Combinatorics and Physics

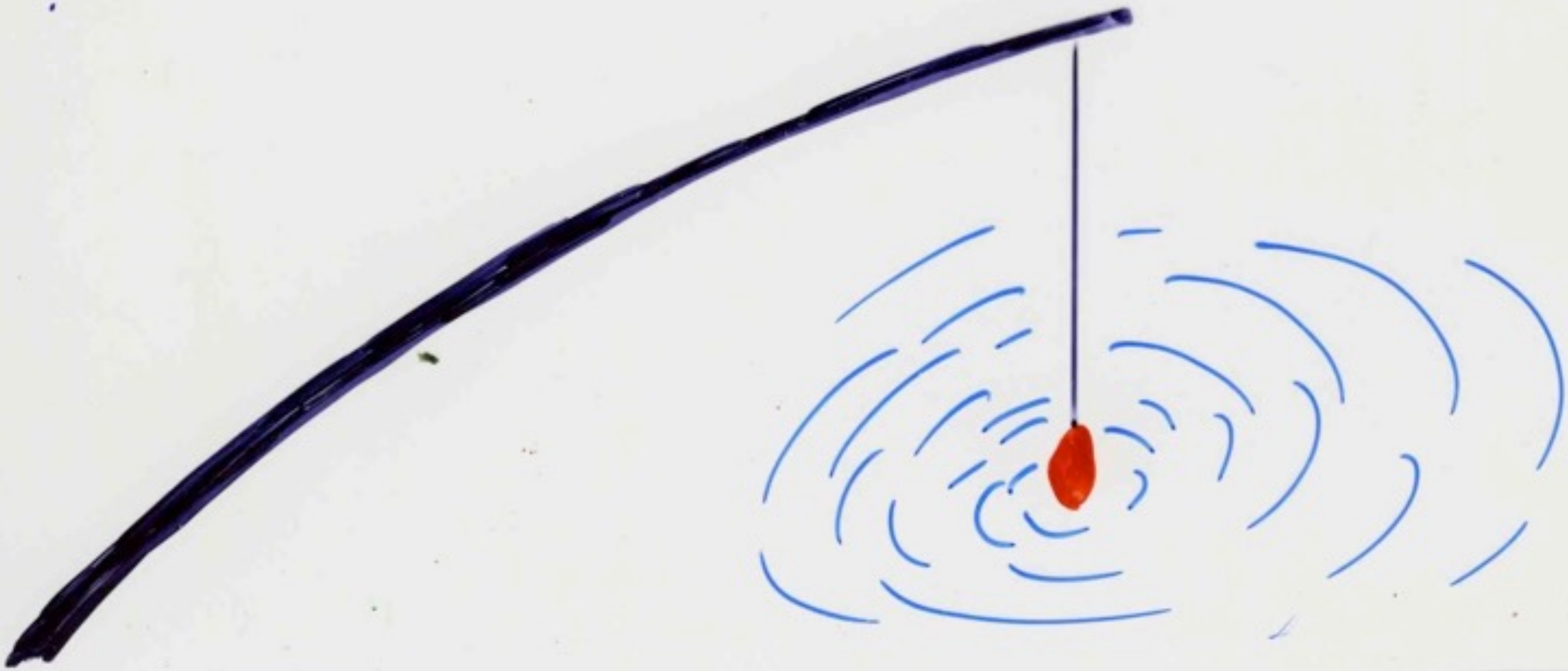
Chapter 0
Introduction
Overview of the course
(part 5)

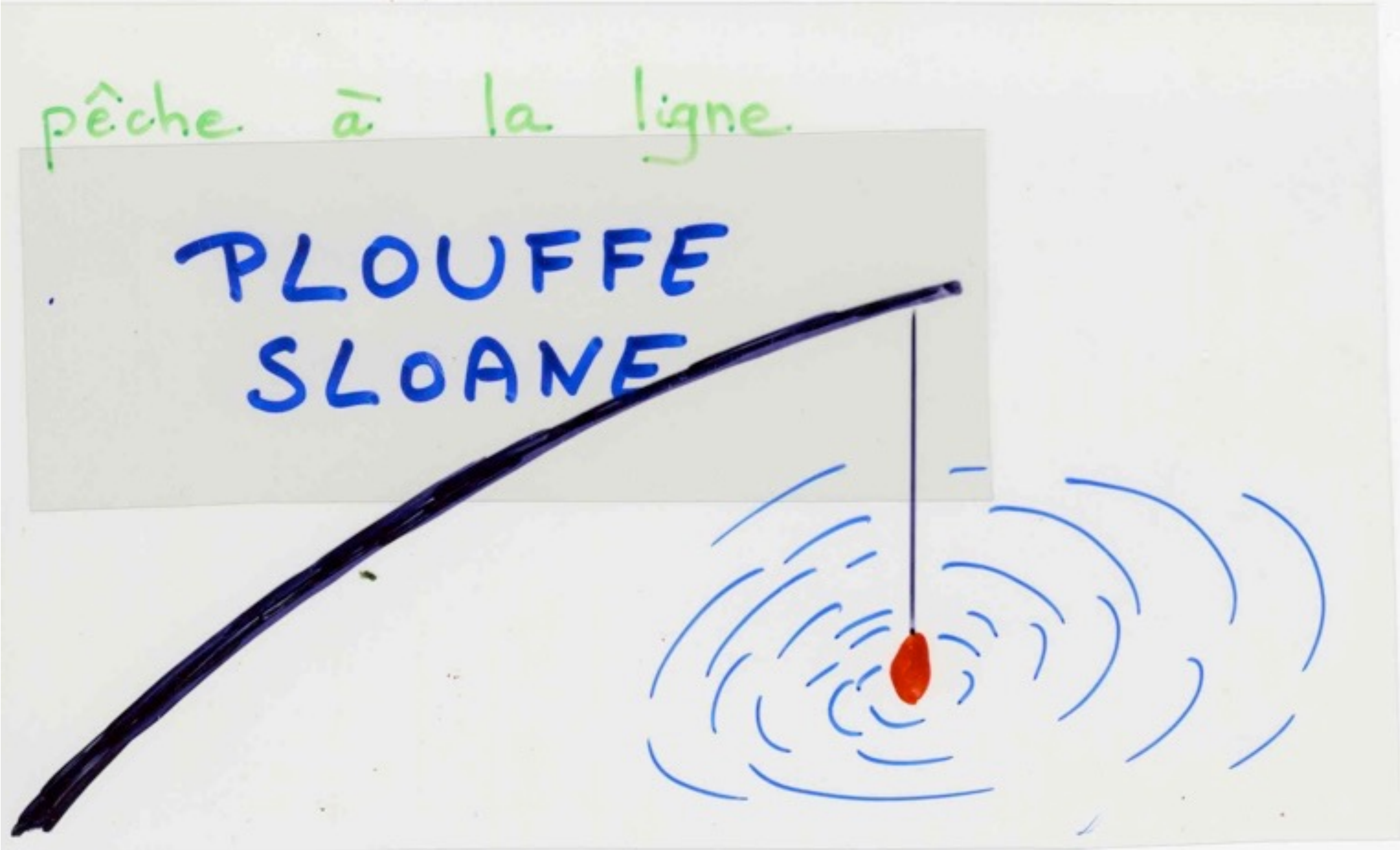
IIT-Madras
14 January 2015

Xavier Viennot
CNRS, LaBRI, Bordeaux

experimental combinatorics

pêche à la ligne




pêche à la ligne
PLOUFFE
SLOANE

THE ENCYCLOPEDIA OF INTEGER SEQUENCES

N. J. A. Sloane
Mathematical Sciences Research Center
AT&T Bell Laboratories
Murray Hill, New Jersey

Simon Plouffe
Départment de Mathématiques et d'Informatique
Université du Québec à Montréal
Montréal, Québec

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M2819 1, 3, 9, 35, 178
Van der Waerden numbers. Ref Loth83 49. [1,2; A5346]

M2820 1, 3, 9, 35, 201, 1827
Coefficients of Bell's formula. Ref NMT 10 65 62. [2,2; A2575, N1134]

M2821 1, 3, 9, 37, 153, 951, 5473, 42729, 353937, 3455083, 30071001, 426685293, 4707929449, 59350096287, 882391484913, 15177204356401, 205119866263713
Sums of logarithmic numbers. Ref TMS 31 79 63. jos. [0,2; A2751, N1135]

M2822 1, 1, 1, 3, 9, 37, 177, 959, 6097, 41641, 325249, 2693691, 24807321, 241586893, 2558036145, 28607094455, 342232522657, 4315903789009, 57569080467073
Expansion of $e^{\tan x}$. Ref JO61 150. [0,4; A6229]

M2823 1, 3, 9, 42, 206, 1352, 10168
Regular semigroups of order n. Ref PL65. MAL 2 2 67. SGF 14 71 77. [1,2; A1427, N1136]

M2824 0, 1, 1, 3, 9, 45, 225, 1575, 11025, 99225, 893025, 9823275, 108056025, 1404728325, 18261468225, 273922023375, 4108830350625, 69850115960625
Expansion of $1 / (1-x)(1-x^2)^{1/2}$. Ref R1 87. [1,4; A0246, N1137]

