

“Des arbres dans les étoiles,
des arbres dans les grains de lumière”

Université de Bordeaux
18 octobre 2012

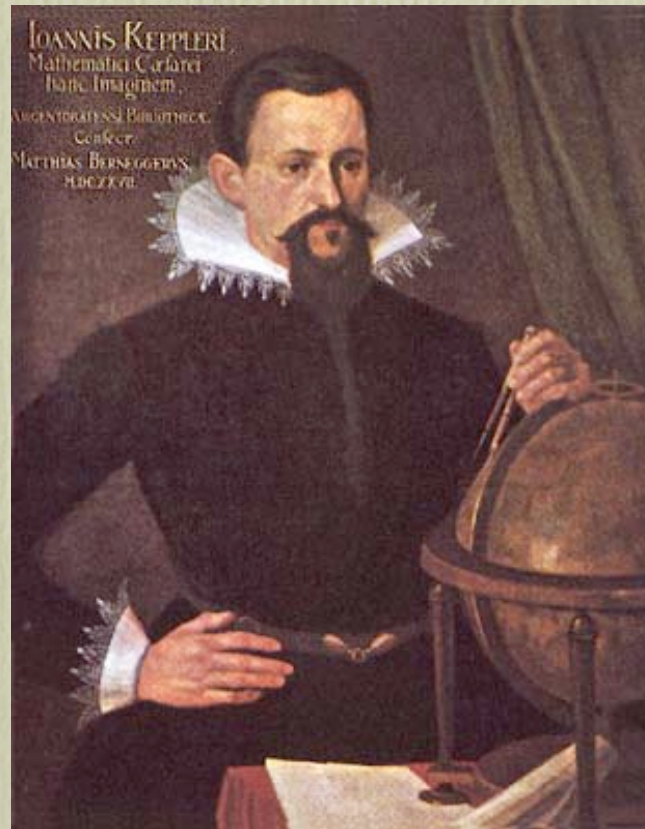
Xavier Viennot
CNRS, LaBRI

Des arbres dans les étoiles ?





