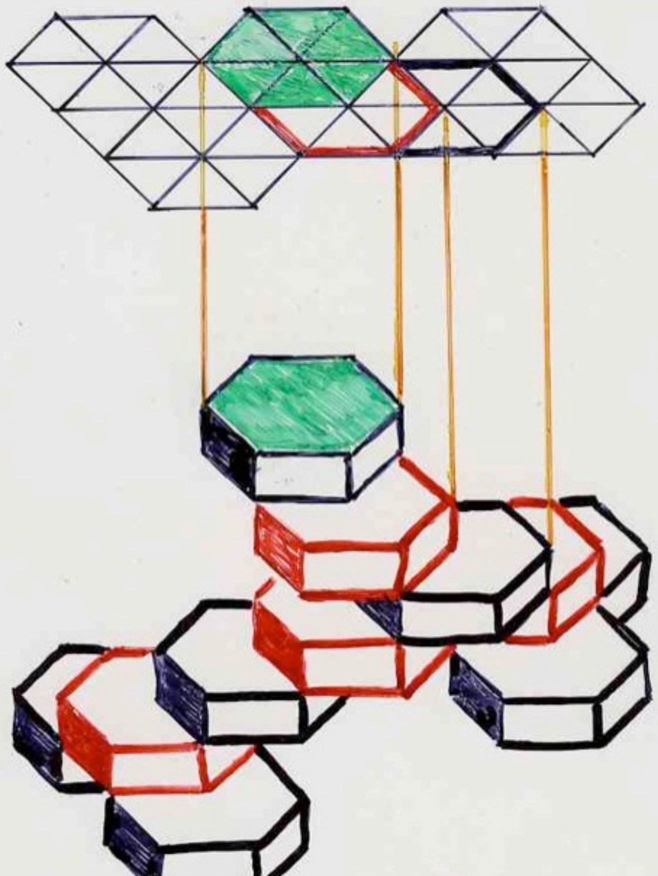
partition function

$$Z_D(t) = \sum_{n \geqslant 0} a_{n,D}\, t^n$$

$$Z(t) = \lim_{\text{"}D \to \infty\text{"}} \left(Z_D(t) \right)^{1/|D|}$$

thermodynamic limit

$$-p(-t) = y$$



KC.

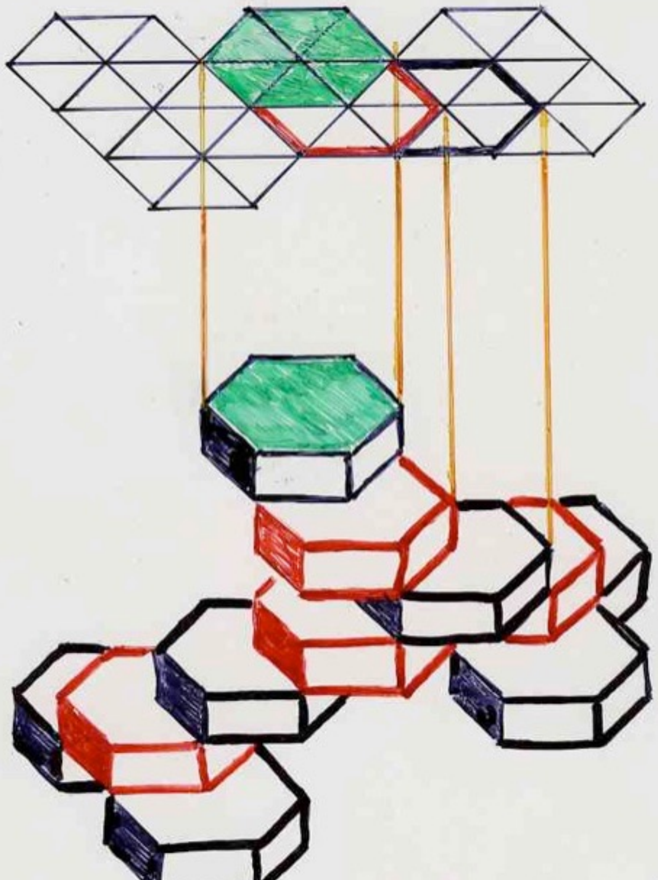
$$R(q) = \prod_{n \geq 0} \frac{(1-q^{5n+1})(1-q^{5n+4})}{(1-q^{5n+3})(1-q^{5n+2})} = \frac{R_{II}}{R_{I}}$$

$$t = -q\left[R(q)\right]^5$$

$$Y(q) = \prod_{n \geq 0} \frac{(1-q^{6n+2})(1-q^{6n+3})^2(1-q^{6n+4})(1-q^{5n+1})^2(1-q^{5n+4})^2(1-q^{5n})^2}{(1-q^{6n+1})(1-q^{6n+5})(1-q^{6n})^2(1-q^{5n+2})^3(1-q^{5n+3})^3}$$