M2825 1, 1, 1, 3, 9, 48, 504, 14188, 1351563
Threshold functions of n variables. Ref PGEC 19 821 70. MU71 38. [0,4; A1530, N1138]

M2826 3, 9, 54, 450, 4725, 59535, 873180, 14594580
Expansion of an integral. Ref C1 167. [2,1; A1194, N1139]

M2827 1, 3, 9, 89, 1705, 67774
Superpositions of cycles. Ref AMA 131 143 73. [3,2; A3225]

M2828 1, 3, 9, 93, 315, 3855, 13797, 182361, 9256395, 34636833, 1857283155, 26817356775, 102280151421, 1497207322929, 84973577874915, 4885260612740877
Fermat quotients: $(2^{p-1}-1)/p$. Ref Well86 70. [0,2; A7663]

M2829 3, 10, 4, 5, 10, 2, 5, 3, 2, 3, 6, 6, 6, 3, 5, 6, 10, 5, 5, 10, 6, 6, 6, 2, 5, 8, 2, 6, 8, 4, 6, 6, 4, 5, 10, 2, 4, 7, 11, 5, 7, 9, 10, 7, 1, 6, 7, 11, 7, 10, 0, 6, 8, 9, 6, 4, 11, 7, 13, 2, 6, 4, 4
Iterations until $3n$ reaches 153 under x goes to sum of cubes of digits map. Ref Robe92 13. [1,1; A3620]

M2830 1, 3, 10, [illegible] 13361, 73011, 597449, 1865358,

Razumov - Stroganov conjecture

Spin chains and combinatorics

A. V. Razumov, Yu. G. Stroganov

Institute for High Energy Physics
142284 Protvino, Moscow region, Russia

(J. R. R. Tolkien)

The XXZ quantum spin chain model with periodic boundary conditions is one of the most popular integrable models which has been investigating by the Bethe Ansatz method during the last 35 years [3]. It is described by the Hamiltonian

$$H_{XXZ} = -\sum_{j=1}^{N} \left\{ \sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta \sigma_j^z \sigma_{j+1}^z \right\}, \qquad \vec{\sigma}_{N+1} = \vec{\sigma}_1. \tag{1}$$

The nonzero wave function components are

$$
\begin{aligned}
N=3:&\quad \psi_{001}=1;\\
N=5:&\quad \psi_{00011}=1,\quad \psi_{00101}=2;\\
N=7:&\quad \psi_{0000111}=1,\quad \psi_{0001101}=\psi_{0001011}=3,\quad \psi_{0010011}=4\quad \psi_{0010101}=7.
\end{aligned}
$$

All components not included in the list can be obtained by shifting. Notice that the components of the ground state are positive in accordance with the Perron–Frobenius theorem.

Let us continue the list. For $N = 9$ the components of the eigenvector with the energy $-27/2$ and $S_z = -1/2$ are

$$
\begin{array}{llll}
\psi_{000001111}=1, & \psi_{000010111}=4, & \psi_{000011011}=6, & \psi_{000100111}=7,\\
\psi_{000101011}=17, & \psi_{000101101}=14, & \psi_{000110011}=12, & \psi_{001001011}=21,\\
 & \psi_{001010011}=25, & \psi_{001010101}=42. &
\end{array}
$$

Let us continue the list. For $N = 9$ the components of the eigenvector with the energy $-27/2$ and $S_z = -1/2$ are

$$
\begin{array}{llll}
\psi_{000001111}=1, & \psi_{000010111}=4, & \psi_{000011011}=6, & \psi_{000100111}=7,\\
\psi_{000101011}=17, & \psi_{000101101}=14, & \psi_{000110011}=12, & \psi_{001001011}=21,\\
 & \psi_{001010011}=25, & \psi_{001010101}=42. &
\end{array}
$$

We omit nonzero components which can be obtained by the reflection of the order of sites since this transformation is a symmetry of our state, as it is for the ground state. For example, we have

$$\psi_{000011101}=\psi_{000010111}=4.$$

1 , 2 , 7 , 42 , 429 , ...

?

M1791 0, 1, 2, 7, 32, 181, 1214, 9403, 82508, 808393, 8743994, 103459471, 1328953592, 18414450877, 273749755382, 4345634192131, 73362643649444, 1312349454922513
$a(n)=n.a(n-1)+(n-2)a(n-2)$. Ref R1 188. [0,3; A0153, N0706]

$$\text{E.g.f.: } (1-x)^{-3} e^{-x}.$$

M1792 1, 1, 2, 7, 32, 181, 1232, 9787, 88832, 907081, 10291712, 128445967, 1748805632, 25794366781, 409725396992, 6973071372547, 126585529106432
Expansion of $1/(1-\sinh x)$. Ref ARS 10 138 80. [0,3; A6154]

M1793 0, 1, 1, 2, 7, 32, 184, 1268, 10186, 93356, 960646, 10959452, 137221954, 1870087808, 27548231008, 436081302248, 7380628161076, 132975267434552
Stochastic matrices of integers. Ref DUMJ 35 659 68. [0,4; A0987, N0707]

M1794 1, 2, 7, 33, 192
Permutations of length n with n in second orbit. Ref C1 258. [2,2; A6595]

M1795 1, 2, 7, 34, 209, 1546, 13327, 130922, 1441729, 17572114, 234662231, 3405357682, 53334454417, 896324308634, 16083557845279, 306827170866106
$a(n)=2n.a(n-1)-(n-1)^2 a(n-2)$. Ref SE33 78. [0,2; A2720, N0708]

M1796 1, 2, 7, 34, 257, 2606, 32300, 440564, 6384634
Polyhedra with n nodes. Ref GR67 424. UPG B15. Dil92. [4,2; A0944, N0709]

M1797 2, 7, 35, 219, 1594, 12935, 113945, 1070324, 10586856, 109259633, 1168384157, 12877168147, 145656436074, 1685157199175, 19886174611045
Two-rowed truncated monotone triangles. Ref JCT A42 277 86. Zei93. [1,1; A6947]

M1798 1, 1, 2, 7, 35, 228, 1834, 17382, 195866, 2487832, 35499576, 562356672, 9794156448, 186025364016, 3826961710272, 84775065603888, 2011929826983504
Coefficients of iterated exponentials. Ref SMA 11 353 45. [0,3; A0154, N0710]

M1799 1, 2, 7, 35, 228, 1834, 17582, 195866, 2487832, 35499576, 562356672, 9794156448, 186025364016, 3826961710272, 84775065603888, 2011929826983504
Expansion of $\ln(1+\ln(1+x))$. [0,2; A3713]

M1800 1, 0, 1, 2, 7, 36, 300, 3218, 42335, 644808
Circular diagrams with n chords. Ref BarN94. [0,4; A7474]

M1801 1, 2, 7, 36, 317, 5624, 251610, 33642660, 14685630688
$n \times n$ binary matrices. Ref CPM 89 217 64. SLC 19 79 88. [0,2; A2724, N0711]

M1802 2, 7, 37, 216, 1780, 32652
Semigroups of order n with 2 idempotents. Ref MAL 2 2 67. SGF 14 71 77. [2,1; A2787, N0712]

M1803 1, 2, 7, 37, 266, 2431, 27007, 353522, 5329837, 90960751, 1733584106, 36496226977, 841146804577, 21065166341402, 569600638022431
$a(n)=(2n-1)a(n-1)+a(n-2)$. Ref RCI 77. [0,2; A1515, N0713]

M1804 1, 1, 2, 7, 38, 291, 2932, 36961, 561948, 10026505, 205608536, 4767440679, 123373203208, 3525630110107, 110284283006640, 3748357699560961
Forests of labeled trees with n nodes. Ref JCT 5 96 68. SIAD 3 574 90. [0,3; A1858, N0714]

M1805 1, 1, 2, 7, 40, 357, 4824, 96428, 2800472, 116473461
n-element partial orders contained in linear order. Ref nbh. [0,3; A6455]

M1806 1, 2, 7, 41, 346, 3797, 51157, 816356, 15050581, 314726117, 7359554632, 190283748371, 5389914888541, 165983936096162, 5521346346543307
Planted binary phylogenetic trees with n labels. Ref LNM 884 196 81. [1,2; A6677]

M1807 1, 1, 2, 7, 41, 376, 5033, 92821, 2257166, 69981919, 2694447797, 126128146156, 7054258103921, 464584757637001, 35586641825705882, 3136942184333040727
Hammersley's polynomial $p_n(1)$. Ref MASC 14 4 89. [0,3; A6846]

M1808 1, 2, 7, 42, 429, 7436, 218348, 10850216, 911835460, 129534272700, 31095744852375, 12611311859677500, 8639383518297652500
Robbins numbers: $\Pi(3k+1)!/(n+k)!$, $k = 0 \, . \, . \; n-1$. Ref MINT 13(2) 13 91. JCT A66 17 94. [1,2; A5130]

M1809 1, 2, 7, 42, 582, 21480, 2142288, 575016219, 415939243032, 816007449011040, 4374406209970747314, 64539836938720749739356
Antisymmetric relations on n nodes. Ref PAMS 4 494 53. MIT 17 23 55. [1,2; A1174, N0715]

