Combinatorics and Physics

Chapter 0
Introduction
Overview of the course
(part 2)

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exponential generating function

une suite de nombres célèbres ....

$$1t + 2\frac{t^3}{3!} + 16\frac{t^5}{5!} + 272\frac{t^7}{7!} + \ldots$$

$$y = tg\ t$$

tangente

$$y = \operatorname{tg} t$$

tangente

$$y = \sum_{n \geq 0} a_{2n+1} \frac{t^{2n+1}}{(2n+1)!}$$

$$y' = 1 + y^2 ; \quad y(0) = 0$$

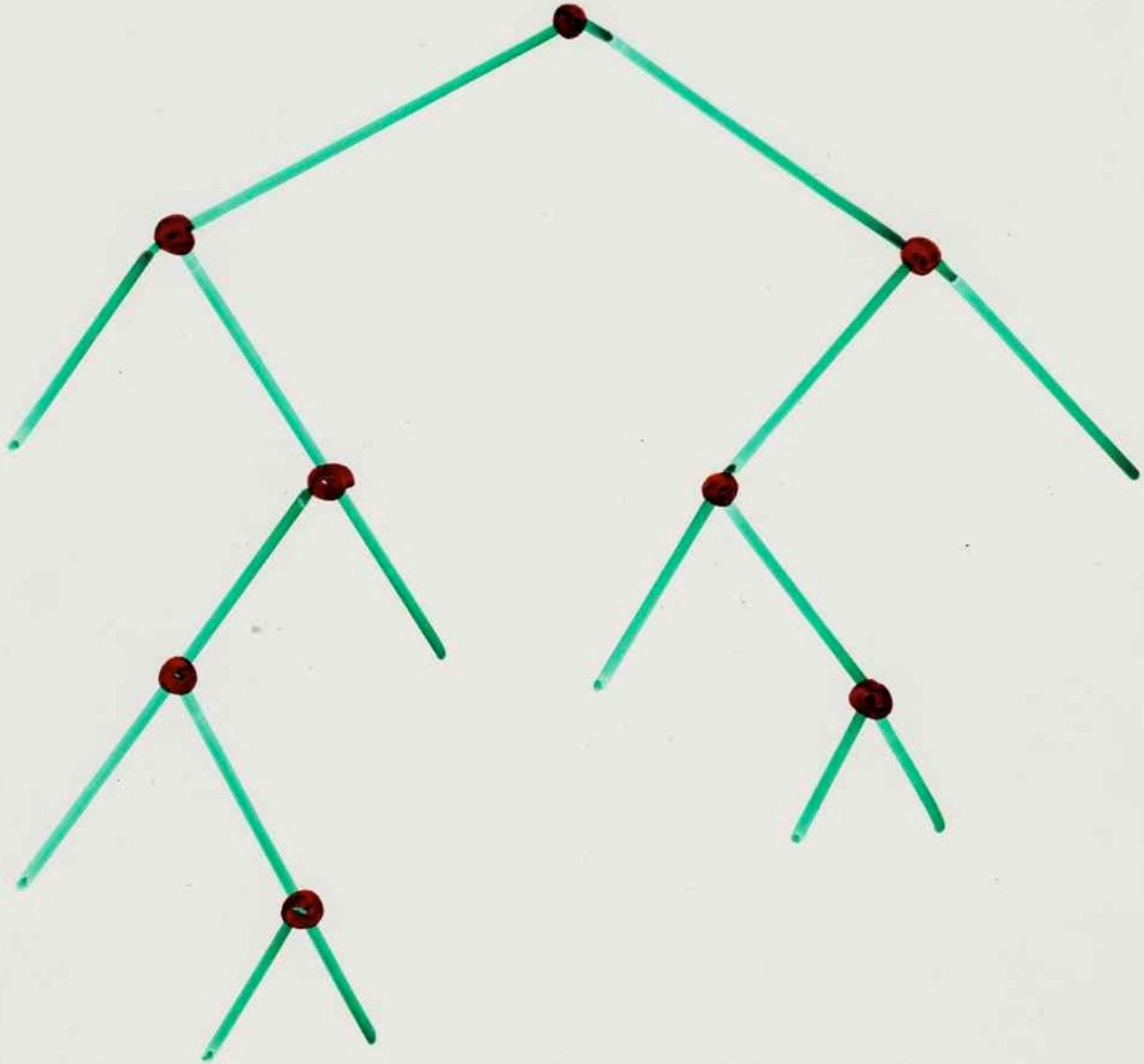
6 \ 2 / 9 7 / 8 \ 4 / 5 \ 1 / 3

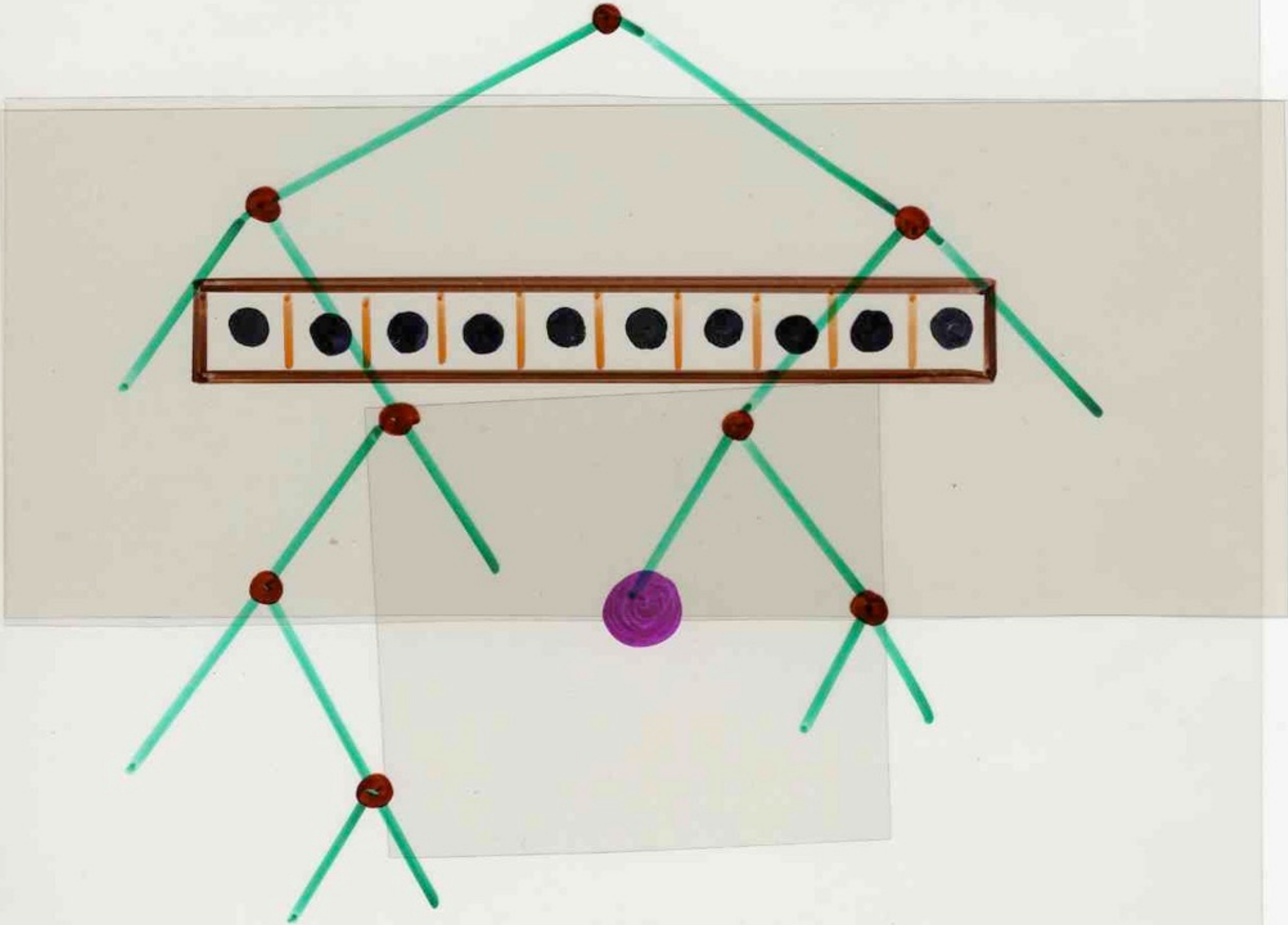
alternating permutations

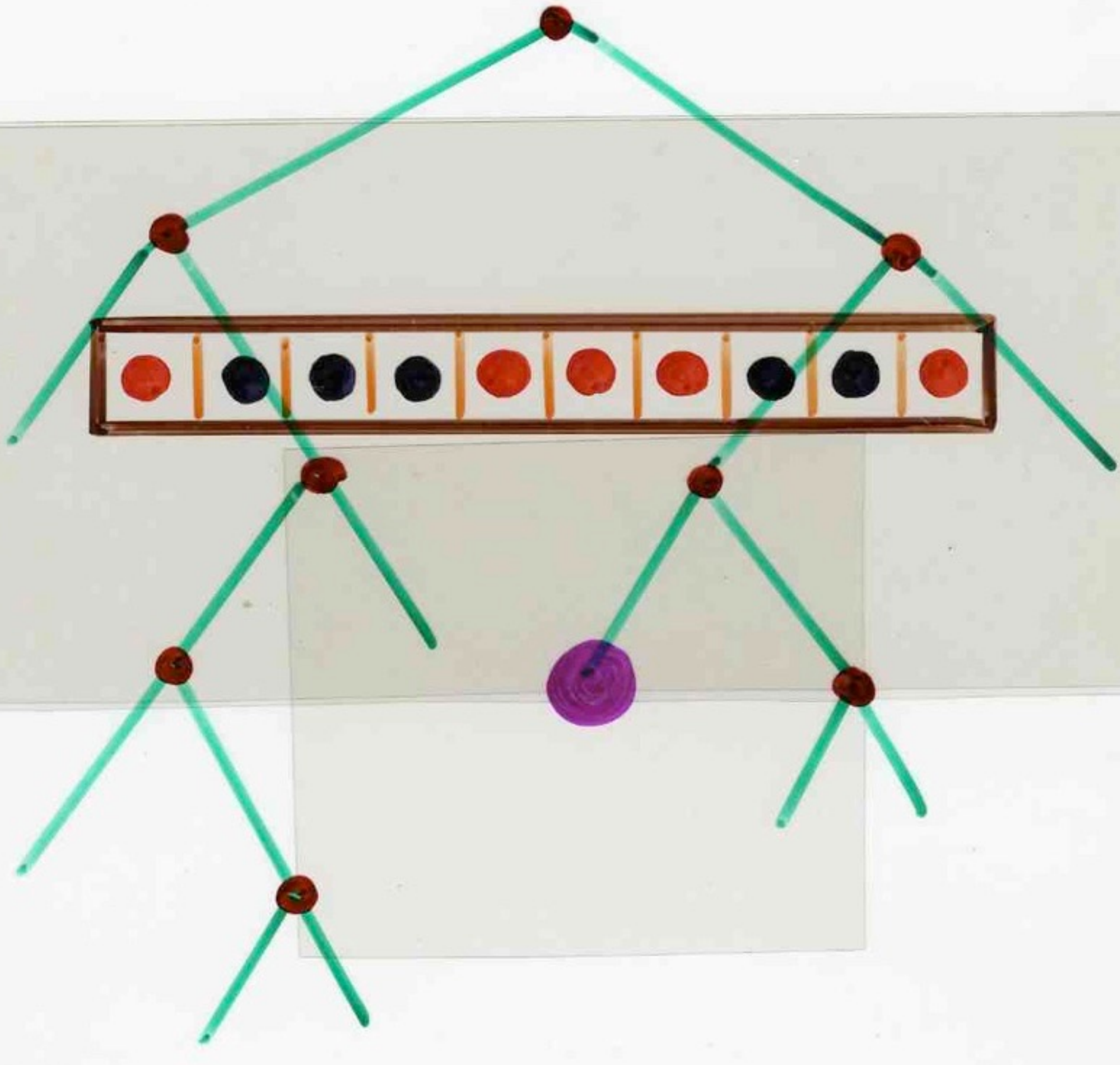
bijective combinatorics

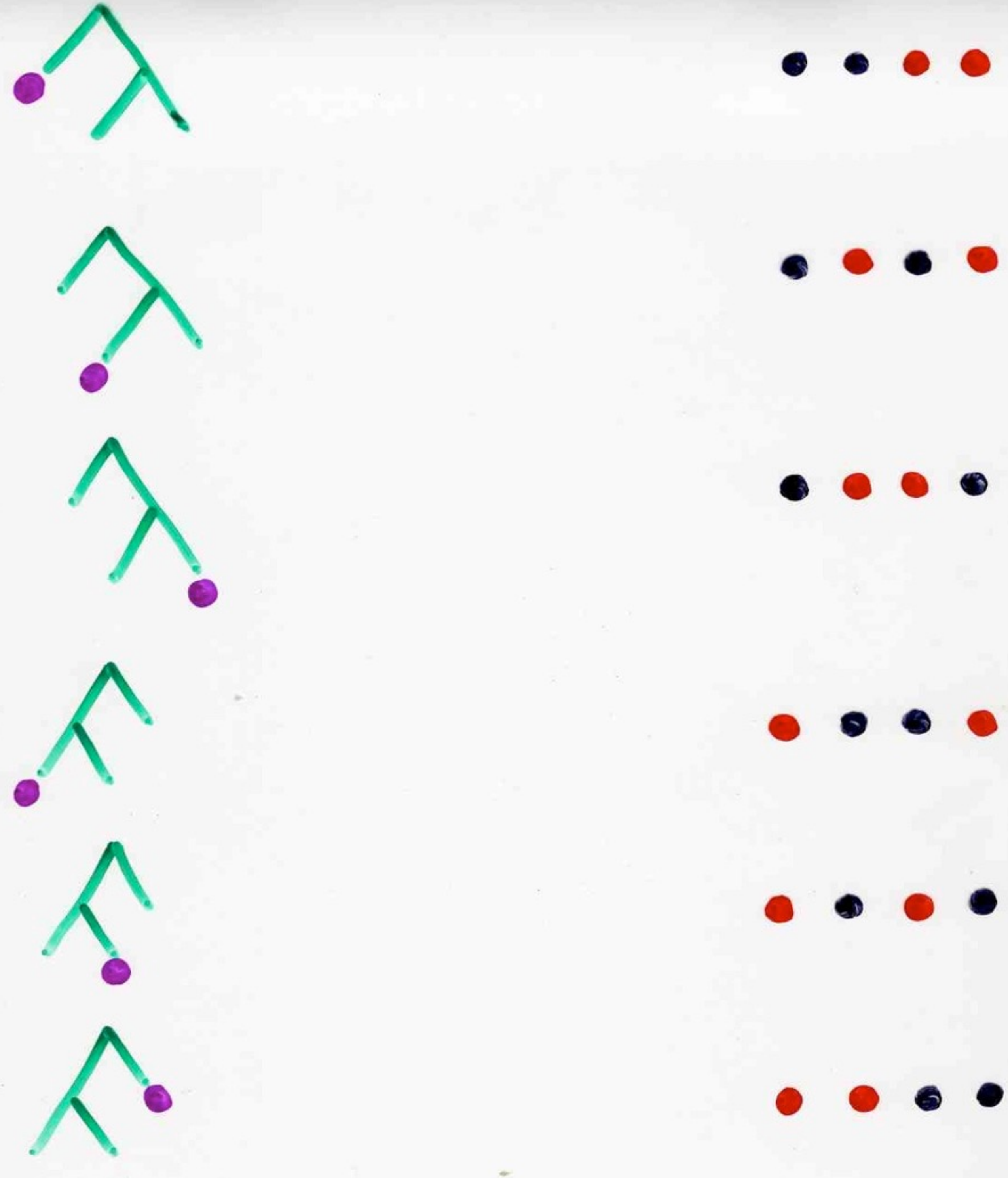
$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