Galileo Galilei
1564-1642



Johannes Kepler
1571 - 1630



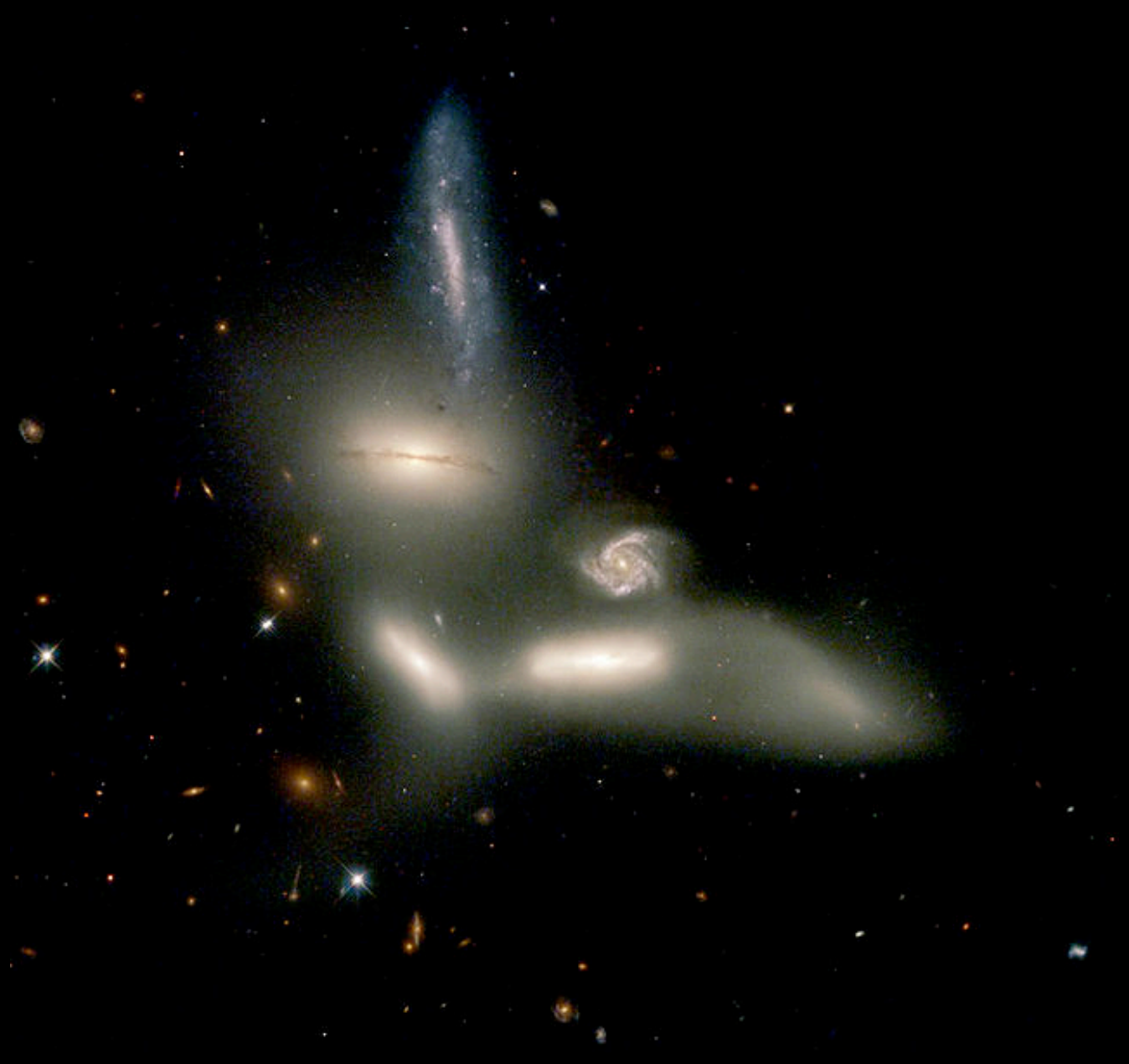
Isaac Newton
1643-1727



Théorie de la relativité restreinte générale

gravitation

Albert Einstein
1879-1955





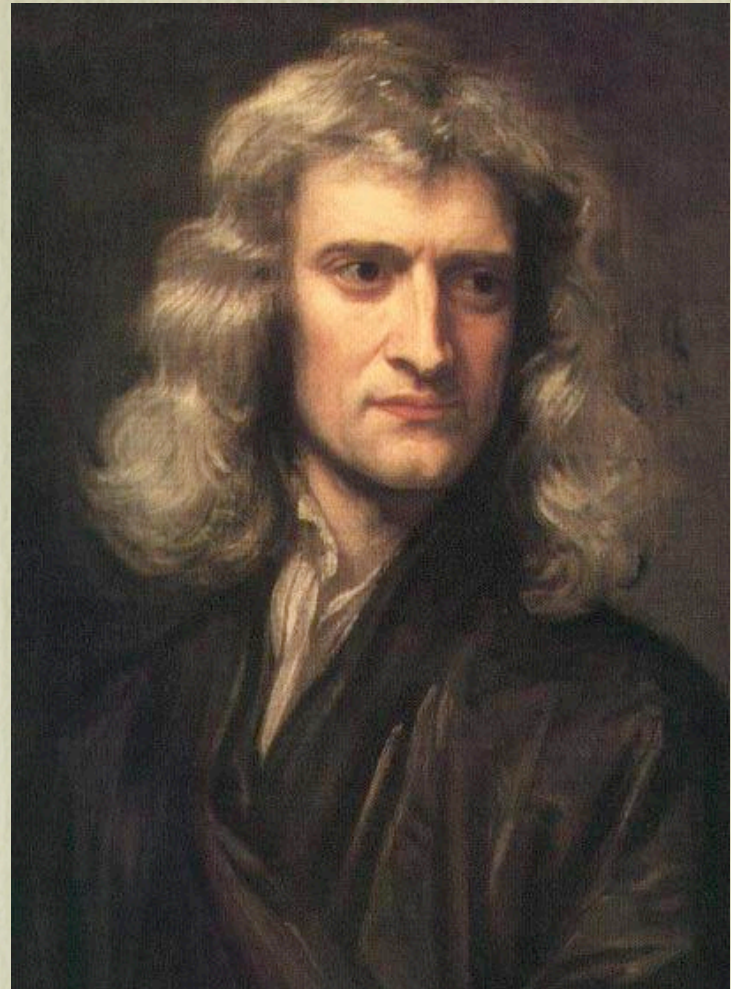
des arbres dans les
grains de lumière ?