M1810 0, 1, 2, 7, 44, 361, 3654, 44207, 622552, 10005041, 180713290, 3624270839, 79914671748, 1921576392793, 50040900884366, 1403066801155039
Modified Bessel function $K_n(1)$. Ref AS1 429. [0,3; A0155, N0716]

M1811 0, 1, 2, 7, 44, 447, 6749, 142176, 3987677, 143698548, 6470422337, 356016927083, 23503587609815, 1833635850492653, 166884365982441238
$a(n)=n(n-1)a(n-1)/2+a(n-2)$. [0,3; A1046, N0717]

M1812 1, 2, 7, 44, 529, 12278, 565723, 51409856, 9371059621, 3387887032202, 2463333456292207, 3557380311703796564, 10339081666350180289849
Sum of Gaussian binomial coefficients $[n,k]$ for $q=4$. Ref TU69 76. GJ83 99. ARS A17 328 84. [0,2; A6118]

M1813 2, 7, 52, 2133, 2590407, 3374951541062, 5695183504479116640376509, 16217557574922386301420514191523784895639577710480
Free binary trees of height n. Ref JCIS 17 180 92. [1,1; A5588]

M1814 1, 1, 2, 7, 56, 2212, 2595782, 3374959180831, 5695183504489239067484387, 16217557574922386301420531277071365103168734284282
Planted 3-trees of height n. Ref RSE 59(2) 159 39. CMB 11 87 68. JCIS 17 180 92. [0,3; A2658, N0718]

M1807 1, 1, 2, 7, 41, 376, 5033, 92821, 2257166, 69981919, 2694447797, 126128146156, 7054258103921, 464584757637001, 35586641825705882, 3136942184333040727
Hammersley's polynomial $p_n(1)$. Ref MASC 14 4 89. [0,3; A6846]

M1808 1, 2, 7, 42, 429, 7436, 218348, 10850216, 911835460, 129534272700, 31095744852375, 12611311859677500, 8639383518297652500
Robbins numbers: $\Pi(3k+1)!/(n+k)!$, $k = 0 \,.\,.\, n-1$. Ref MINT 13(2) 13 91. JCT A66 17 94. [1,2; A5130]

M1809 1, 2, 7, 42, 582, 21480, 2142288, 575016219, 415939243032, 816007449011040, 4374406209970747314, 64539836938720749739356
Antisymmetric relations on n nodes. Ref PAMS 4 494 53. MIT 17 23 55. [1,2; A1174, N0715]

M1807 1, 1, 2, 7, 41, 376, 5033, 92821, 2257166, 69981919, 2694447797, 126128146156, 7054258103921, 464584757637001, 35586641825705882, 3136942184333040727
Hammersley's polynomial $p_n(1)$. Ref MASC 14 4 89. [0,3; A6846]

M1808 1, 2, 7, 42, 429, 7436, 218348, 10850216, 911835460, 129534272700, 31095744852375, 12611311859677500, 8639383518297652500
Robbins numbers: $\Pi(3k+1)!/(n+k)!$, $k = 0 \,..\, n-1$. Ref MINT 13(2) 13 91. JCT A66 17 94. [1,2; A5130]

M1809 1, 2, 7, 42, 582, 21480, 2142288, 575016219, 415939243032, 816007449011040, 4374406209970747314, 64539836938720749739356
Antisymmetric relations on n nodes. Ref PAMS 4 494 53. MIT 17 23 55. [1,2; A1174, N0715]

ASM

Alternating sign matrices

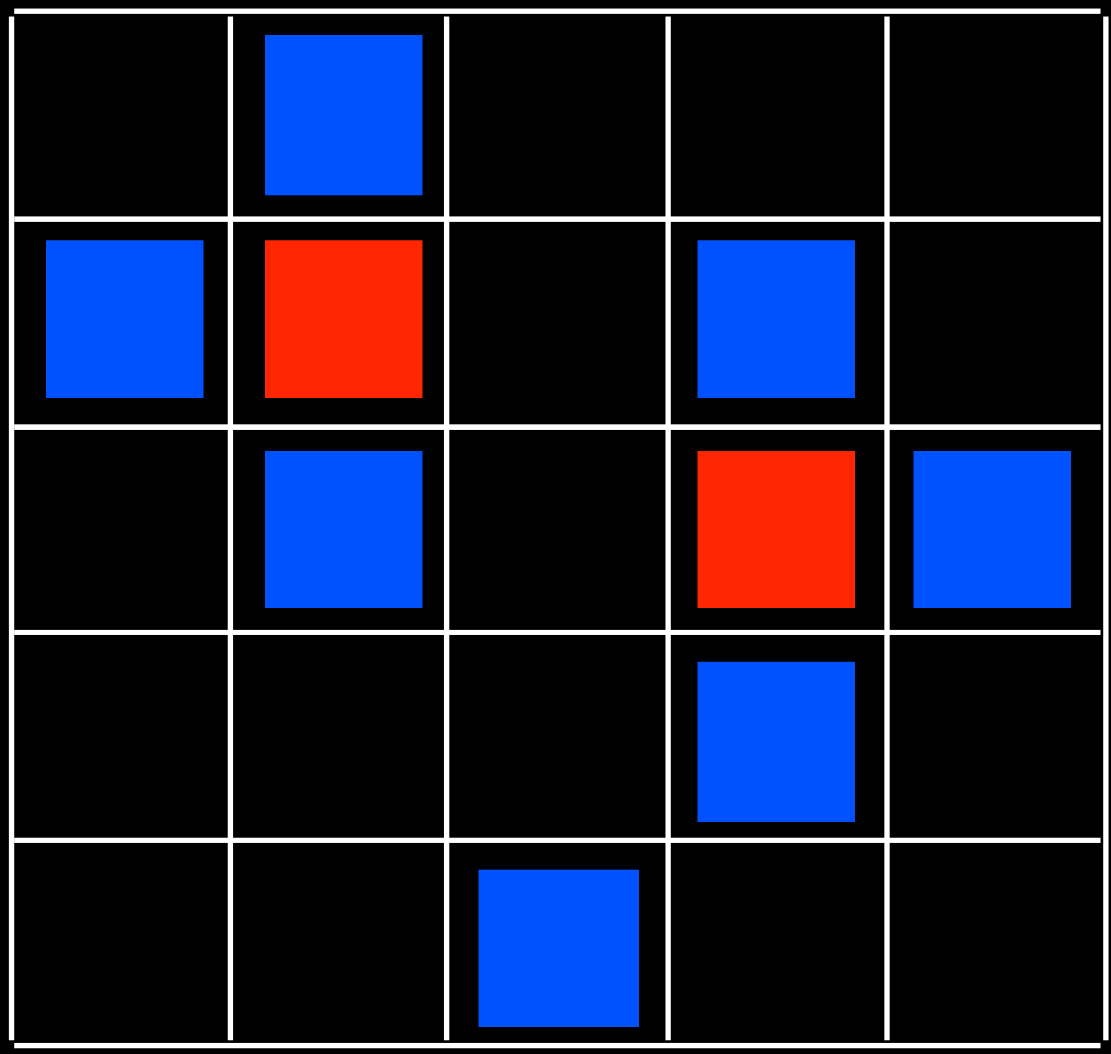
"Matrices à signes alternés"

(alternating sign matrices)

- entrées : 0, 1, −1
- somme des entrées, ligne, colonne = 1
- entrées ≠ 0 alternent en signe ligne, colonne

ex :

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



enumeration of ASM

1, 2, 7, 42, 429, ...

$$\frac{1!\ 4!\ \cdots\ (3n-2)!}{n!\ (n+1)!\ \cdots\ (n+n-1)!}$$

?

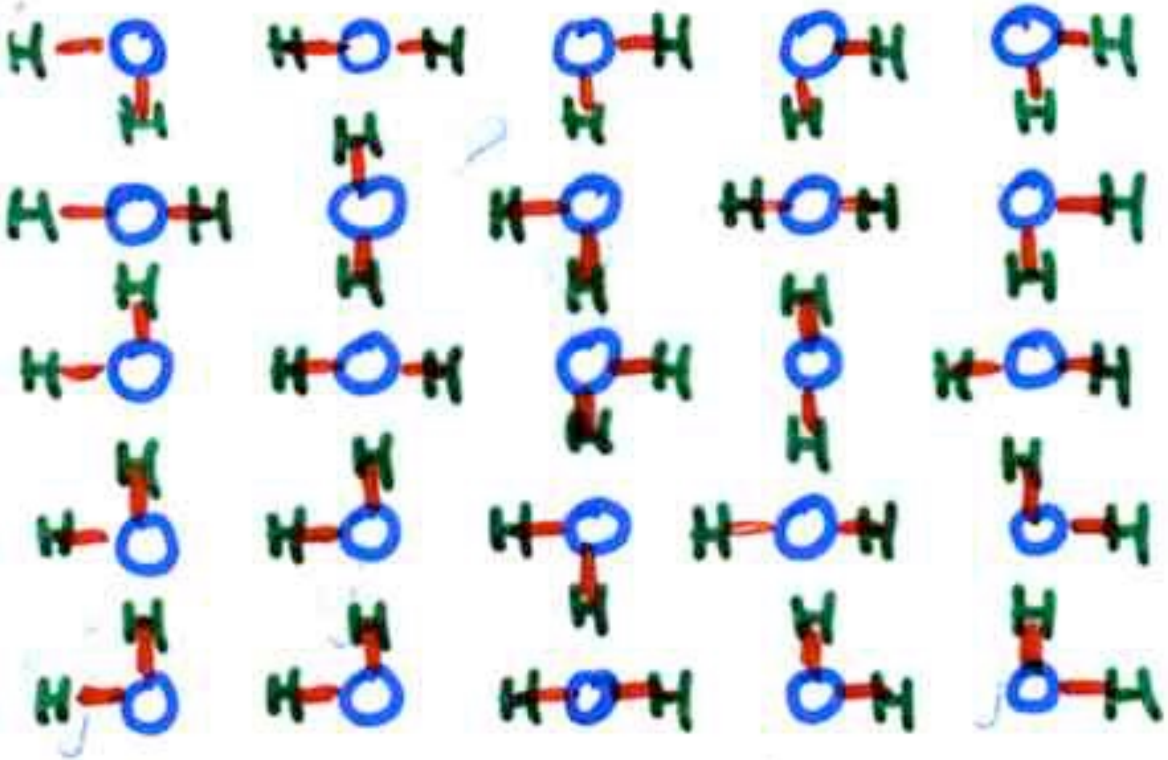
alternating sign matrices conjecture

Mills, Robbins, Rumsey (1982)