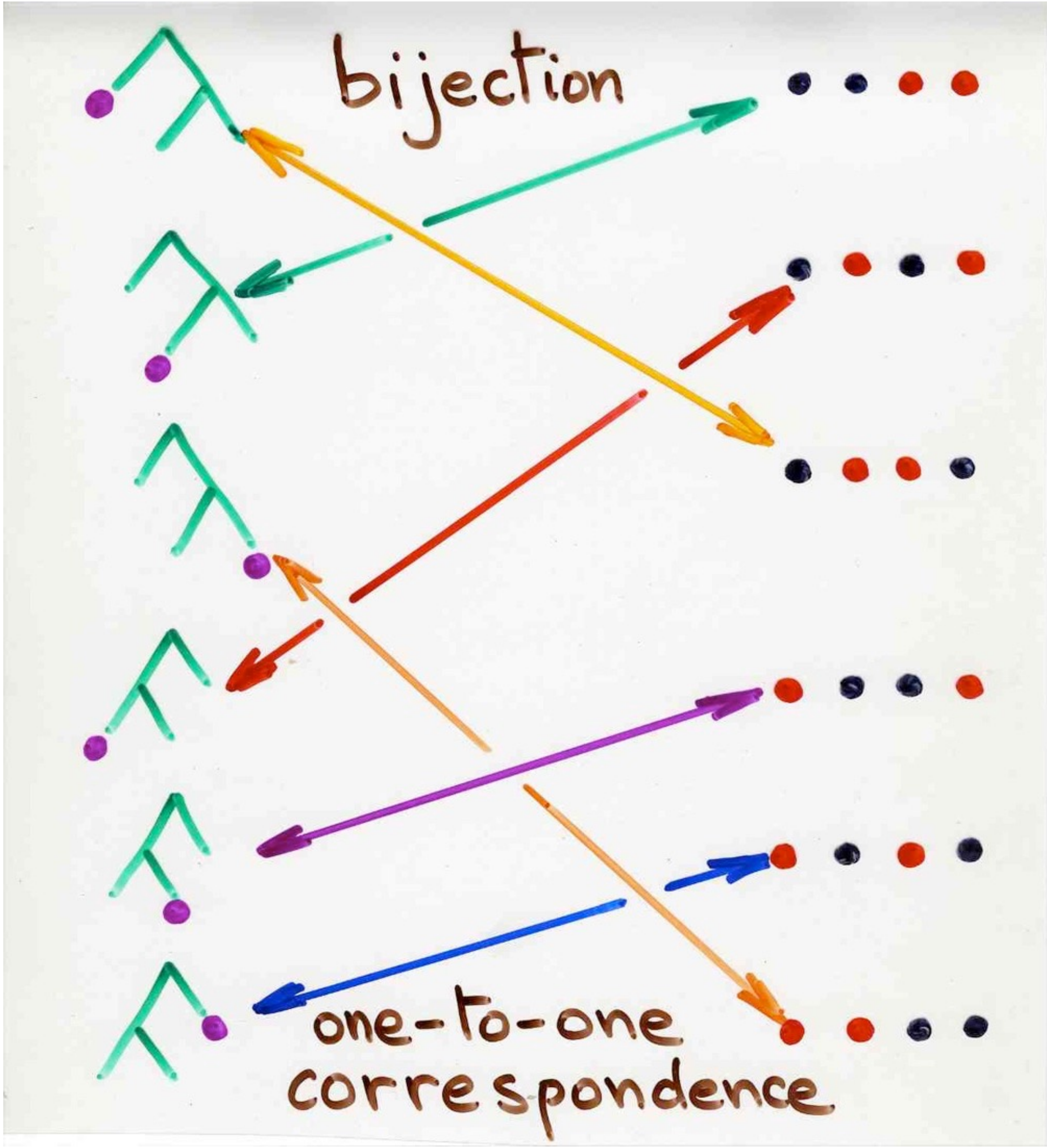
$$(n+1)\,C_n = \binom{2n}{n}$$



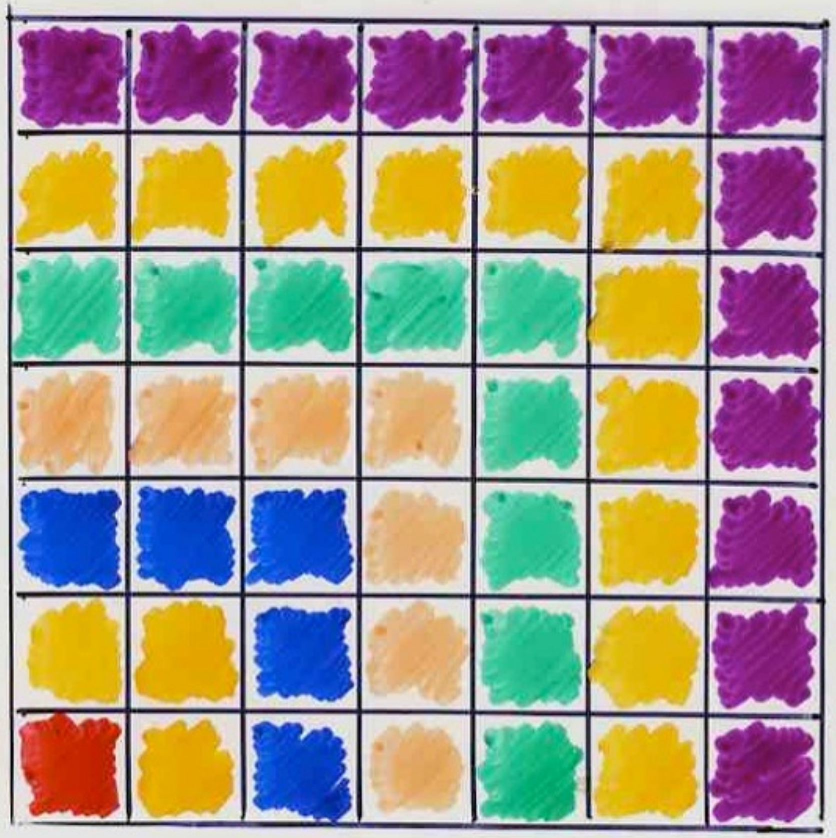




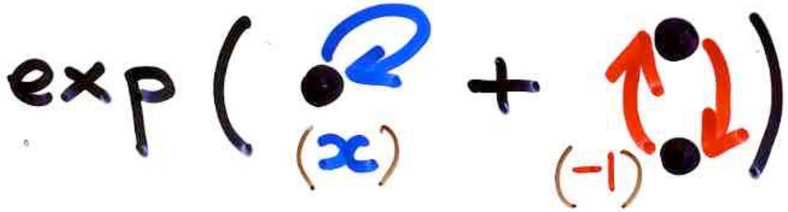



bijection
one-to-one
correspondence

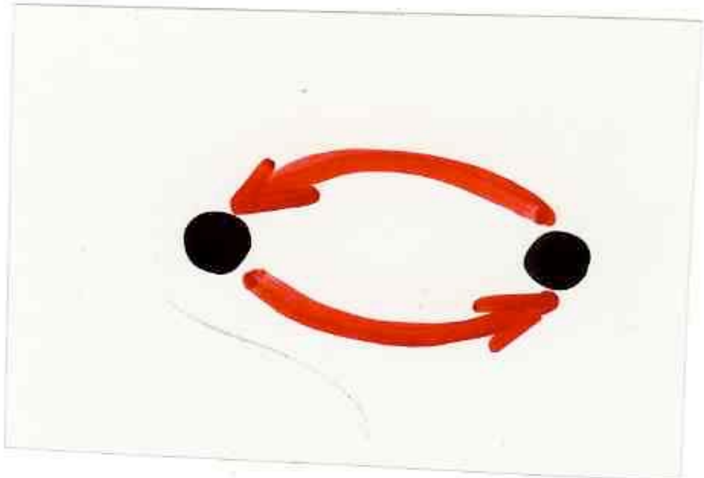
“The bijective paradigm”



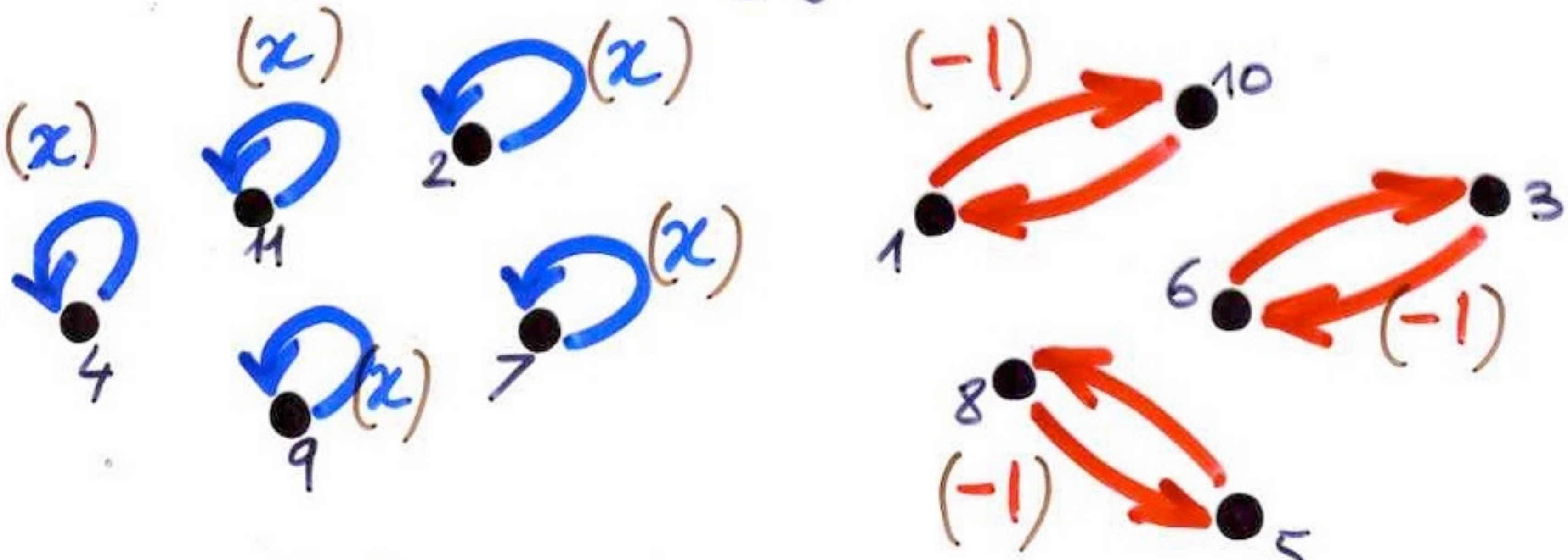
$$n^2 = 1 + 3 + \ldots + (2n-1)$$



$$\sum_{n\geqslant 0} H_n(x)\frac{t^n}{n!} = \exp\left(x t - \frac{t^2}{2}\right)$$



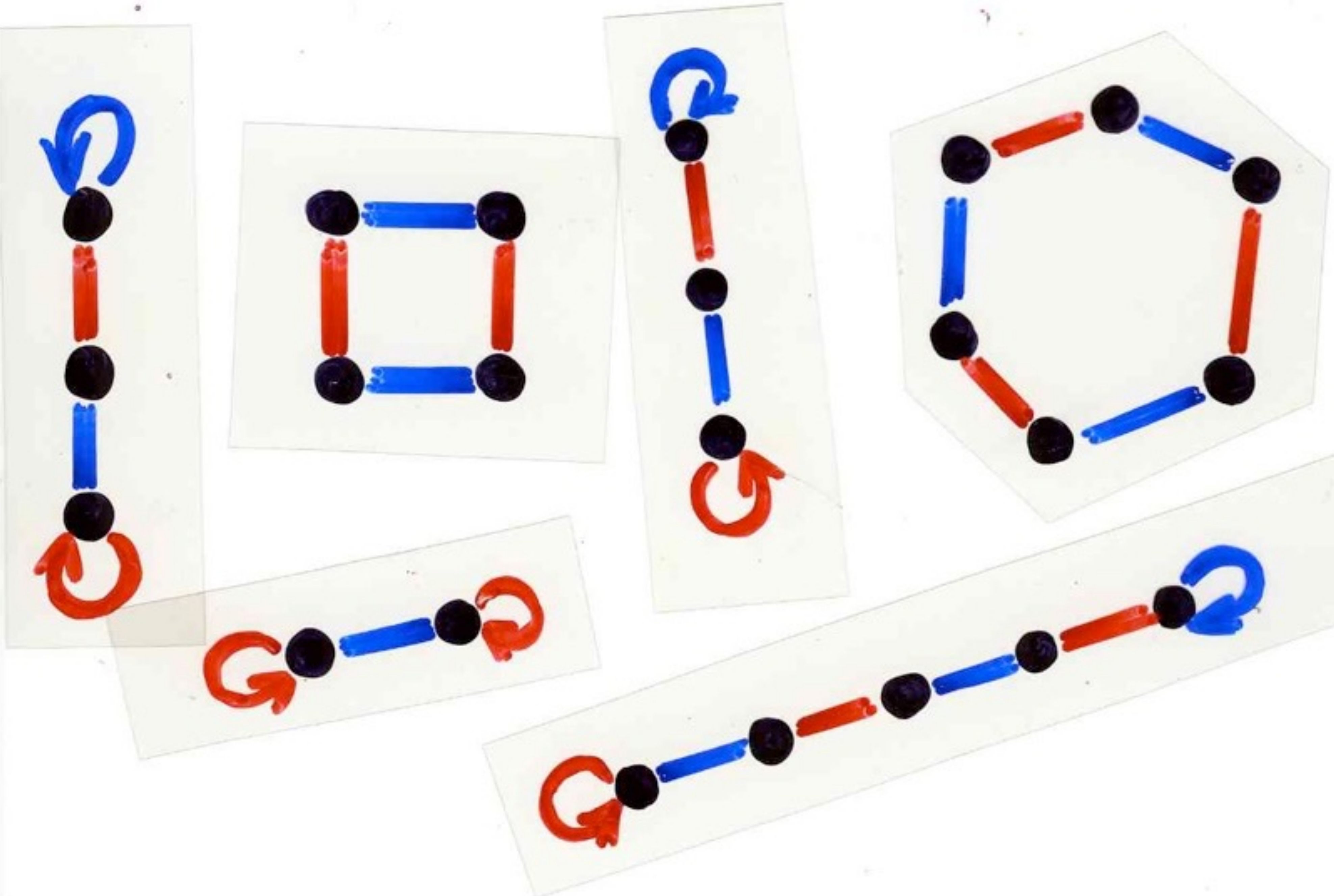
Hermite configurations



weight (x)