collégiale Notre-Dame Vernon



Christian Huygens
1629-1695

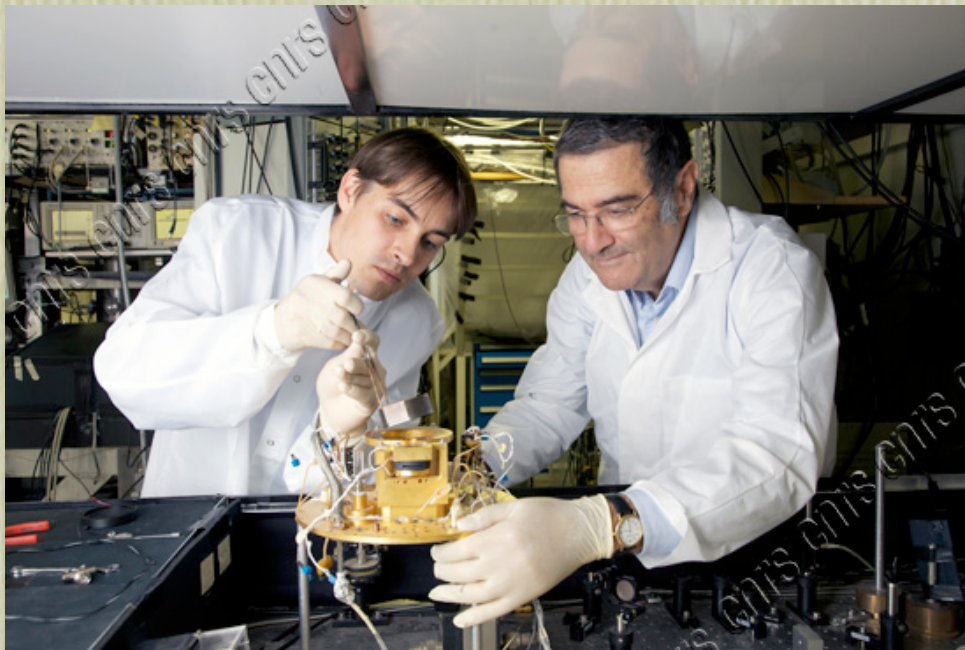


Isaac Newton
1643-1727



Daniel B. Holeman

le monde quantique



Serge Haroche