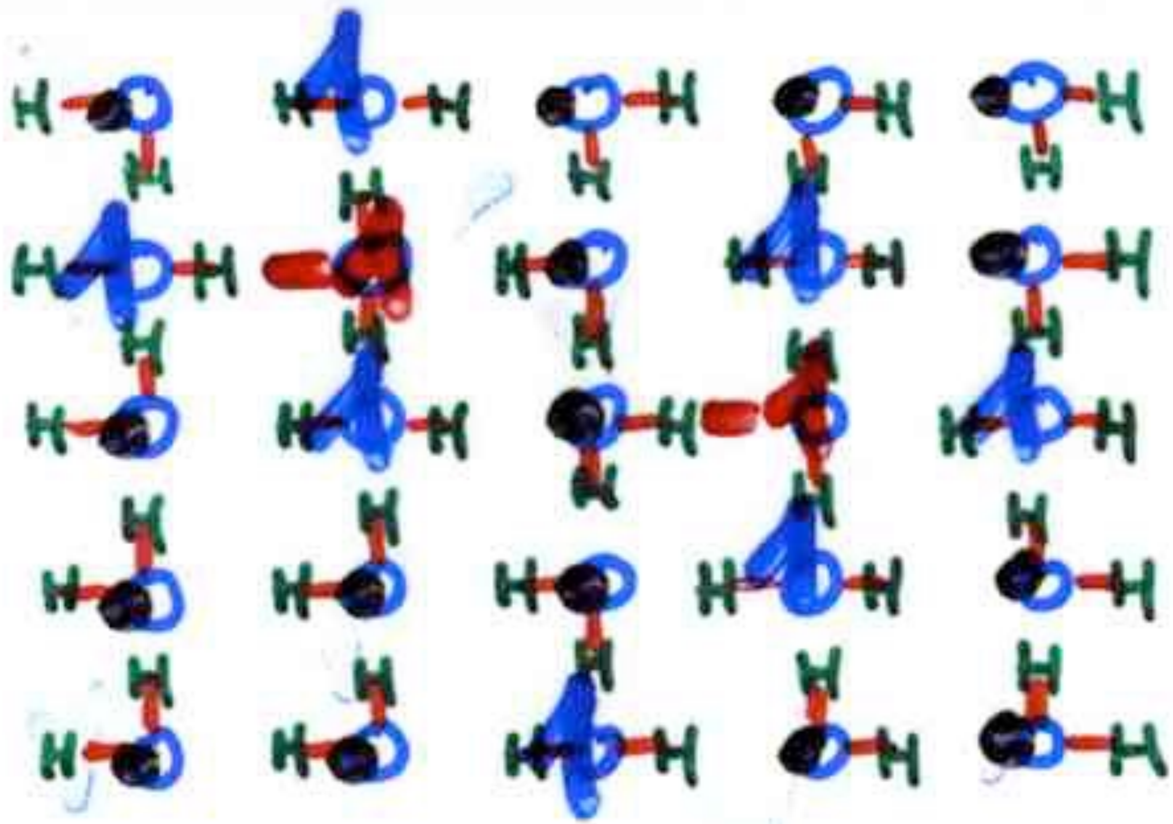
Robbins

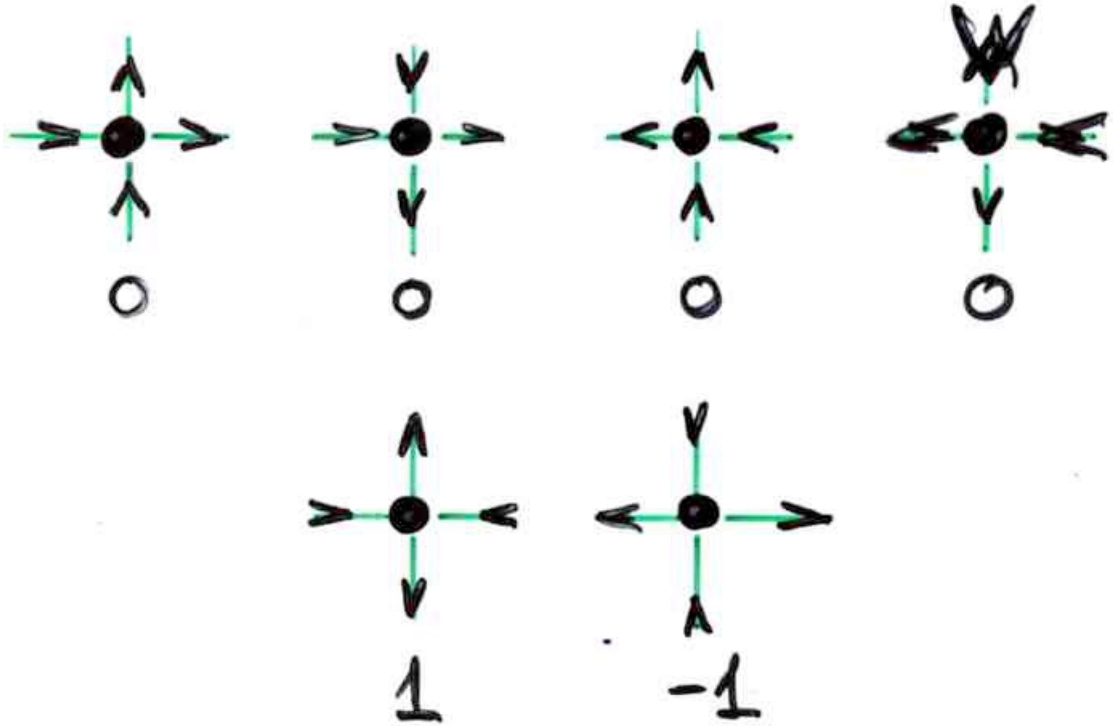
The Mathematical Intelligencer (1991)

“These conjectures are of such compelling simplicity that it is hard to understand how any mathematician can bear the pain of living without understanding why they are true”



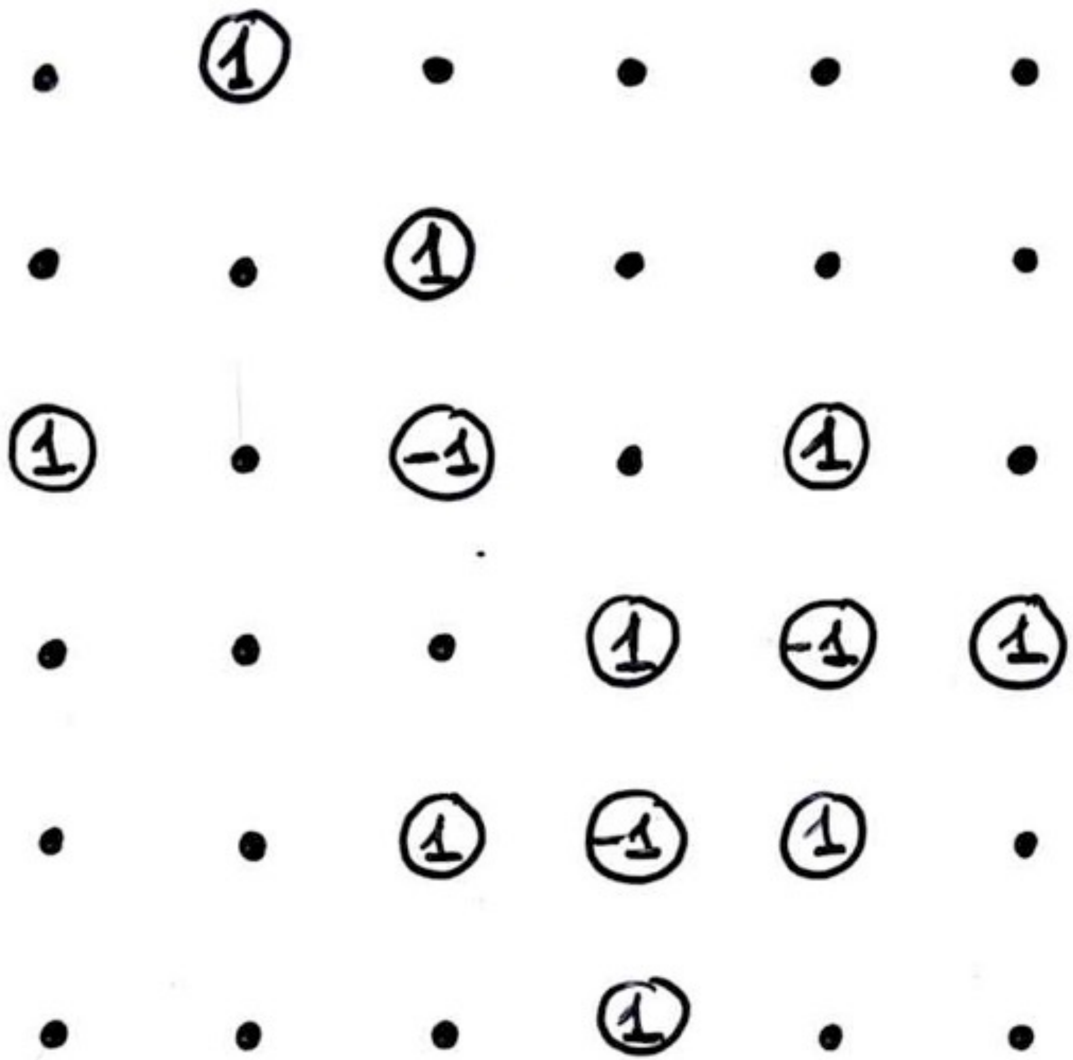
·	1	·	·	·
1	-1	·	1	·
·	1	·	-1	1
·	·	·	1	·
·	·	1	·	·