(-1)

$$\sum_{n \geqslant 0} H_n(x) H(y) \frac{t^n}{n!} = (1-4t^2)^{-1/2} \exp\left[\frac{4xyt - 4(x^2+y^2)t^2}{1-4t^2}\right]$$



drawing calculus

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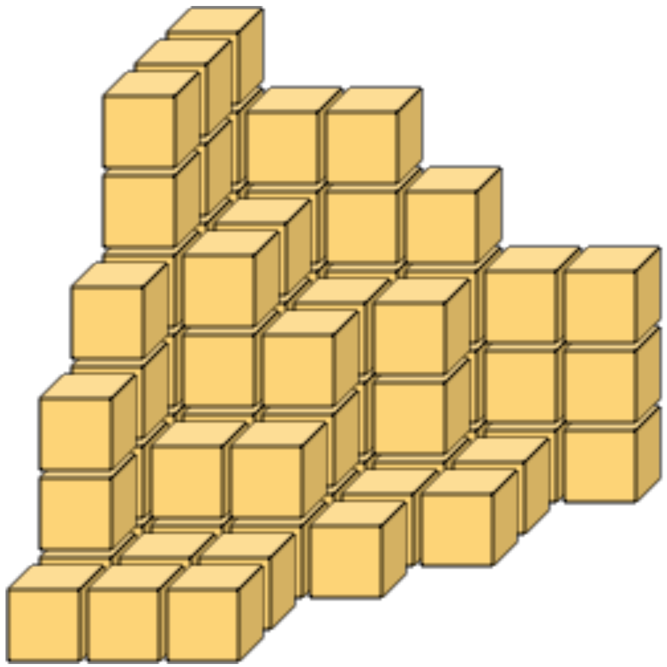
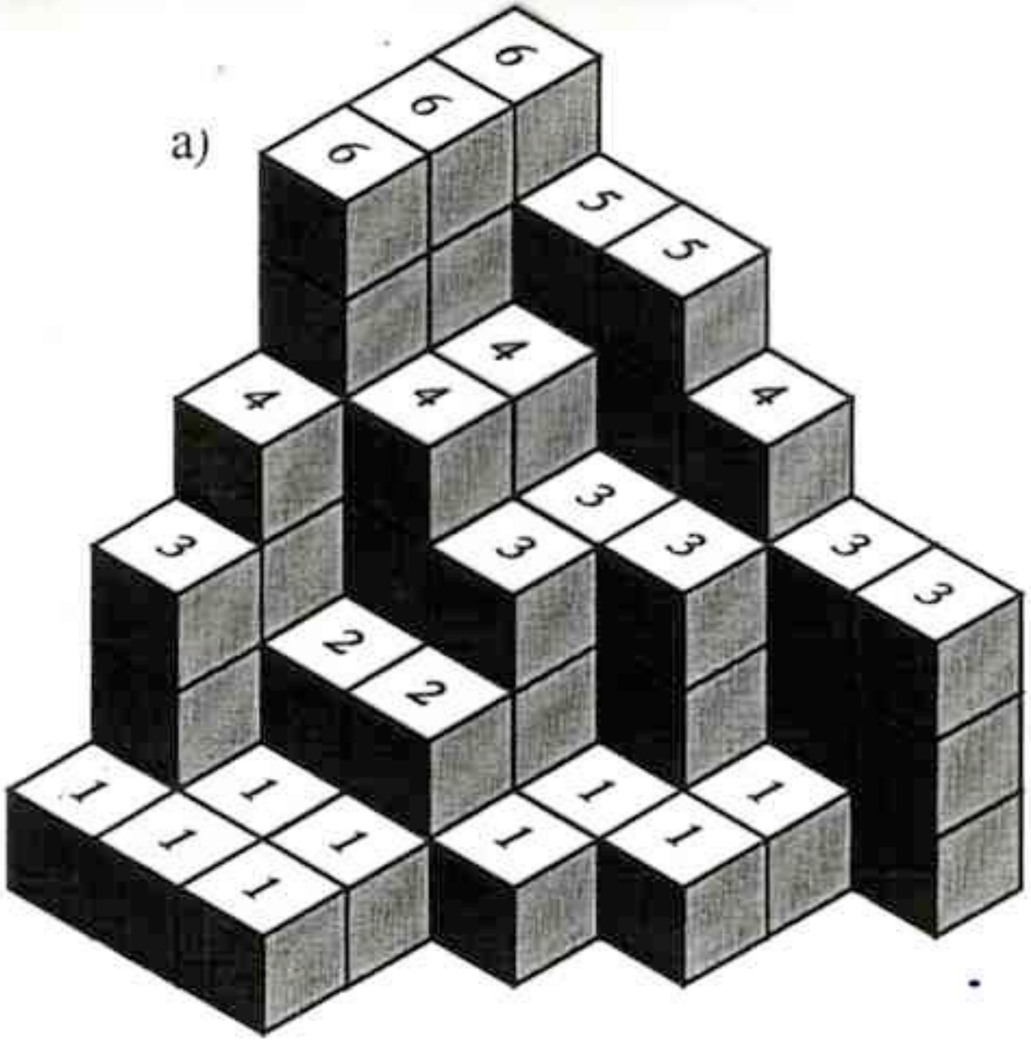
computing drawings

better understanding

plane partitions

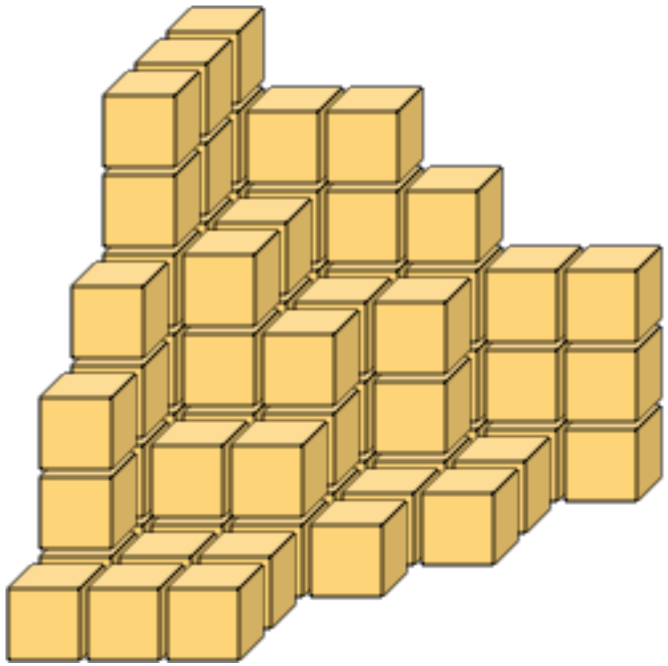


a)



b)