Nobel de physique 2012

Arbres de la nature,
des arbres partout





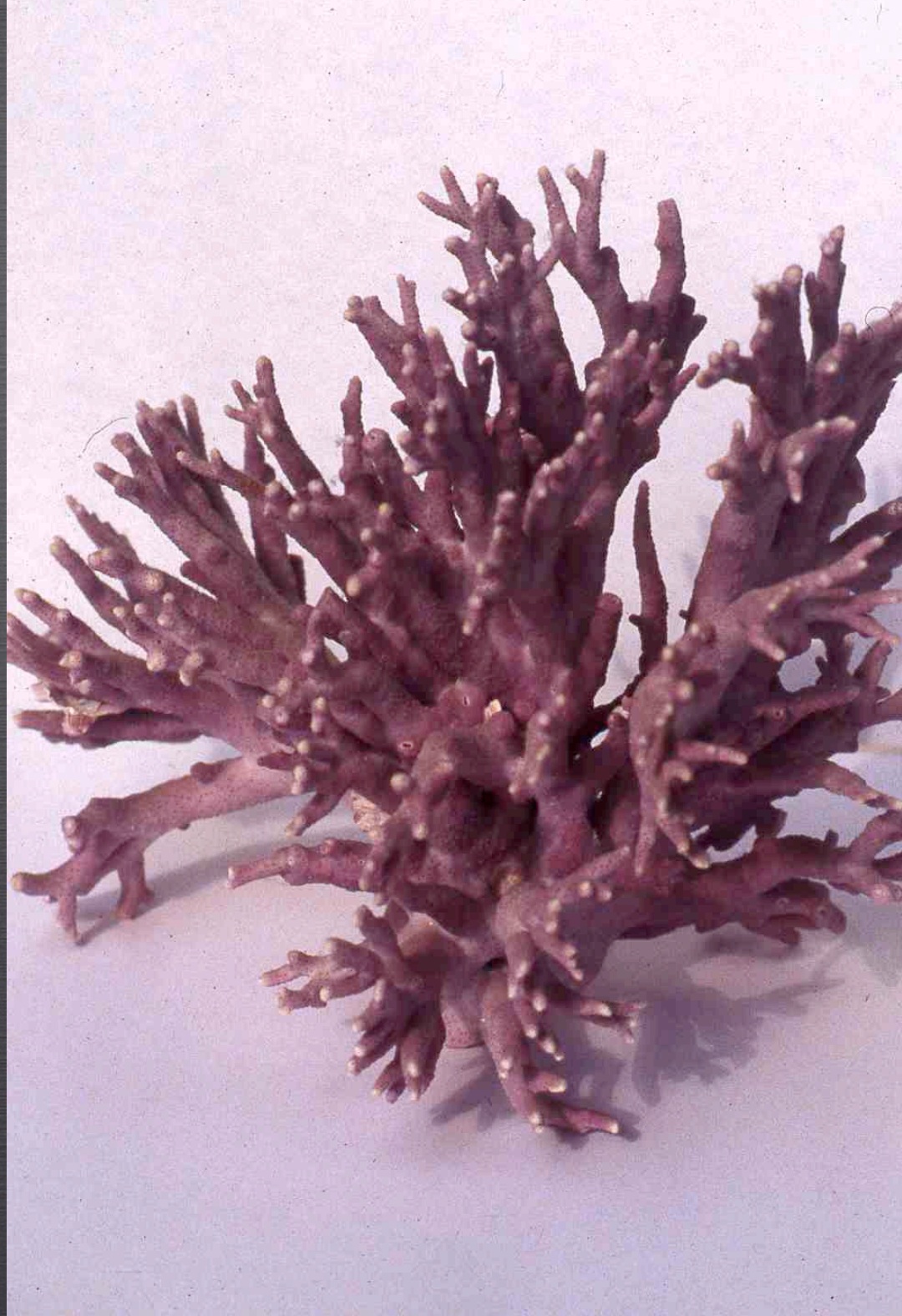












CORAIL



DECHARGE ELECTRIQUE



DEPOTS ELECTROLYTIQUES

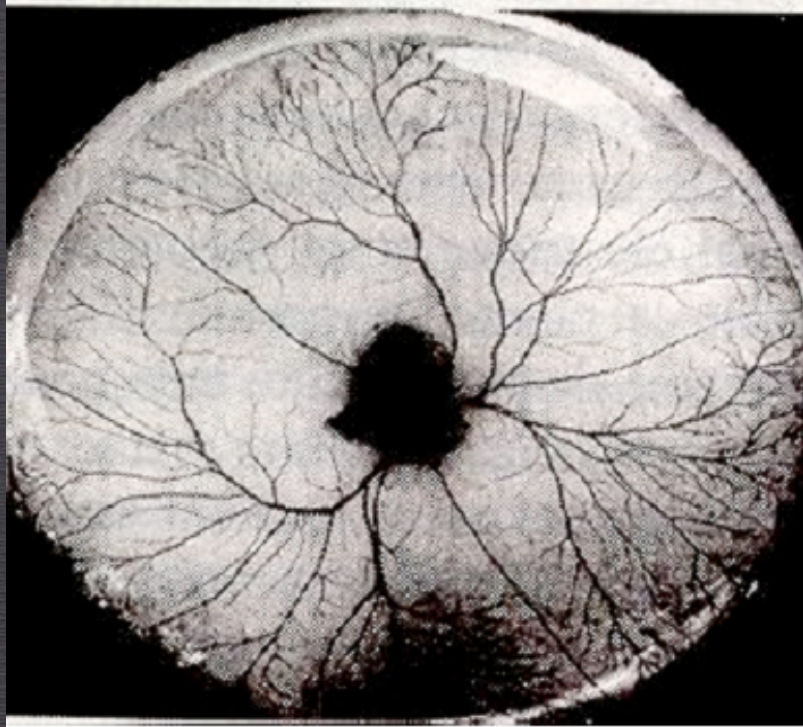