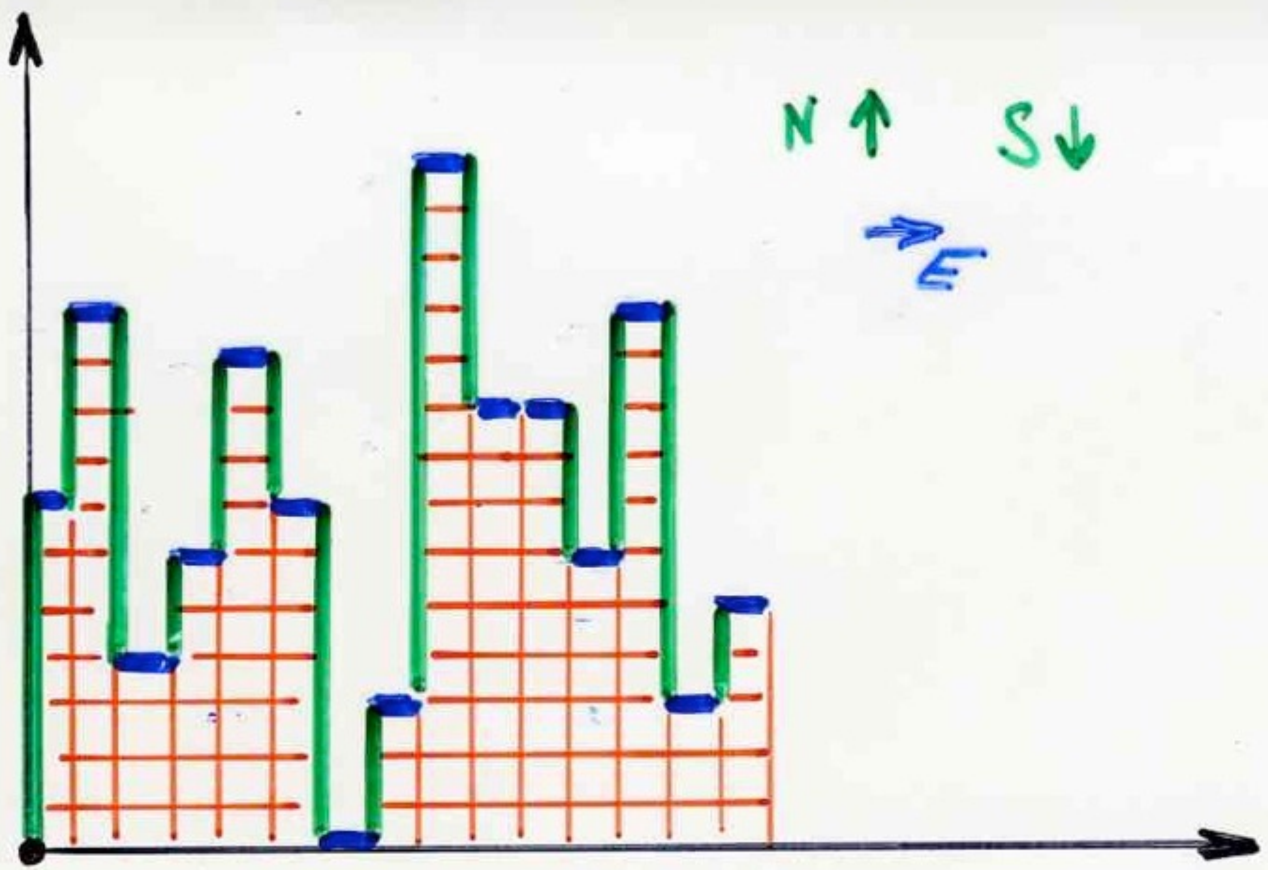
$$Z(t) = Y(q(t))$$

$$-p(-t) = y$$



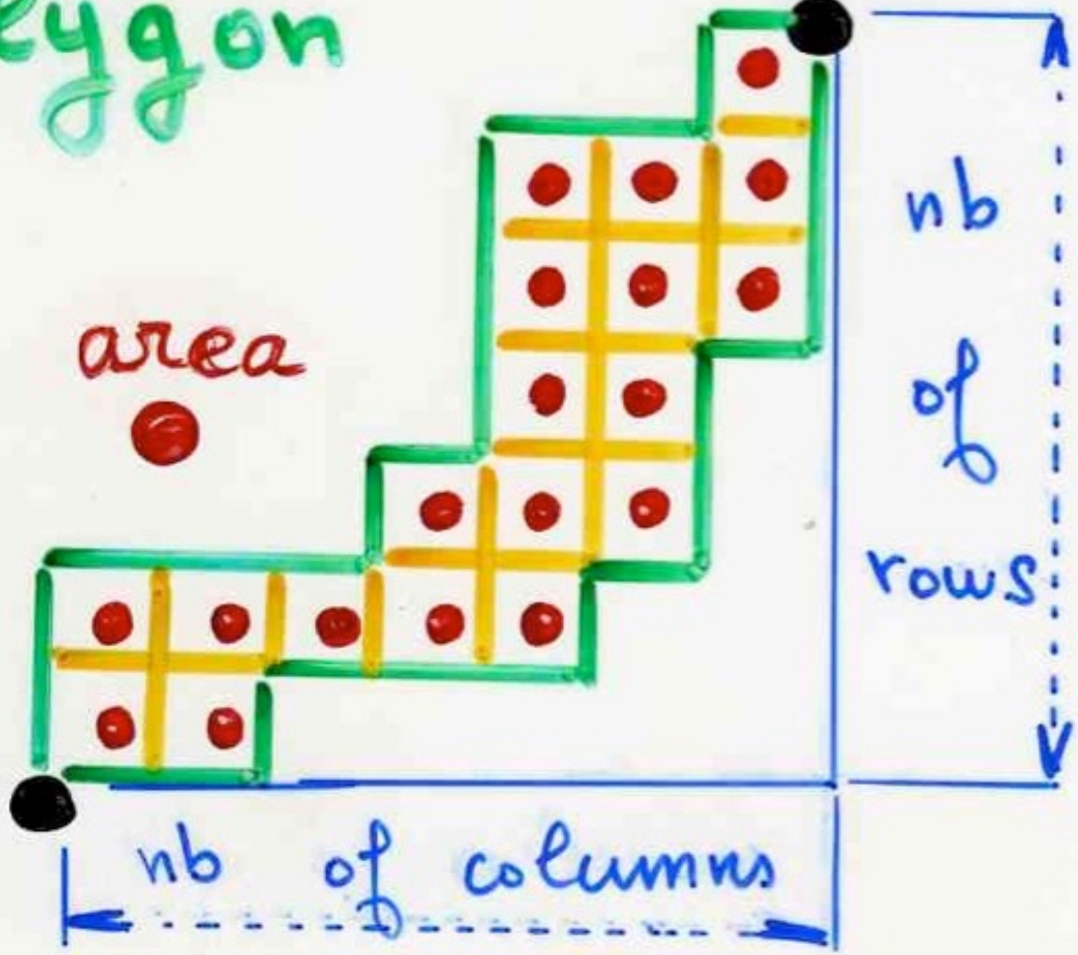
9- Bessel

$$D = \sum_{n \geq 0} \frac{(-1)^n \, t^n q^n \, q^{\binom{n}{2}}}{(1-q)\cdots(1-q^n)(1-uq)\cdots(1-uq^n)}$$