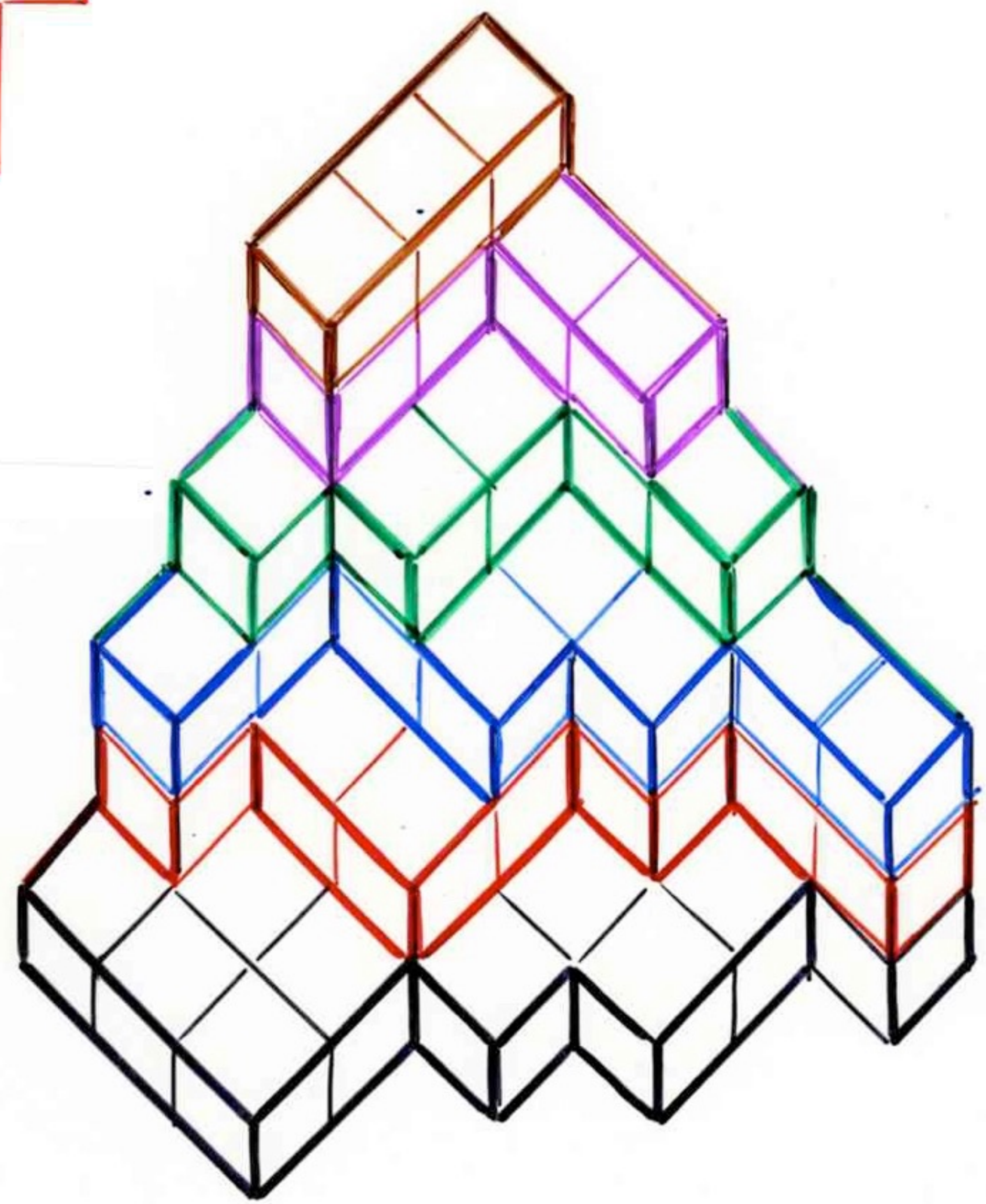
6	5	5	4	3	3
6	4	3	3	1	
6	4	3	1	1	
4	2	2	1		
3	1	1			
1	1	1			

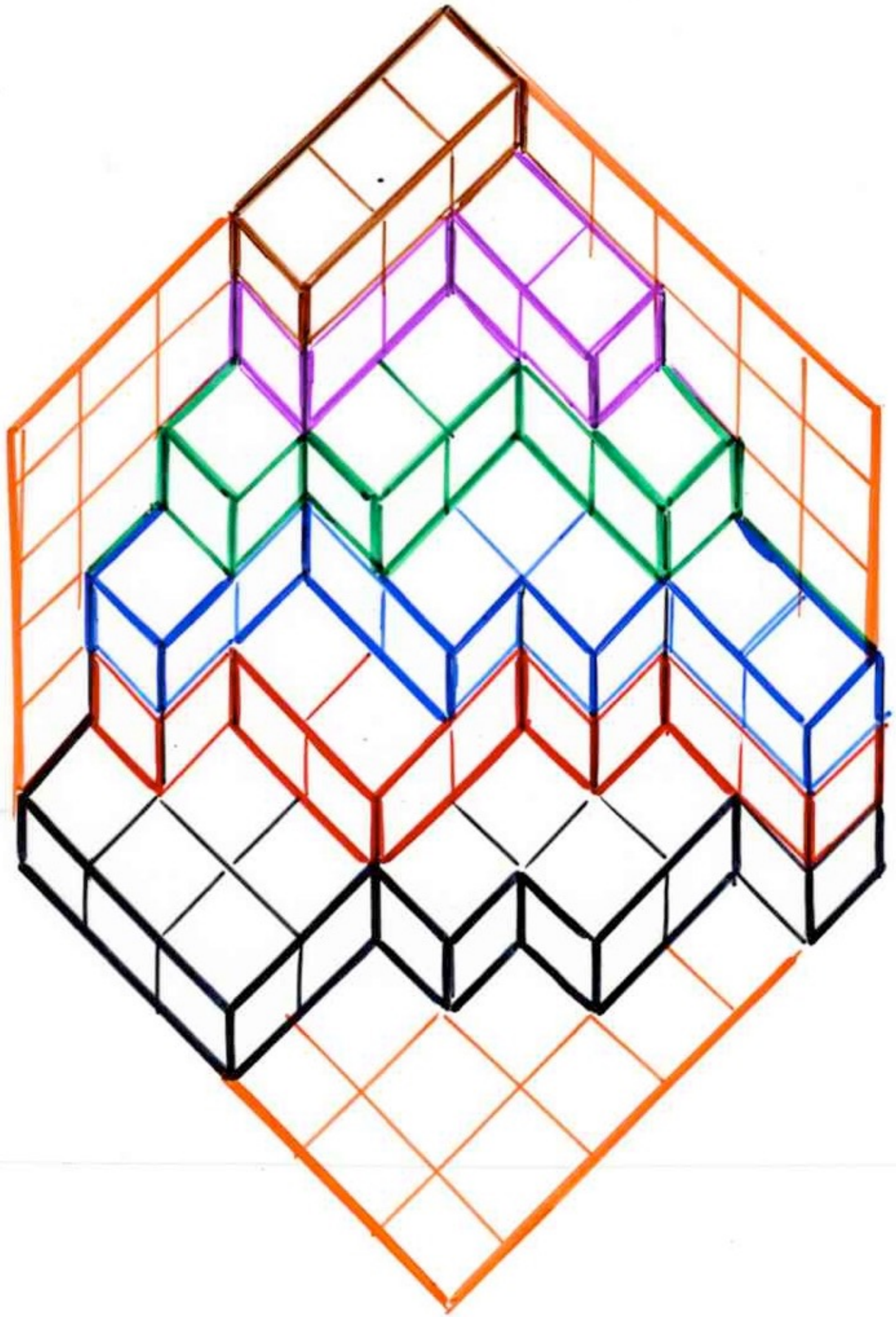


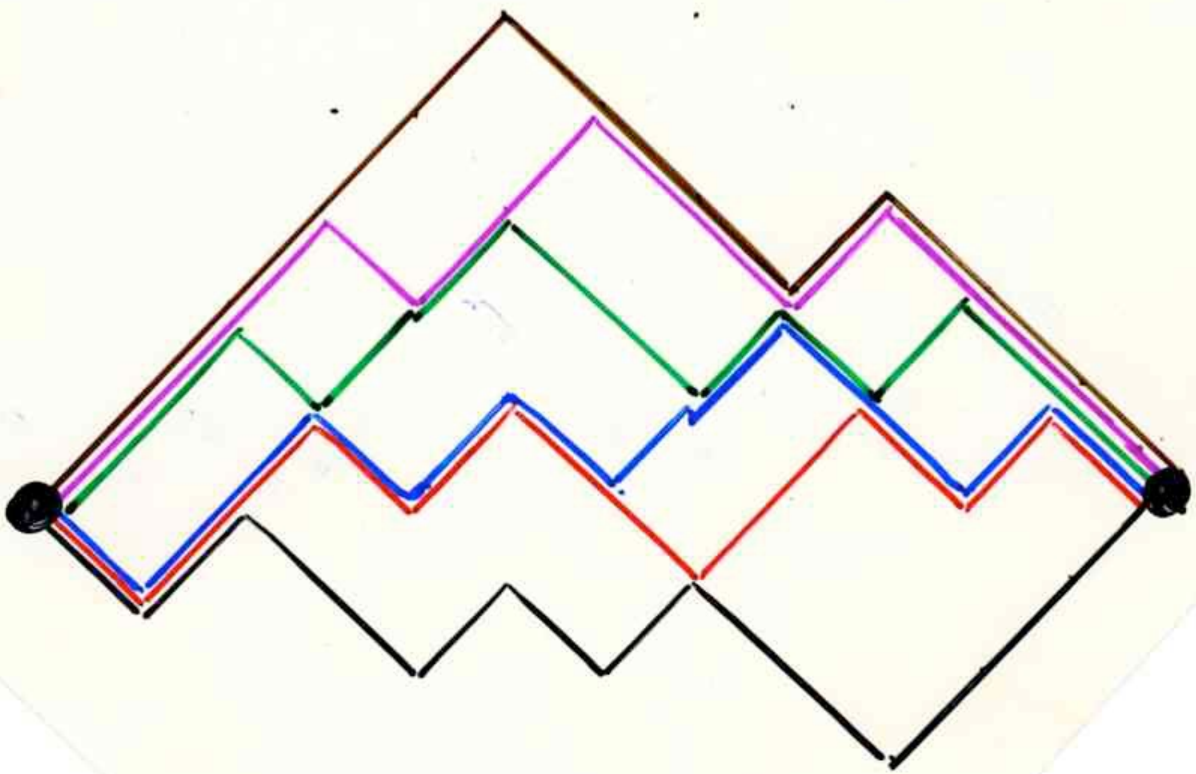
6	5	5	4	3	3
6	4	3	3	1	
6	4	3	1	1	
4	2	2	1		
3	1	1			
1	1	1			