VINCENT FLEURY



DIGITATION VISQUEUSE

INJECTION D'HUILE ENTRE
DEUX PLAQUES

POUMON



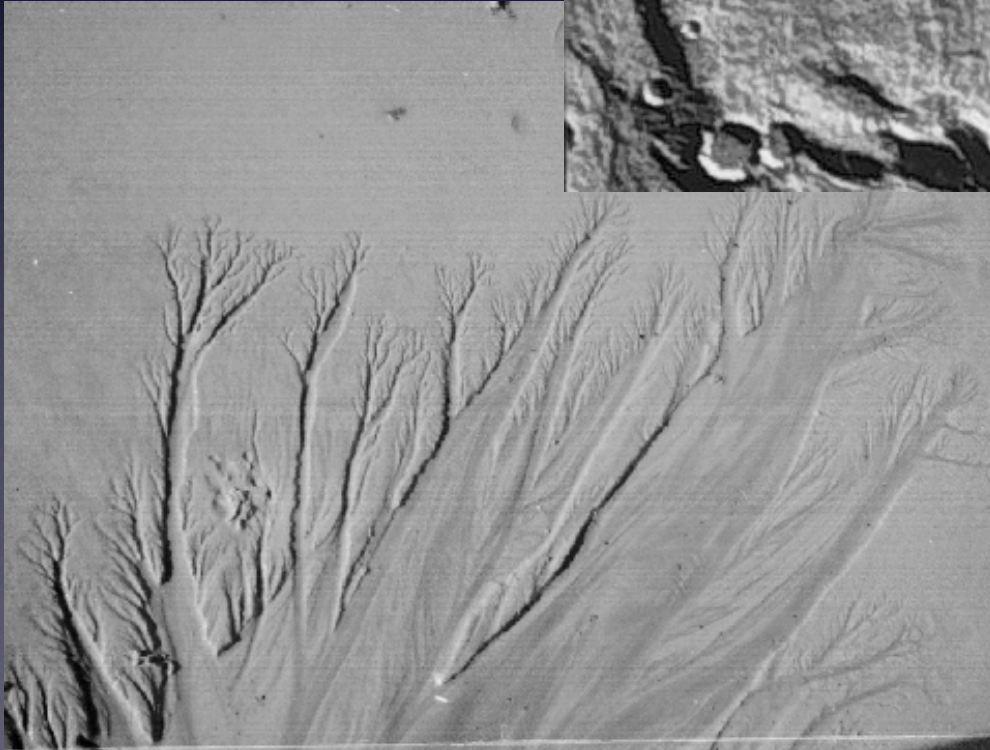
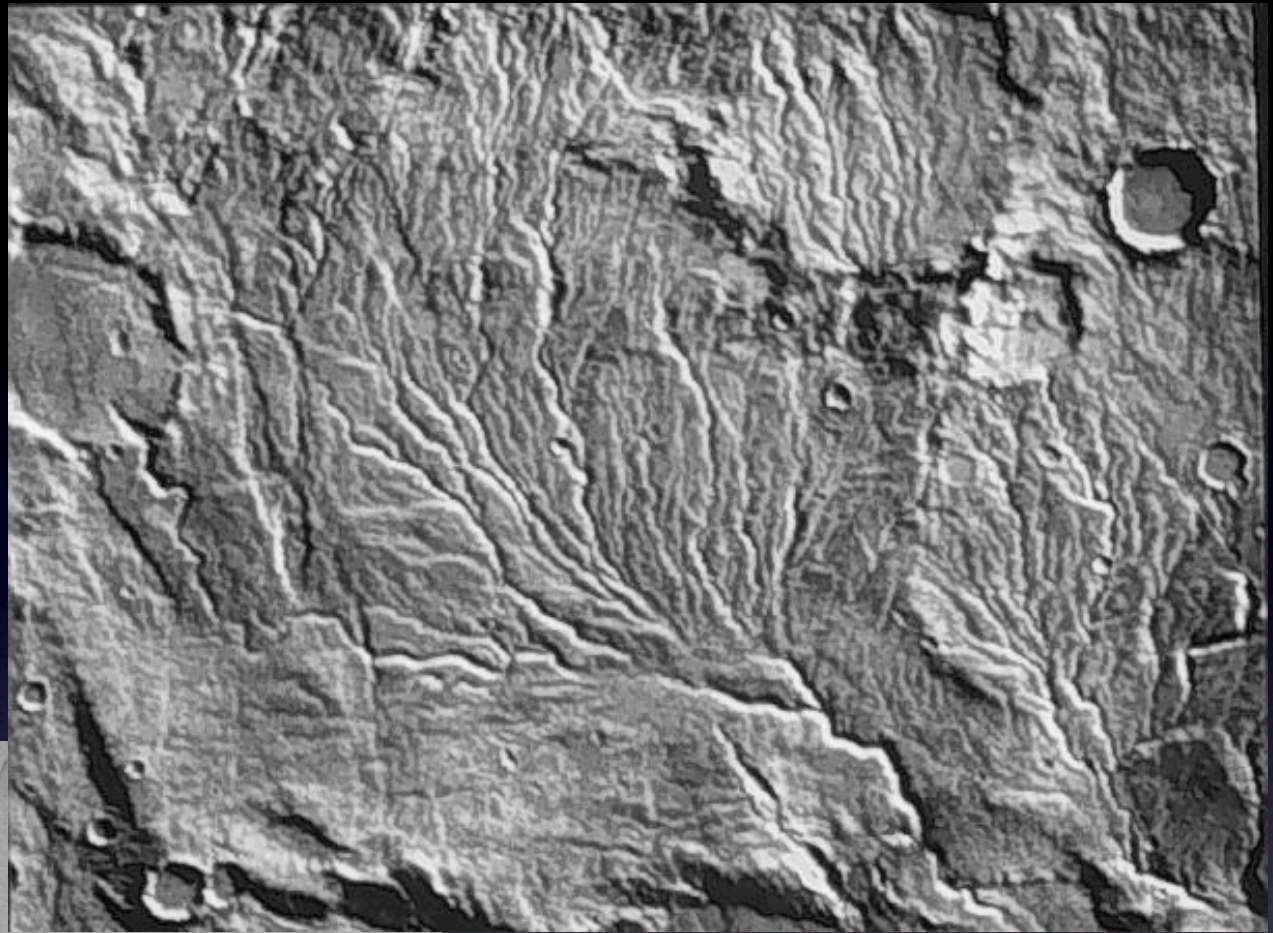
OEUF