Partially directed
self-avoiding walks
(paths)

staircase polygon

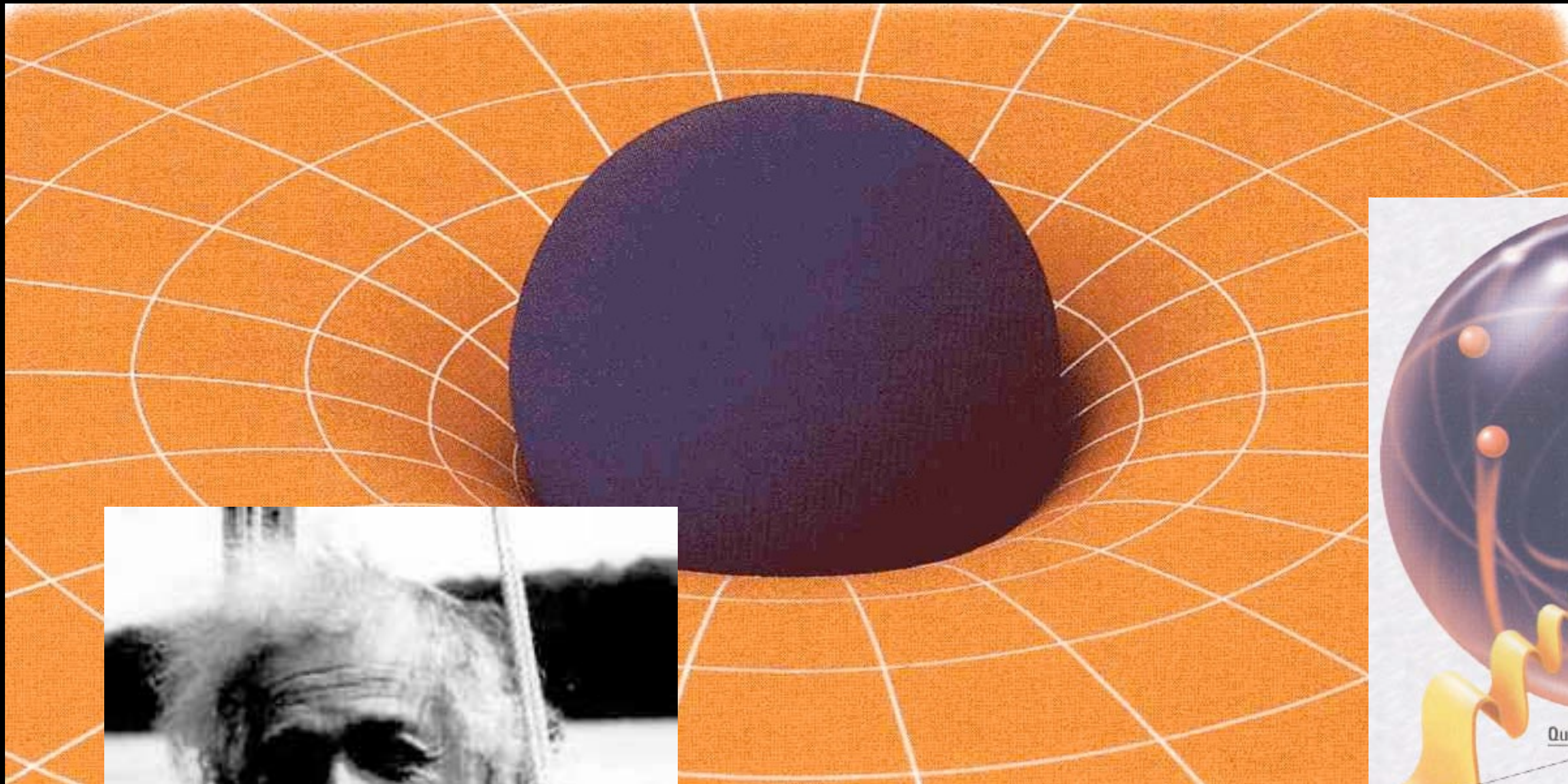


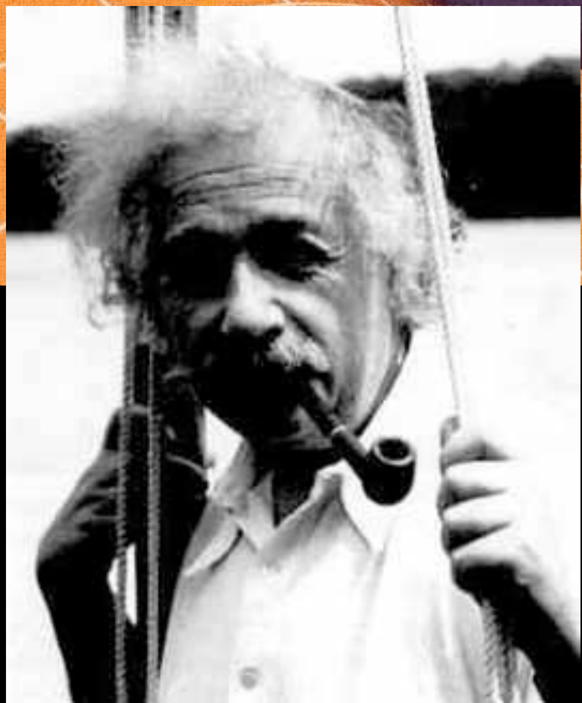
$$a_{m,n,p}$$

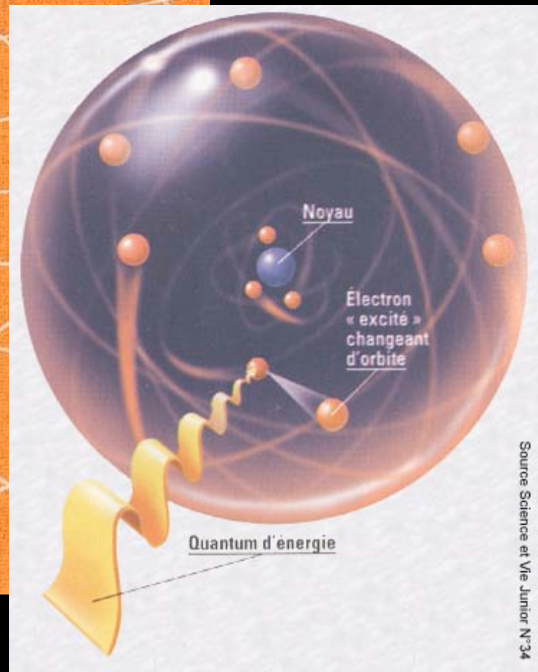
2D Lorentzian quantum gravity