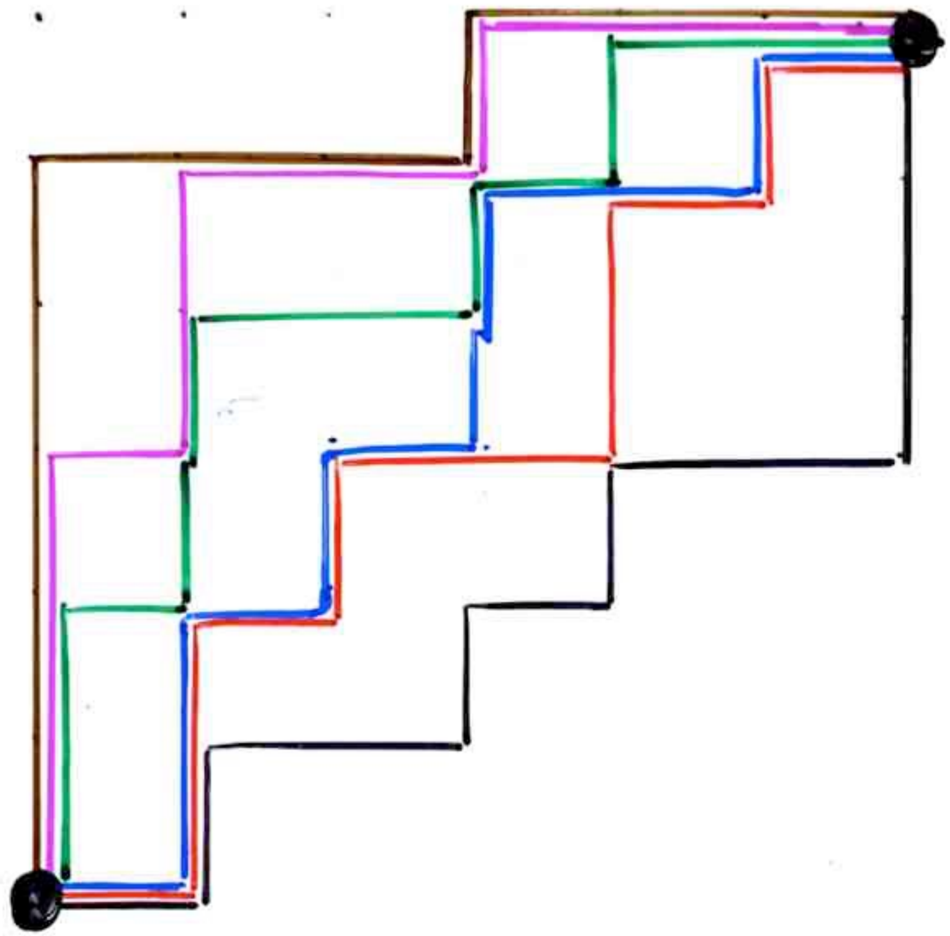
$$\prod_{\substack{1 \leq i \leq a \\ 1 \leq j \leq b \\ 1 \leq k \leq c}} \frac{i+j+k-1}{i+j+k-2}$$

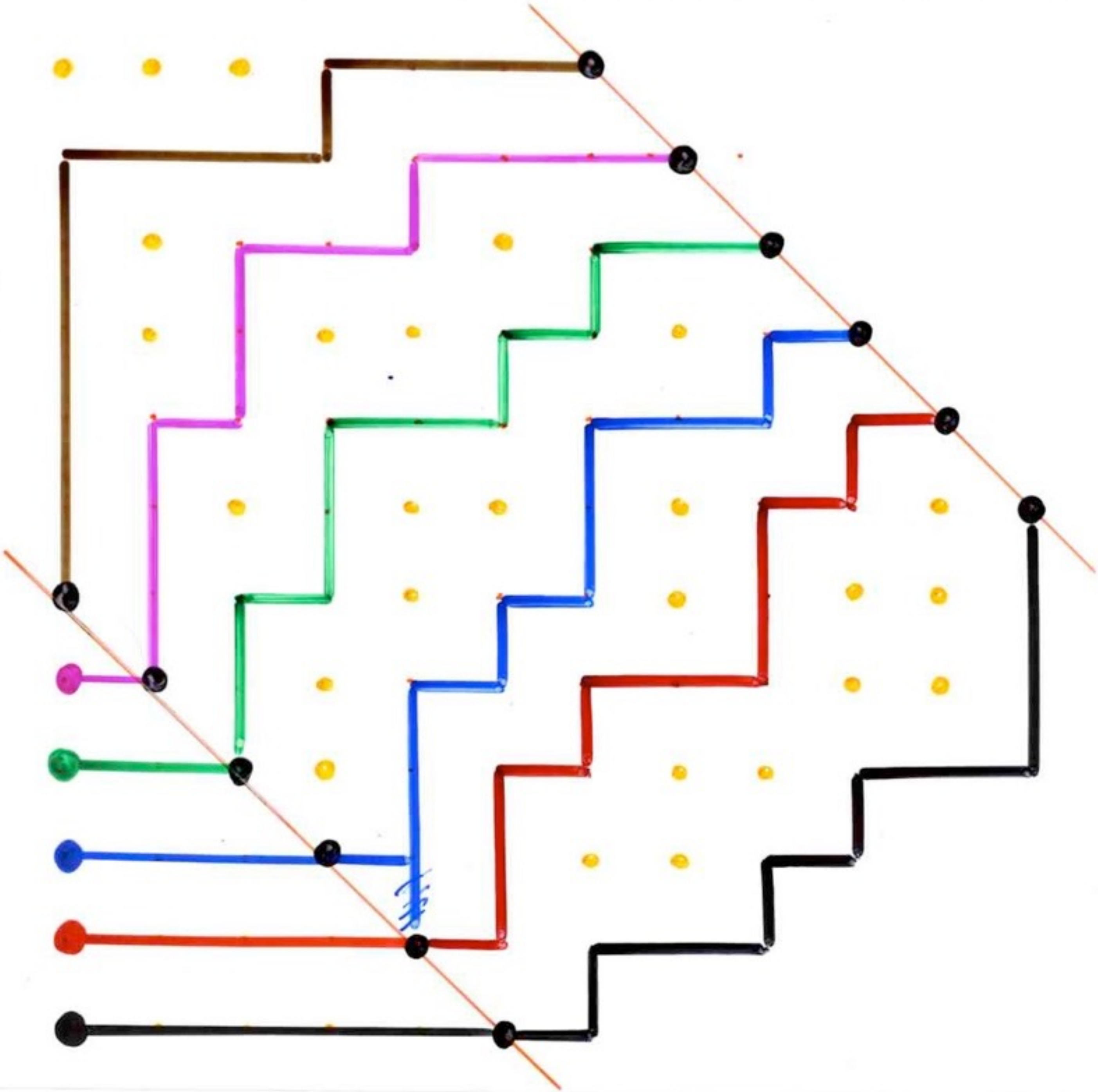
6	5	5	4	3	3
6	4	3	3	1	
6	4	3	1	1	
4	2	2	1		
3	1	1			
1	1	1			