NATIONAL GEOGRAPHIC

SUR MARS



**SUR TERRE
SUR UNE PLAGE**



DES ARBRES
DES ARBORESCENCES
PARTOUT

Des arbres naturels ...
aux arbres mathématiques

arbre botanique
structure ramifiée



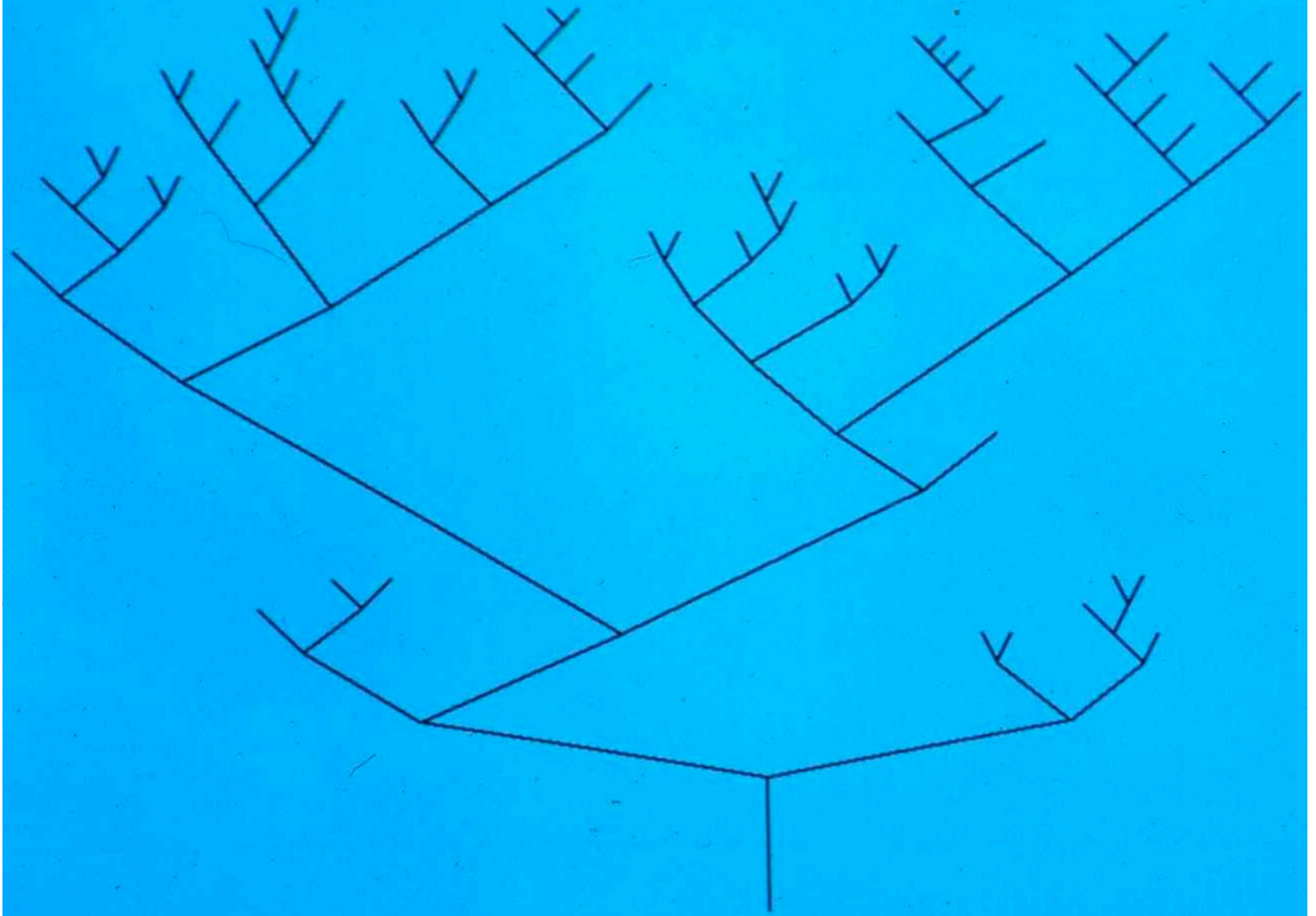
arbre binaire
combinatoire
(topologique)