relativité générale

mécanique quantique







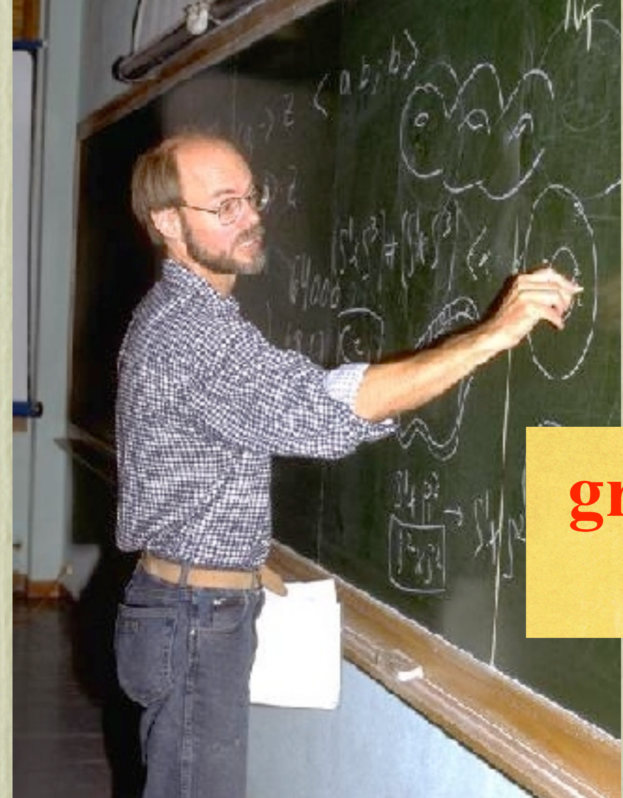
gravitation quantique

- J. Ambjørn, R. Loll, "Non-perturbative Lorentzian quantum gravity and topology change", Nucl. Phys. B536 (1998) 407-436
  arXiv: hep-th/9805108