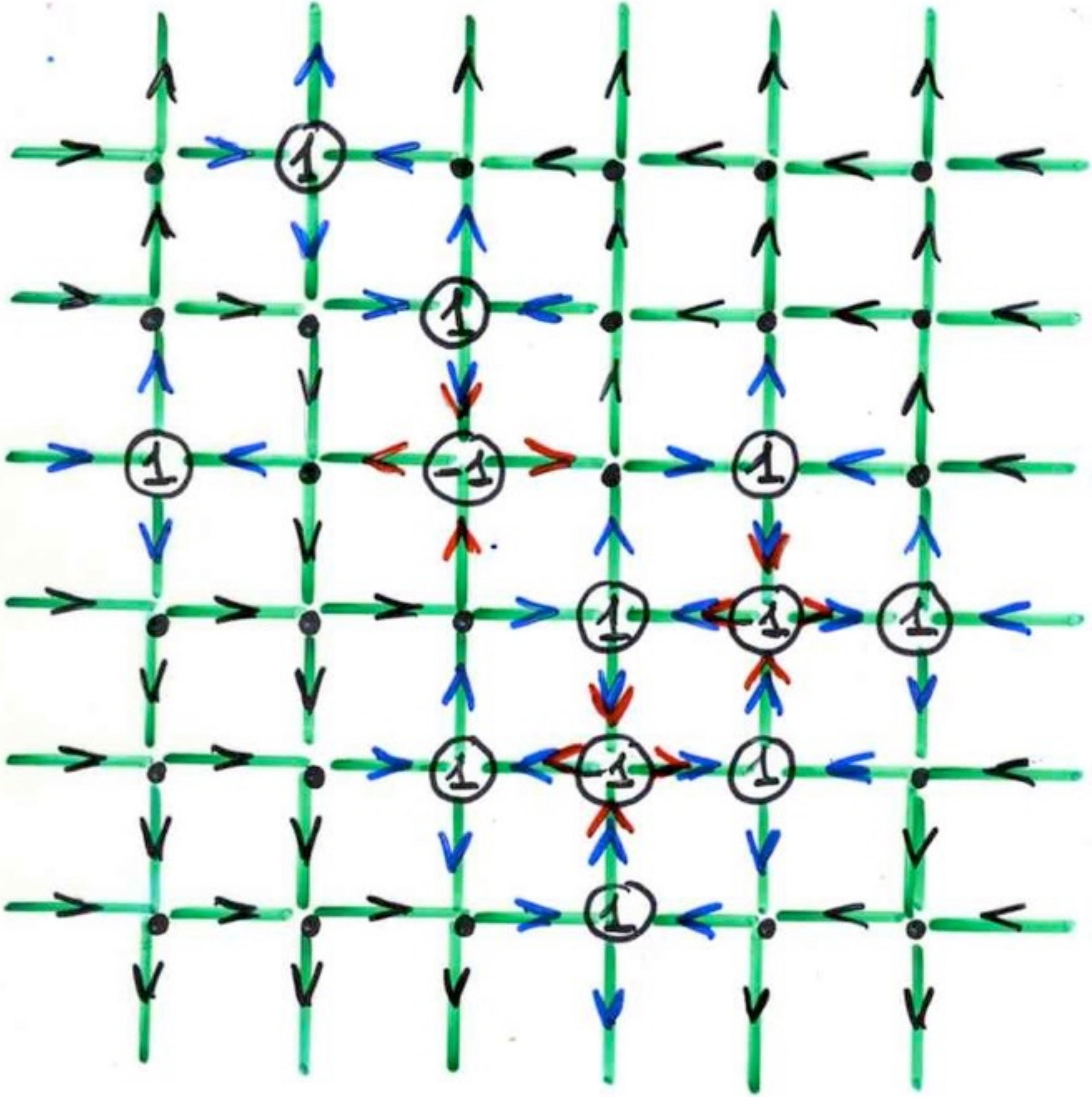


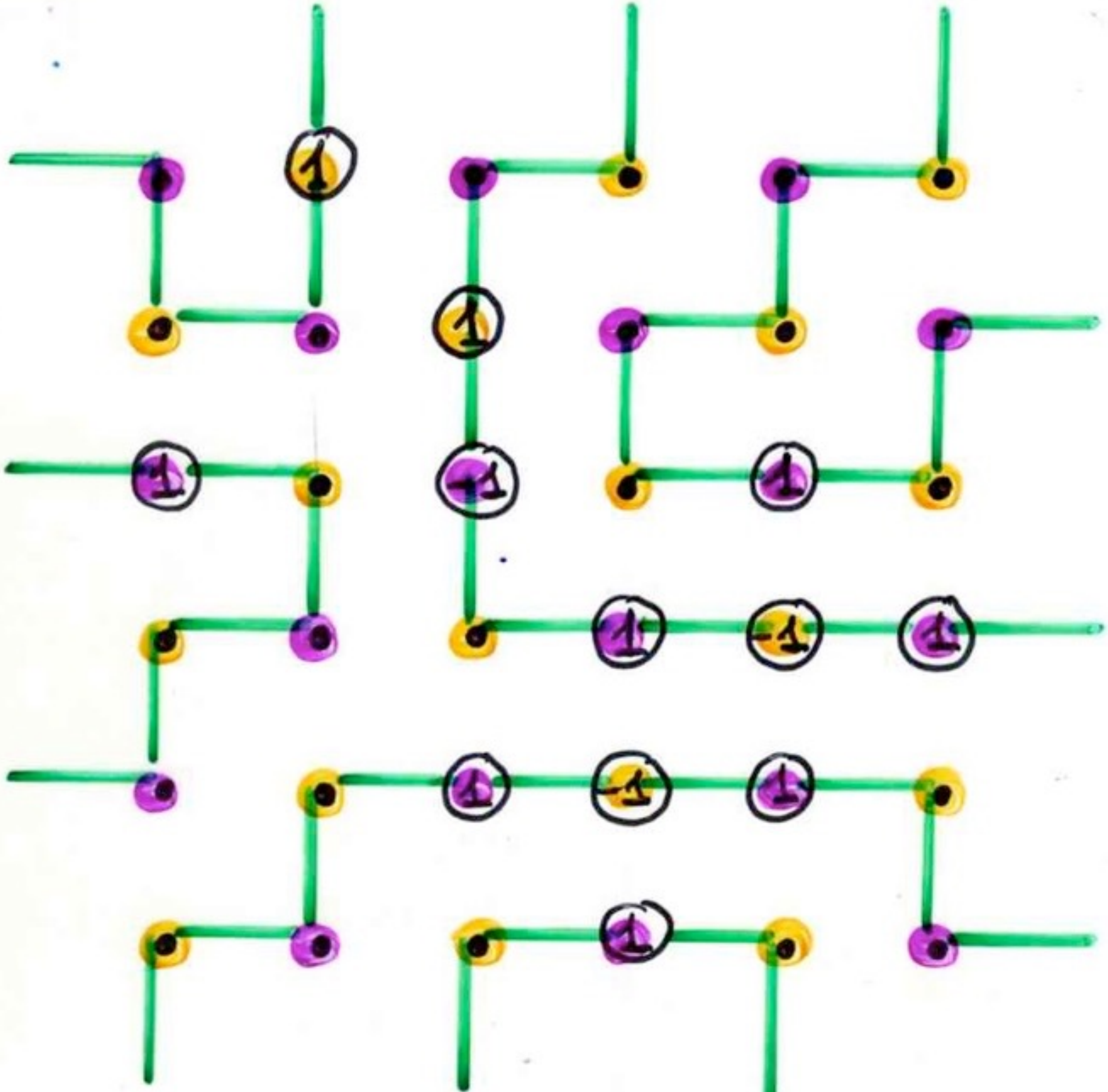
6-vertex model

The bijection AMS FPL

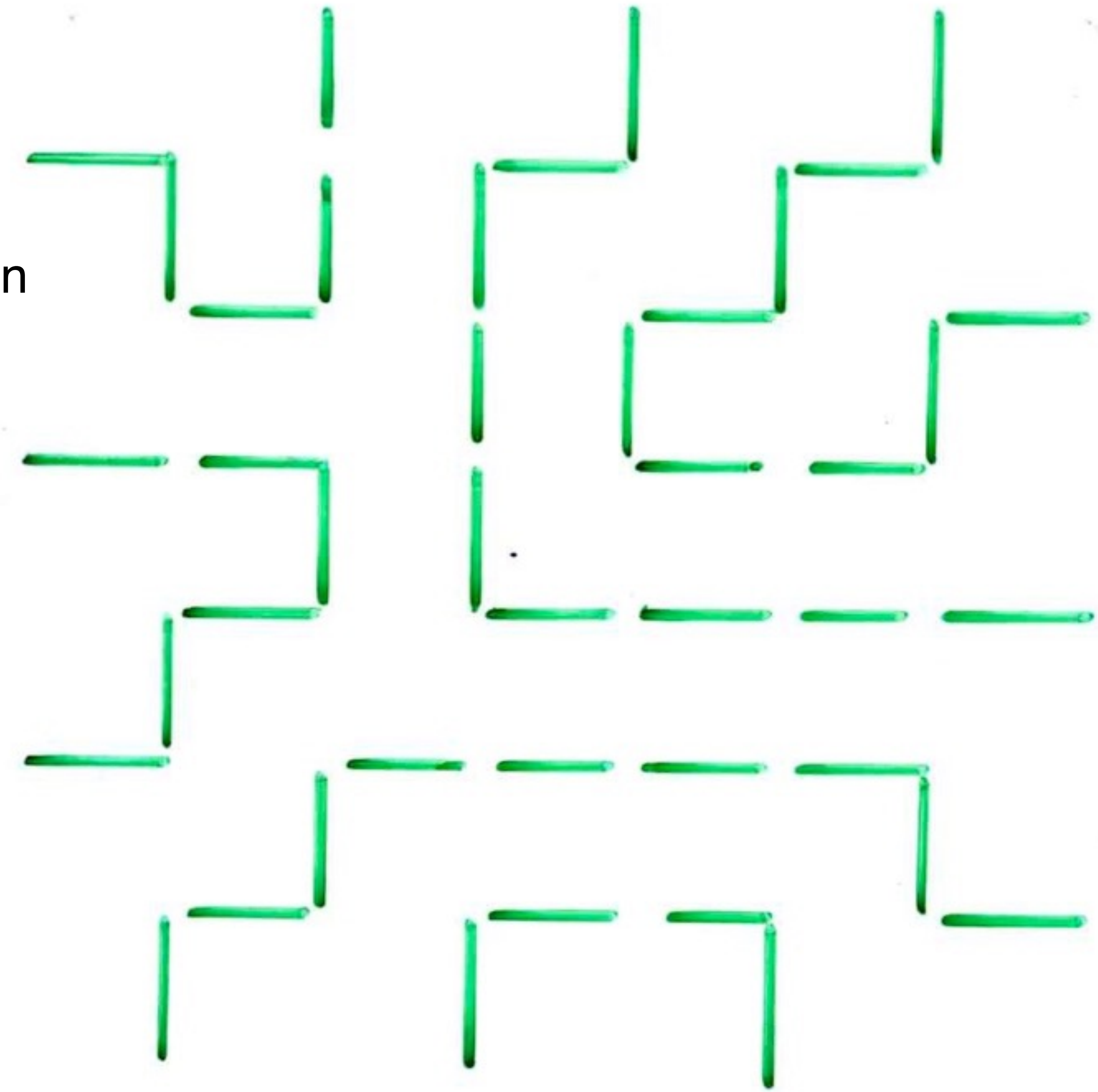


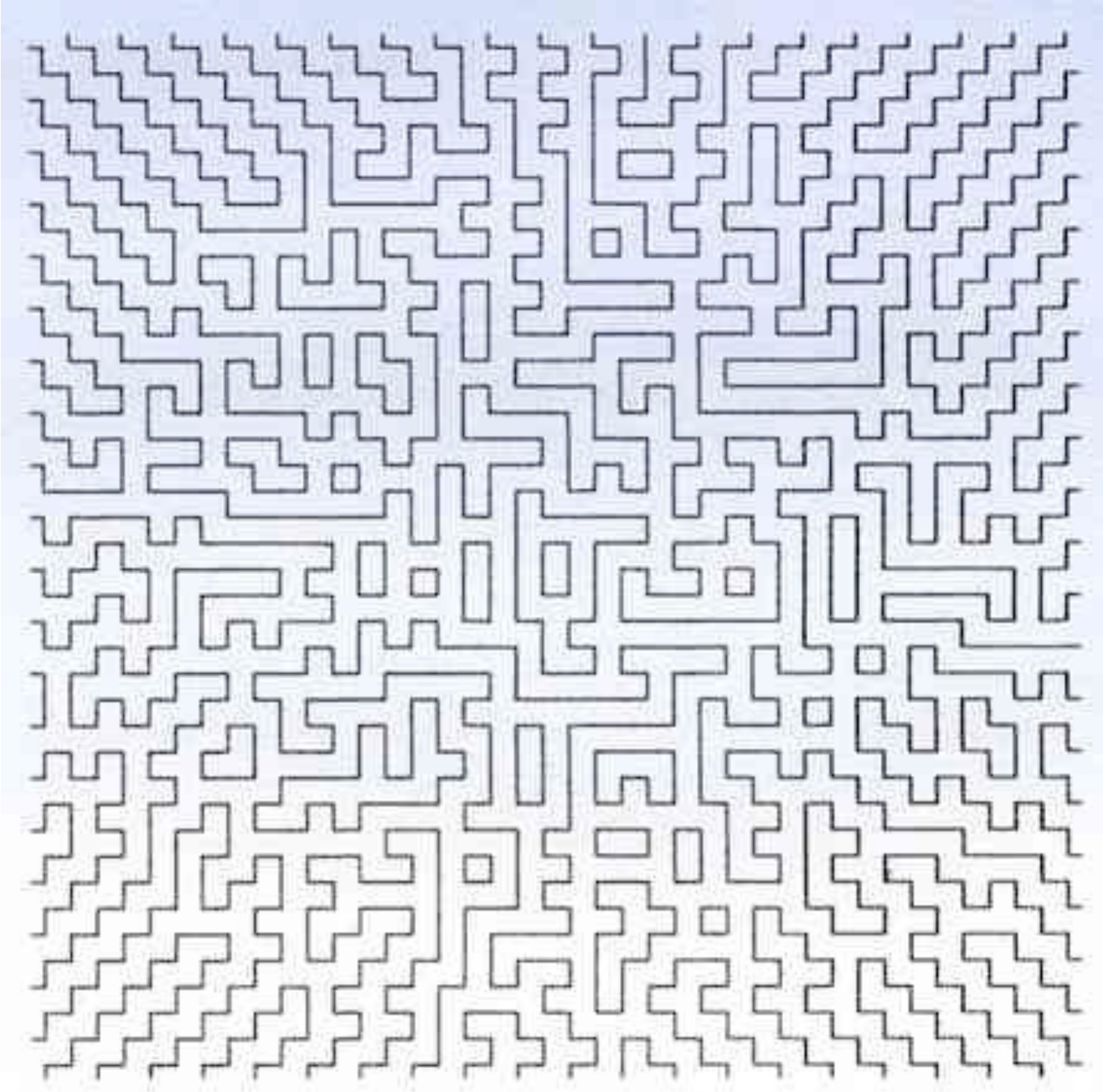
The 6-vertex model