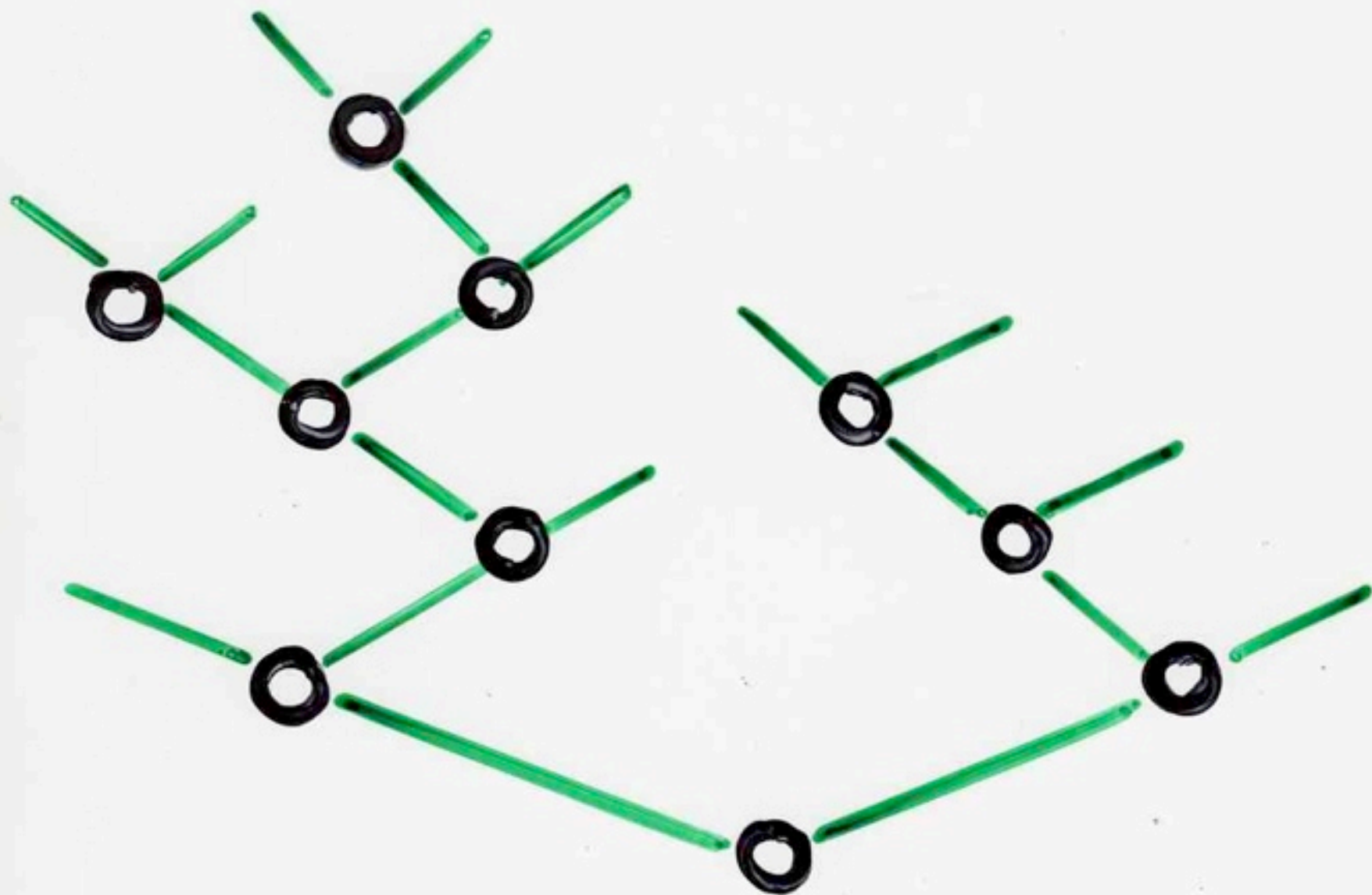