The LGV Lemma

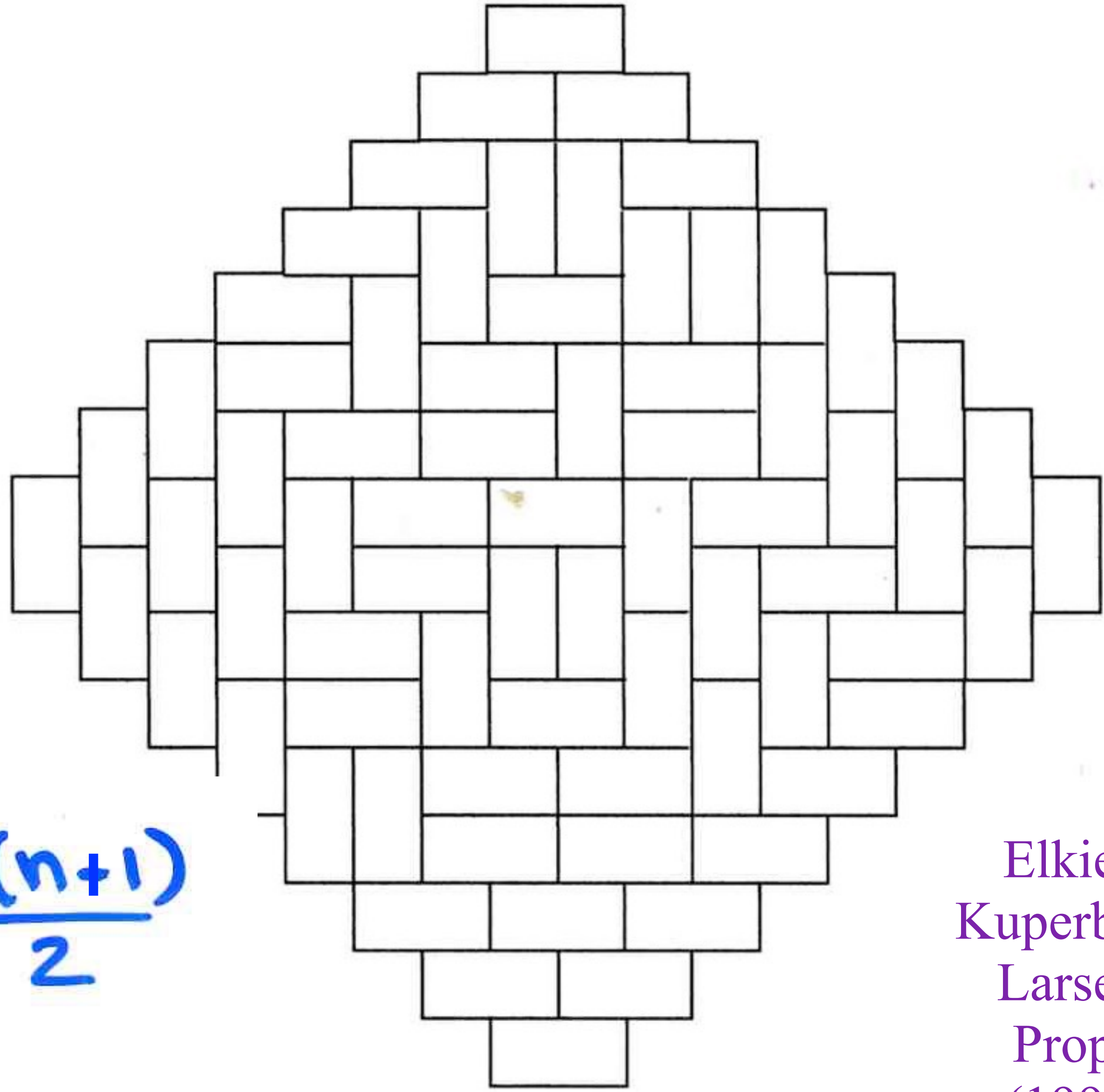
non-crossing paths

≈

determinants

Aztec tilings

$2^{(1+2+3+4+....+n)}$



$2^{\frac{n(n+1)}{2}}$

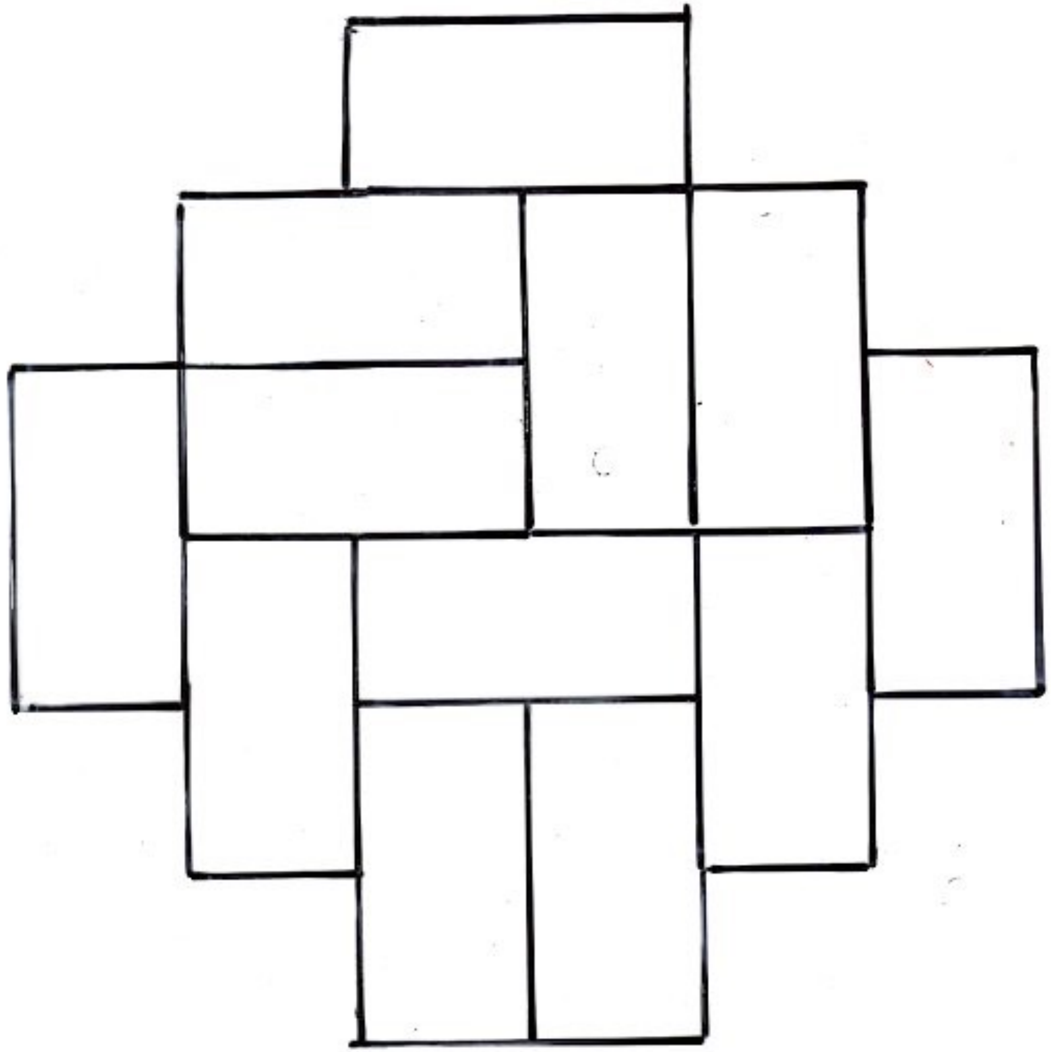
Elkies, Kuperberg, Larsen, Propp (1992)