FPL
“Fully
Packed
Loop”
configuration





"nostalgic combinatorics"

NOVI COMMENTARII ACADEMIAE SCIENTIARVM IMPERIALIS PETROPOLITANAE

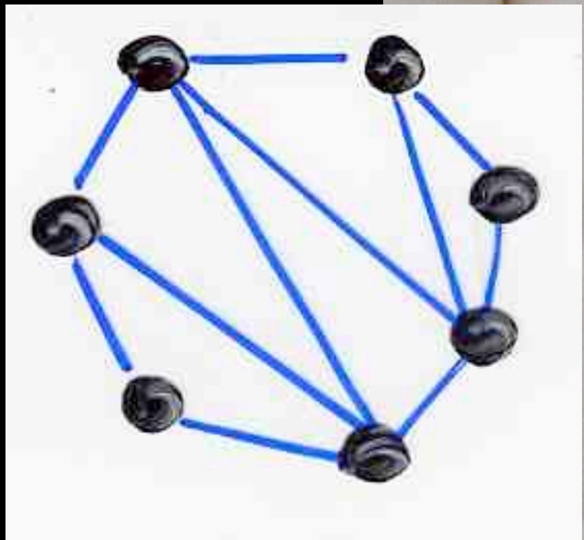
TOM. VII.

pro Annis MDCCLVIII. et MDCCLIX.