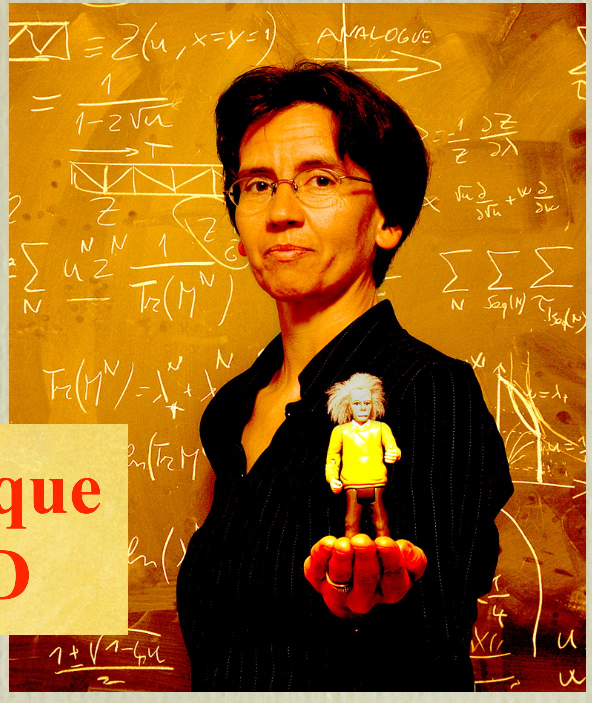
- P. Di Francesco, E. Guitter, C. Kristjansen, Integrable 2D Lorentzian gravity and random walks", Nucl. Phys. B 567 (2000) 515-553
  arXiv: hep-th/9907084