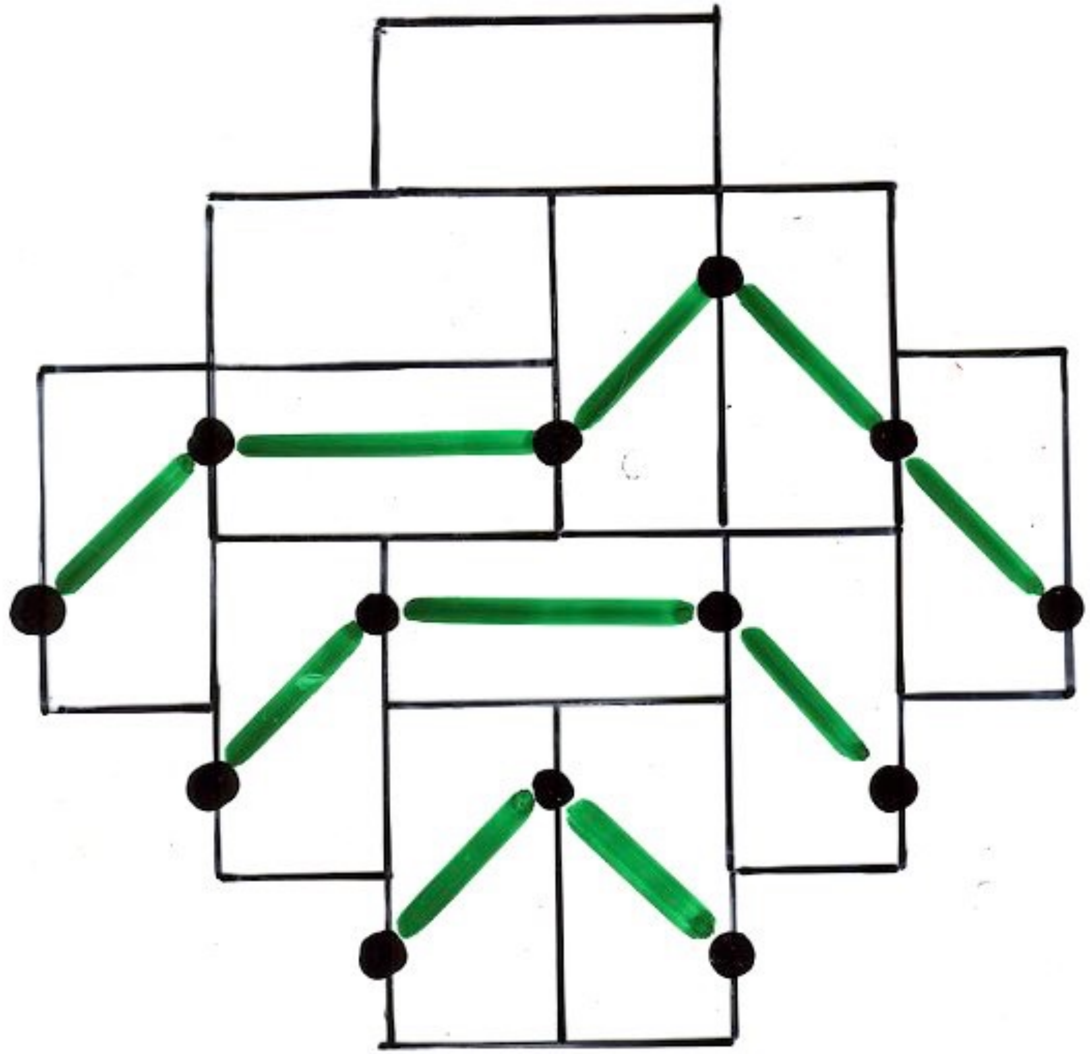
bijection

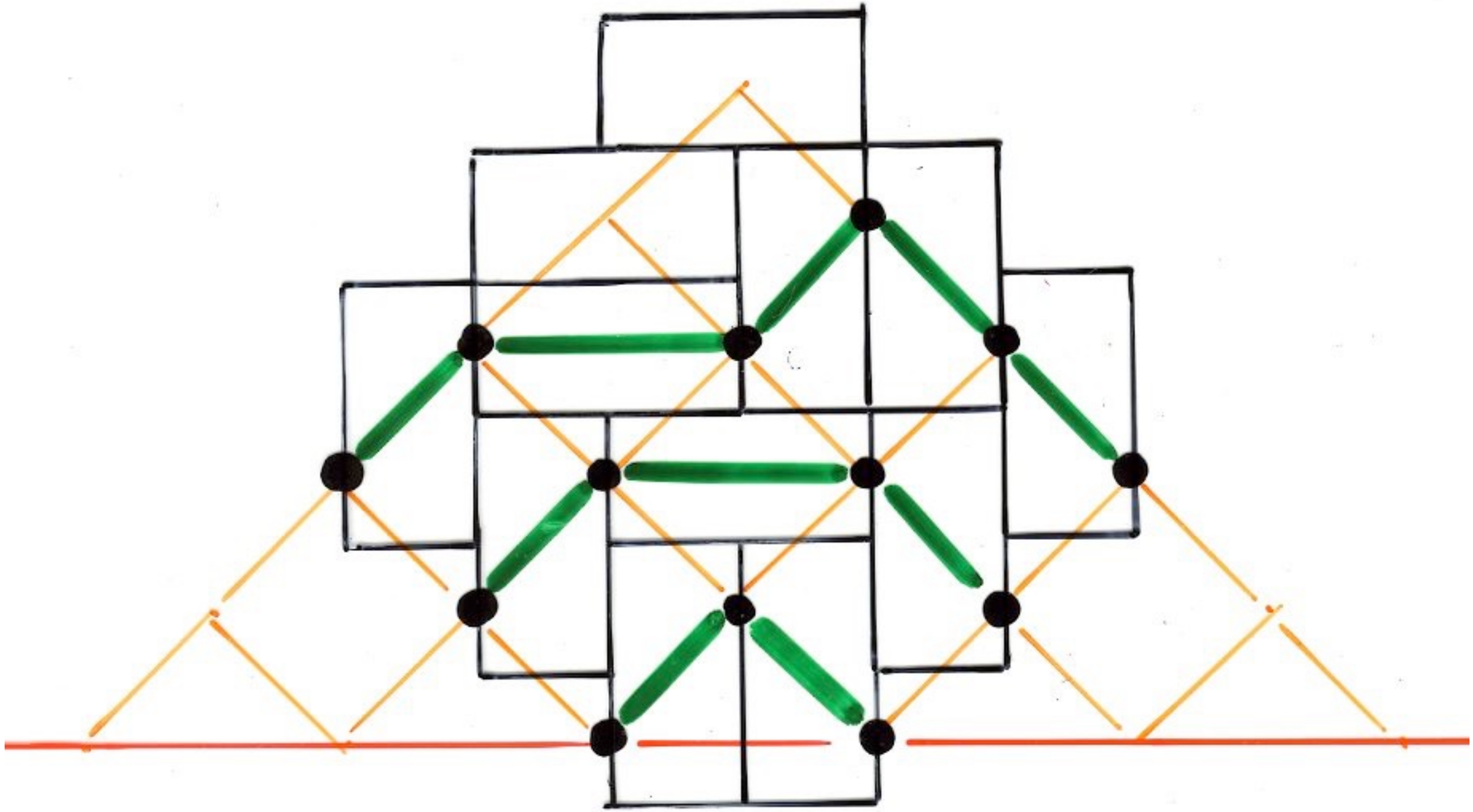
tiling Aztec tilings

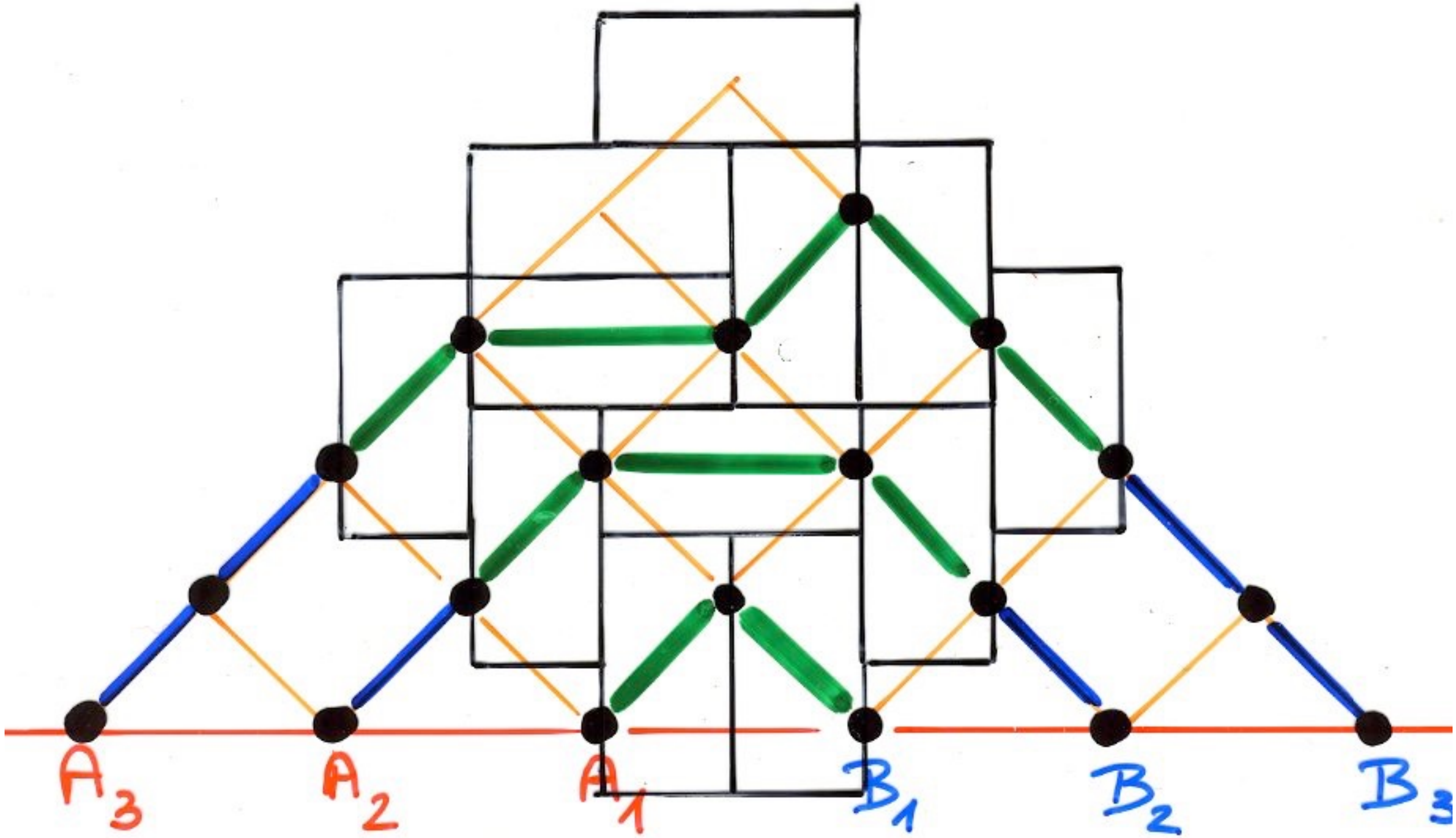
$\updownarrow$

configurations
of non-crossing paths



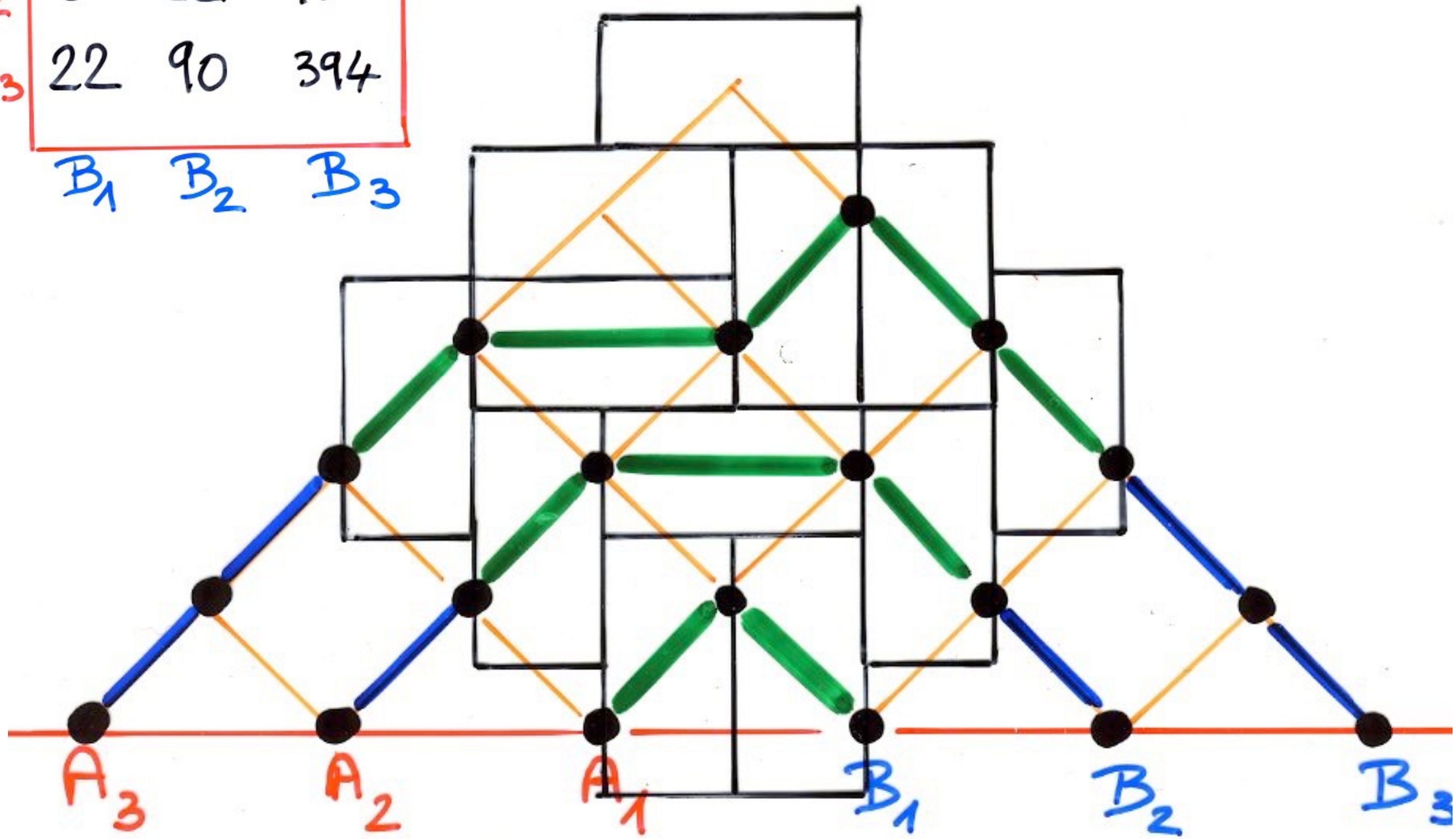






A3
A2
A1
B1
B2
B3

	B_1	B_2	B_3
A_1	2	6	22
A_2	6	22	90
A_3	22	90	394



- introduction to enumerative and bijective combinatorics
- non-crossing paths, tilings, determinants and Young tableaux. The LGV Lemma.