PETROPOLI

TYPIS ACADEMIAE SCIENTIARVM

MDCCLXI.



ENVMERATIO MODORVM QVIBVS FIGVRAE PLANAE RECTILINEAE PER DIAGONALES DIVIDVNTVR IN TRIANGVLA.

Auctore

IOH. ANDR. DE SEGNER.

Triangulum per diagonalem in alia ſolui non poſſe, vtpote quod ex ſe ipſo vno tantum modo componitur, notum eſt: quadrilaterum autem ita diuidi duplicem in modum poſſe, mox apparet. Neque difficulter perſpicitur, modos, quibus quinque laterum figura per diagonales in triangula ſoluitur, quinque eſſe, quorum quilibet diſcrepat ab altero. *S*ex autem, vel plurium laterum figurae, quot modis ita ſoluantur enumerare difficilius eſt; eoque difficilius, quo plura, ſunt figurae latera. Soluitur autem hexagonum in triangula 14 modis diuerſis, heptagonum 42, ogdogonum 132, enneagonum 429; quos numeros mecum beneuolus communicauit ſummus *Eulerus*; modo, quo eos reperit, atque progreſſionis ordine, celatis. Vtrumque perſpiciendi cupido, poſt tentamina quaedam inania, eos numeros eliciendi methodus occurrit adeo ſimplex, vt in ea acquieſcendum mihi putauerim, quam hic proponam.

Triangulum prima ordine eſt figurarum planarum rectilinearum, quadrilaterum ſecunda, et ita deinceps, ſic merorum exponit, ac rigoroſe demonſtrat, dum docet, quomodo pro quouis polygono reſolutionum numerus, ex cognita reſolutione polygonorum ſimpliciorum, quae paucioribus conſtant lateribus, colligi debeat. Hac ratione, ſi a ſimpliciſſimis incipiamus, hanc inueſtigationem continuo ad polygona plurium laterum extendere licet, ſicque Ill. Auctor ſub finem tabellam adiecit, in qua iſtiusmodi reſolutiones ad polygona 20 laterum vsque exhibet. Liceat autem nobis, a ſummo quodam Geometra, qui eandem hanc tabulam calculo ſubiecit, admonitis, obſeruare, hanc tabulam, ob quendam calculi errorem, tantum vsque ad polygona 15 laterum eſſe iuſtam, quippe pro hoc polygono reſolutionum numerus non eſt 751900, vt tabella habet, ſed 742900, ſequentes quoque numeri, dum forte nouus error irrepſit, primo ad 17 vsque latera nimis ſunt magni, deinde vero nimis parui, dum pro 20 lateribus reſolutionum numerus eſt 477638700. Facilius haec apparent, ſi ex lege primum obſeruata, qua quilibet numerus ex omnibus praecedentibus colligitur, alia ad computum facilior eliciatur, cuius ope quilibet numerus ex ſolo praecedente definiatur. Ita ſi pro polygono n laterum numerus reſolutionum ſit P, pro polygono ſequente $n+1$ laterum numerus reſolutionum erit $\frac{4n-6}{n}$ P. Quin etiam hinc, ſine conſideratione praecedentium, ſtatim indefinite pro polygono n laterum numerus reſolutionum ita per factores exprimitur, vt ſit:

$$\frac{2}{2}\cdot\frac{6}{3}\cdot\frac{10}{4}\cdot\frac{14}{5}\cdot\frac{18}{6}\cdot\frac{22}{7}\cdot\ \ldots\ldots\ \frac{4n-10}{n-1}$$

vbi numeratores quaternario, denominatores vero vnitate creſcunt. Hinc ſequentem nouam tabulam, bene-

vole

vole nobiſcum communicatam, adiungere e re viſum eſt, quod Ill. Auctori huius ſchediasmatis non diſpliciturum eſſe ſperamus.

Num. laterum	numerus reſolutionum.	num. laterum	numerus reſolutionum.
III	1	XV	742900
IV	2	XVI	2674440
V	5	XVII	9694845
VI	14	XVIII	35357670
VII	42	XIX	129644790
VIII	132	XX	477638700
IX	429	XXI	1767263190
X	1430	XXII	6564120420
XI	4862	XXIII	24466267020
XII	16796	XXIV	91482563640
XIII	58786	XXV	343059613650.
XIV	208012		

VII.

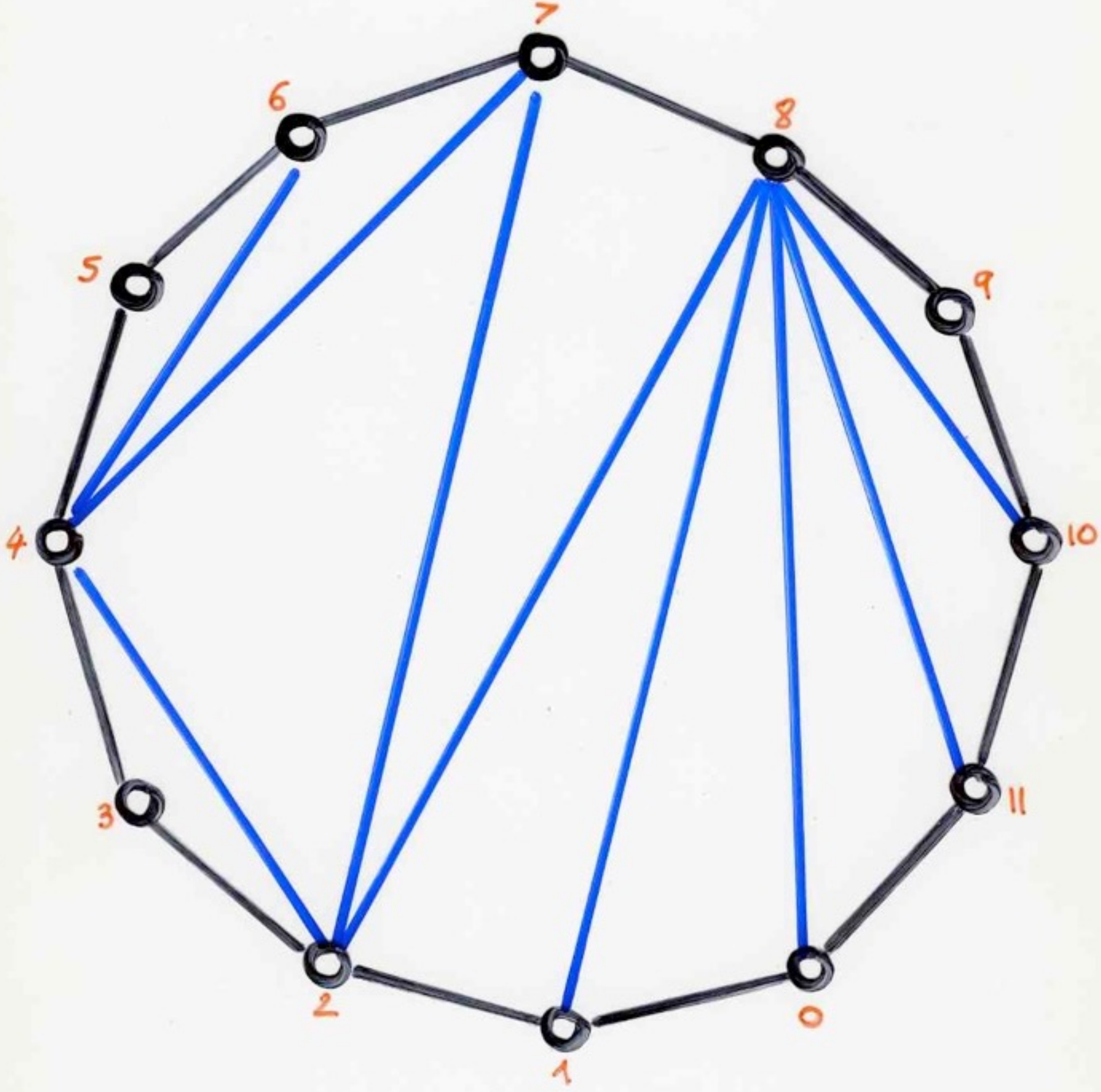
Methodus ſimplex et vniuerſalis omnes omnium aequationum radices detegendi.

$$2(2n+1)\,C_n = (n+2)\,C_{n+1}$$

Complures iam a Geometris excogitatae ſunt methodi aequationum algebraicarum radices, vel accurate, vel proxime ſaltem, determinandi: omnes autem fere poſtulant, vt valores radicum, quae quaeruntur,

$$\frac{1}{n+1}\binom{2n}{n}$$


EULER



7
6
8
5
9
4
10
3
11
2
0
1

getheilet und solches kan auf 5 erley verschiedene Arten geschehen, nemlich durch die Diagonales I. $\frac{ac}{ad}$; II. $\frac{bd}{be}$; III. $\frac{ca}{ce}$; IV. $\frac{db}{da}$; V. $\frac{ec}{eb}$

Ferner kan ein Sechseck durch 3 Diagonales in 4 Triangula zerschnitten und dieses kan auf 14 verschiedene Arten geschehen.