quantum gravity

gravitation quatique Lorentzienne 2D



J.Ambjørn

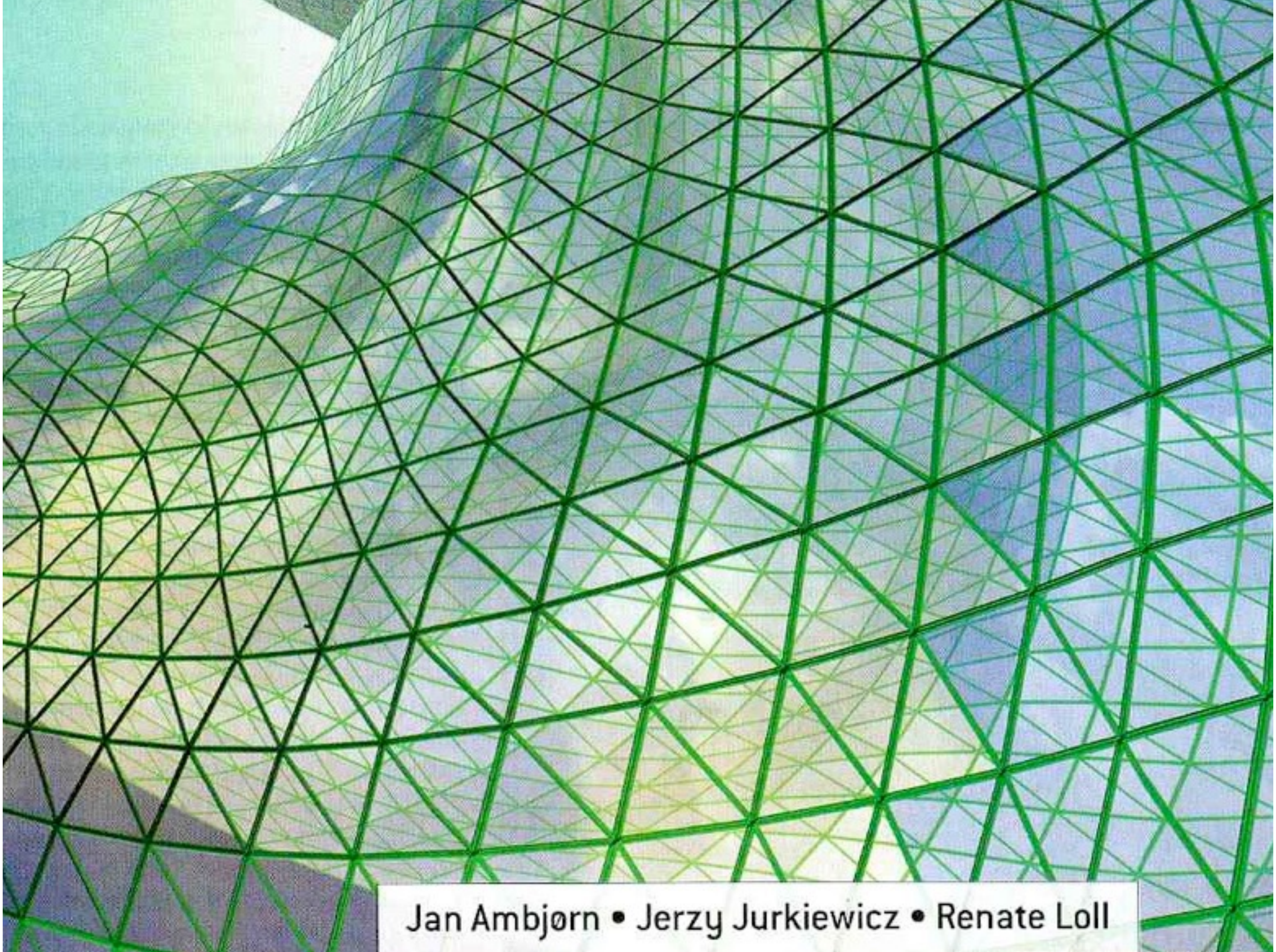


R. Loll

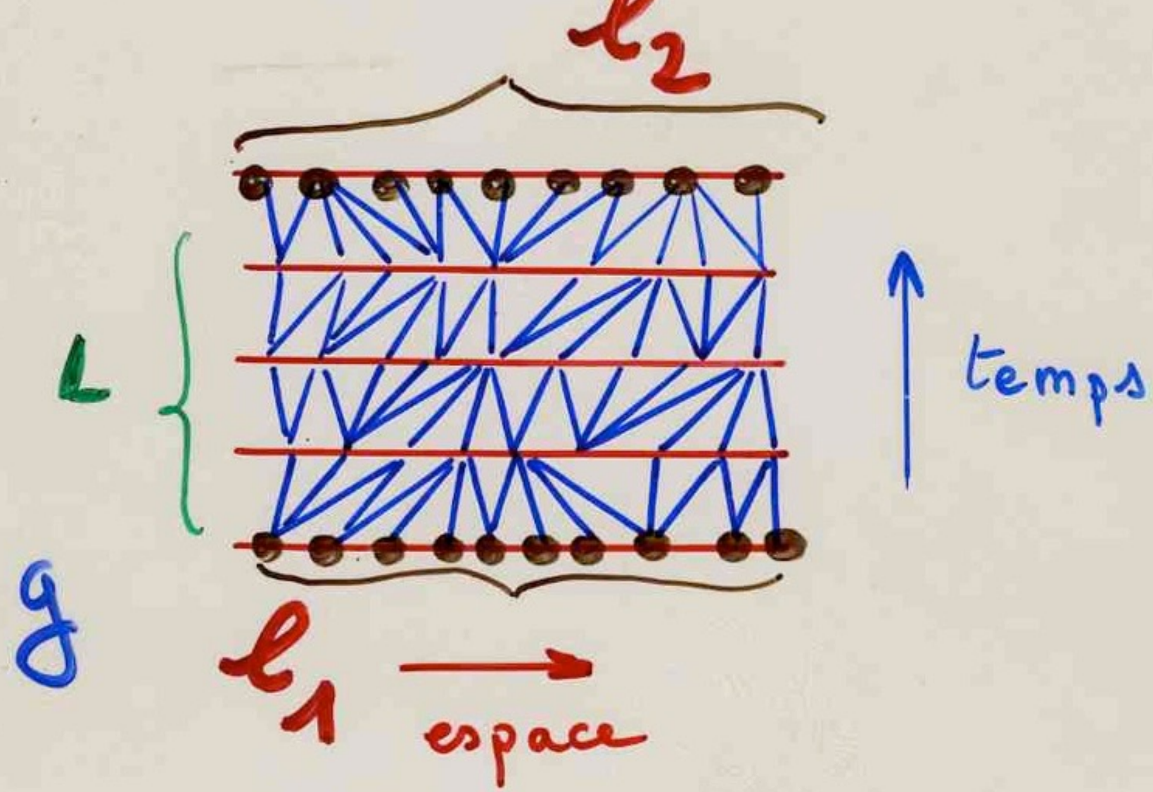
P. Di Francesco

E.Guitter

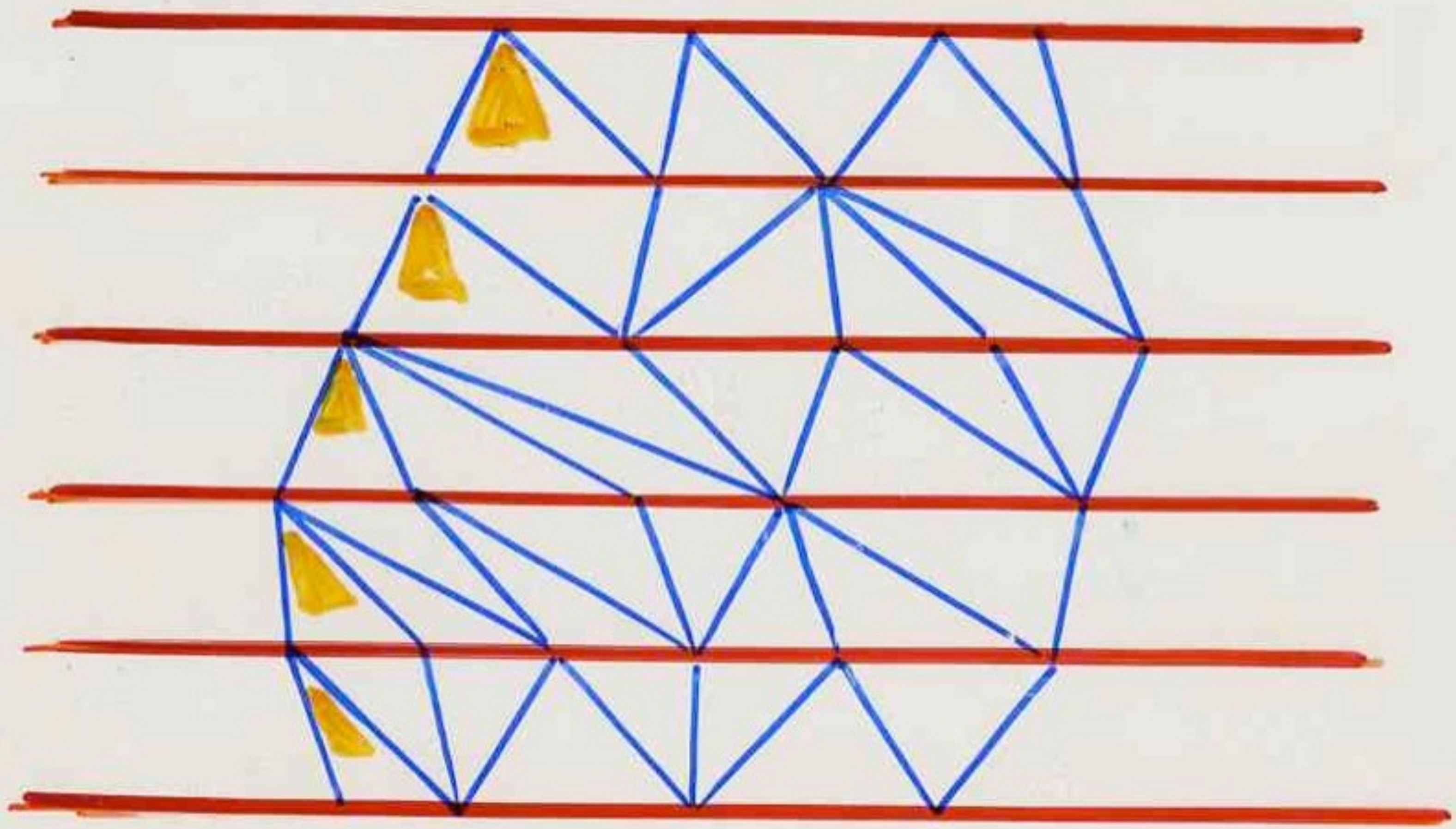
C. Kristjansen



Jan Ambjørn • Jerzy Jurkiewicz • Renate Loll



Path integral amplitude for the propagation from geometry ℓ_1 to ℓ_2



Catalan number !

$$C_n = \frac{1}{(n+1)}\binom{2n}{n}$$

representation of Lie algebras
with full heaps

R. Green

totally commutative elements
in Coxeter groups

- introduction to enumerative and bijective combinatorics

- non-crossing paths, tilings, determinants and Young tableaux. The LGV Lemma.

- introduction to the theory of heaps of pieces: the 3 basics lemma
- heaps of pieces and statistical mechanics: directed animals, gas models, q-Bessel functions in physics
- heaps of pieces and 2D Lorentzian quantum gravity