Nun ist die Frage generaliter, da ein Polygonum von n Seiten durch $n-3$ Diagonales in $n-2$ Triangula zerschnitten wird, auf wie vielerley verschiedene Arten solches geschehen könne.

Setze ich nun die Anzahl dieser verschiedenen Arten $= x$ so habe ich per Inductionem gefunden

wann $n = 3.\ 4.\ 5.\ 6.\ 7.\ 8.\ 9.\ 10$

so ist $x = 1,\ 2,\ 5,\ 14,\ 42,\ 132,\ 429,\ 1430$

Hieraus habe ich nun den Schluss gemacht, dass generaliter sey

$$x = \frac{2.6.10.14.18.22.\ \ldots\ (4n-10)}{2.3.4.5.6.7\ \ldots\ (n-1)}$$

oder es ist

$1 = \frac{2}{2}$; $2 = 1.\frac{6}{3}$; $5 = 2.\frac{10}{4}$; $14 = 5.\frac{14}{5}$; $42 = 14.\frac{18}{6}$; $132 = 42.\frac{22}{7}$;

dass also aus einer jeden Zahl die folgende leicht gefunden wird. Die Induction aber, so ich gebraucht, war ziemlich mühsam, doch zweifle ich nicht, dass diese Sach nicht sollte weit leichter entwickelt werden können. Ueber die Progression der Zahlen 1, 2, 5, 14, 42, 132, etc. habe ich auch diese Eigenschaft angemerket, dass

$$1 + 2a + 5a^2 + 14a^3 + 42a^4 + 132a^5 + \text{etc.} = \frac{1-2a-\sqrt{(1-4a)}}{2aa}$$

also wann $a = \frac{1}{4}$ so ist $1 + \frac{2}{4} + \frac{5}{4^2} + \frac{14}{4^3} + \frac{42}{4^4} + \text{etc.} = 4$.

Ew. Hochwohlgeb. meinige bitte ich zu Dero Hochwohlgeb. beständigen Gewogenheit gehorsamst empfehlen, und habe die Ehre mit der schuldigsten Hochachtung lebenslang zu beharren

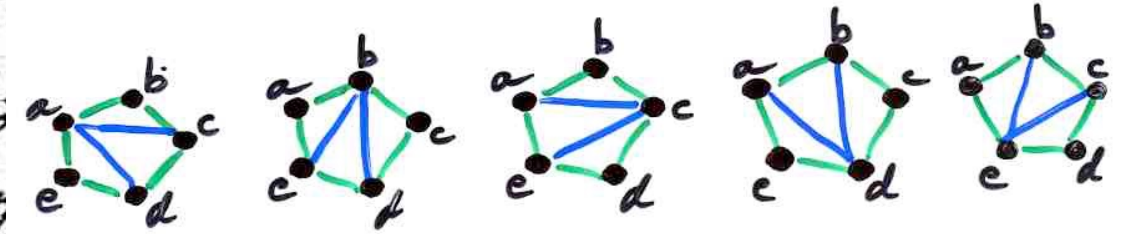
Ew. Hochwohlgebohrnen

Berlin d. 4. Sept 1751.

gehorsamster Diener

L. Euler

I. ac ad; II. bd be; III ca ce; IV db da; V ec eb



durch $n-3$ Diagonales in $n-2$ Triangula

$= x$

wann $n =$ 1, 2, 5, 14, 42, 132, 429, 1430, ...

so ist $x =$ 1, 2, 5, 14, 42, 132, 429, 1430

generaliter

$$x = \frac{2 \cdot 6 \cdot 10 \cdot 14 \cdot 18 \cdot 22 \cdots (4n-10)}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdots (n-1)} = \frac{(2n)!}{(n+1)!\, n!}$$

$$1 = 2 \cdot \tfrac{6}{\cdot};\quad 5 = 2 \cdot \tfrac{10}{4};\quad 14 = 5 \cdot \tfrac{14}{5};\quad 42 = 14 \cdot \tfrac{18}{6};$$

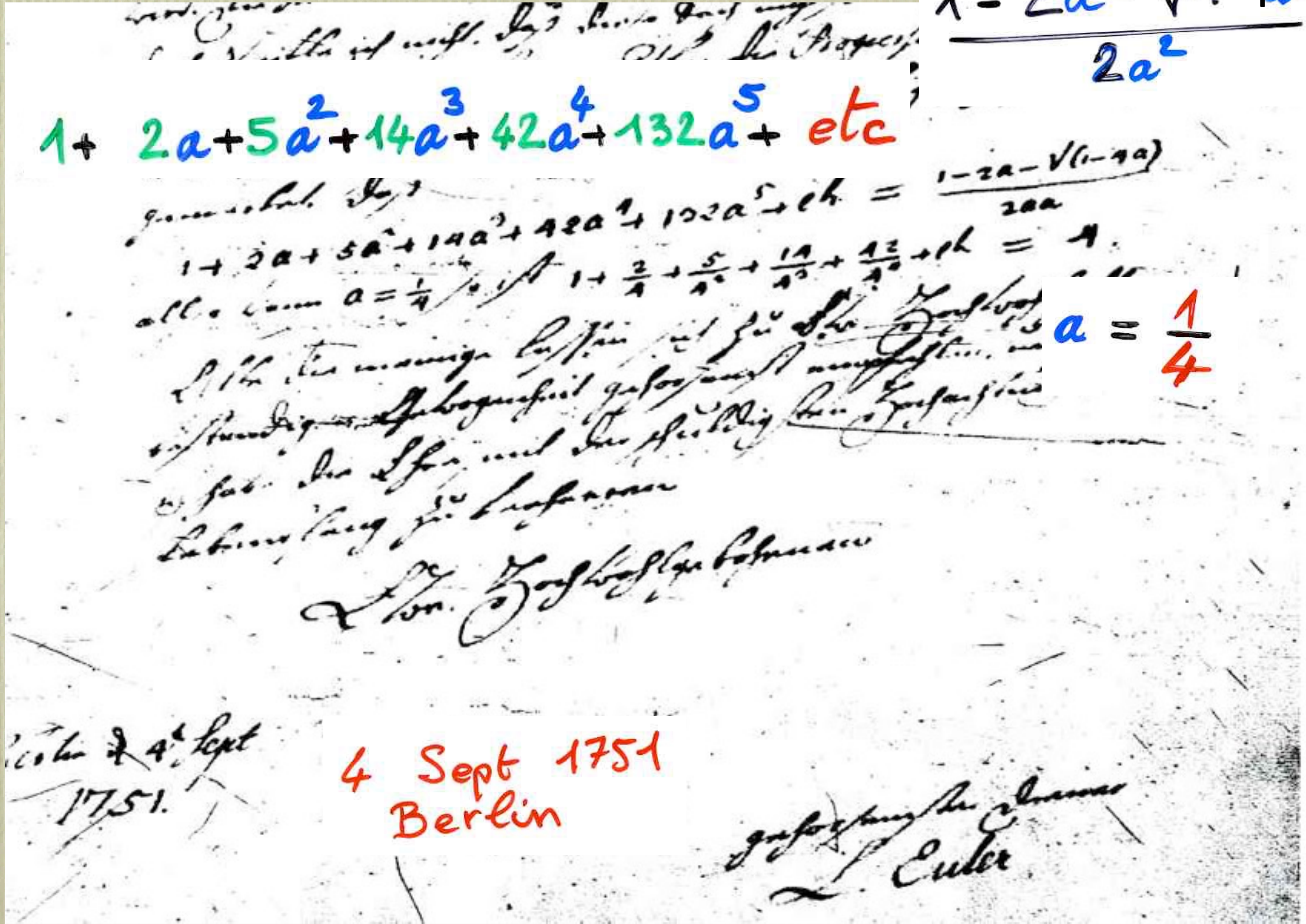
$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

$$n! = 1 \times 2 \times 3 \times \ldots \times n$$

$$\frac{1-2a-\sqrt{1-4a^2}}{2a^2}$$

4

$$1 + 2a + 5a^2 + 14a^3 + 42a^4 + 132a^5 + \text{etc}$$



$$a = \frac{1}{4}$$

4 Sept 1751
Berlin

- introduction to enumerative and bijective combinatorics
- non-crossing paths, tilings, determinants and Young tableaux. The LGV Lemma.
- introduction to the theory of heaps of pieces: the 3 basics lemma
- heaps of pieces and statistical mechanics: directed animals, gas models, q-Bessel functions in physics
- heaps of pieces and 2D Lorentzian quantum gravity
- combinatorics of the PASEP), relation with orthogonal polynomials
- alternating sign matrices, FPL and the (ex)-Razumov-Stroganov conjecture

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Introduction to combinatorics (generating functions)
see: Ch1 and 2, course CEA Saclay 2007 (*)
also: course Frutillar 2009 Ch1,
Tata Bombay 2010 Ch1 and 2

- Heaps of pieces, course Talca 2013/14
- Cellular Ansatz, course IIT Bombay 2013

- Combinatorial theory of orthogonal polynomials
- « Petite école » Bordeaux, 2006/2007 (*)

(*) main website www.xavierviennot.org, page « cours »