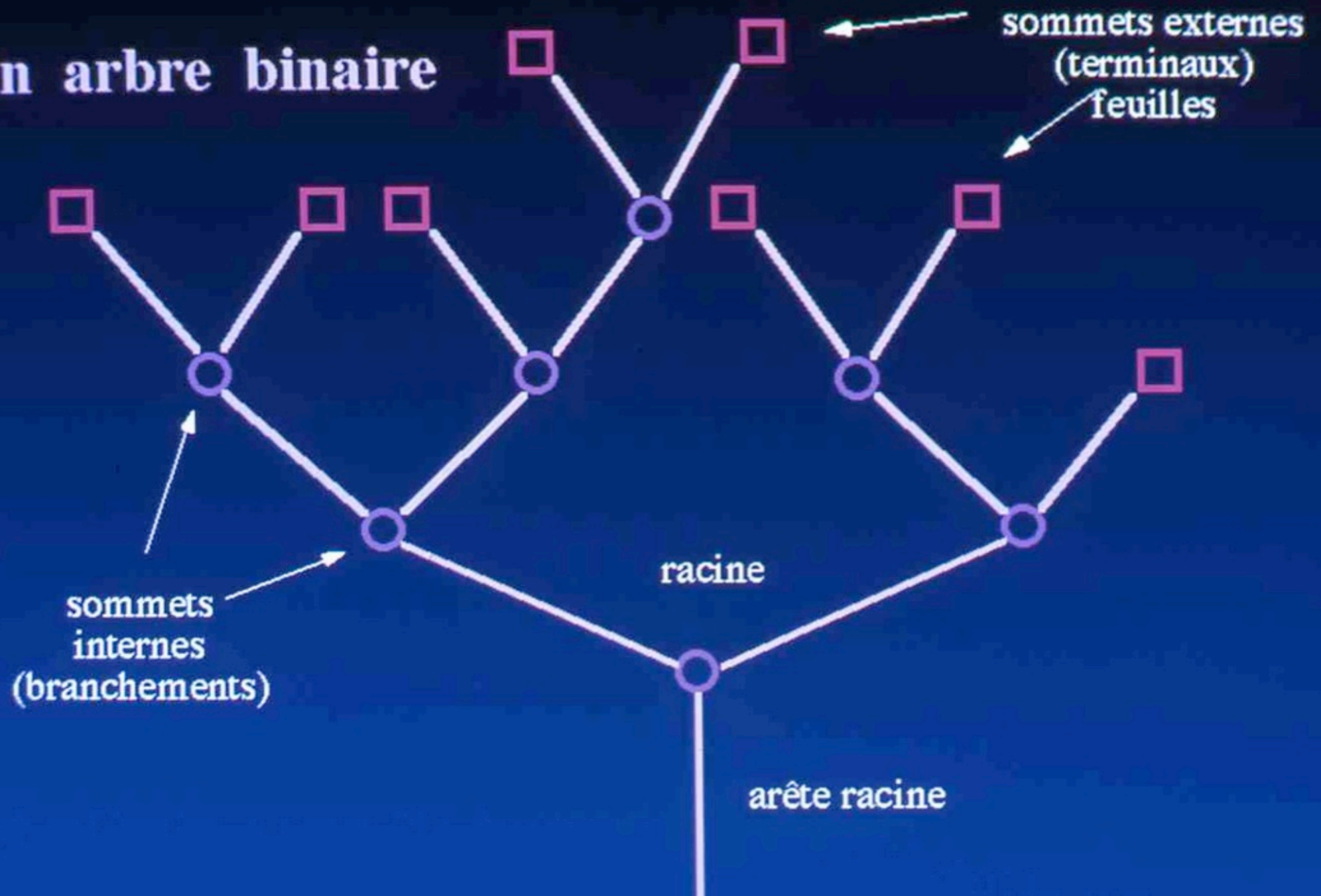


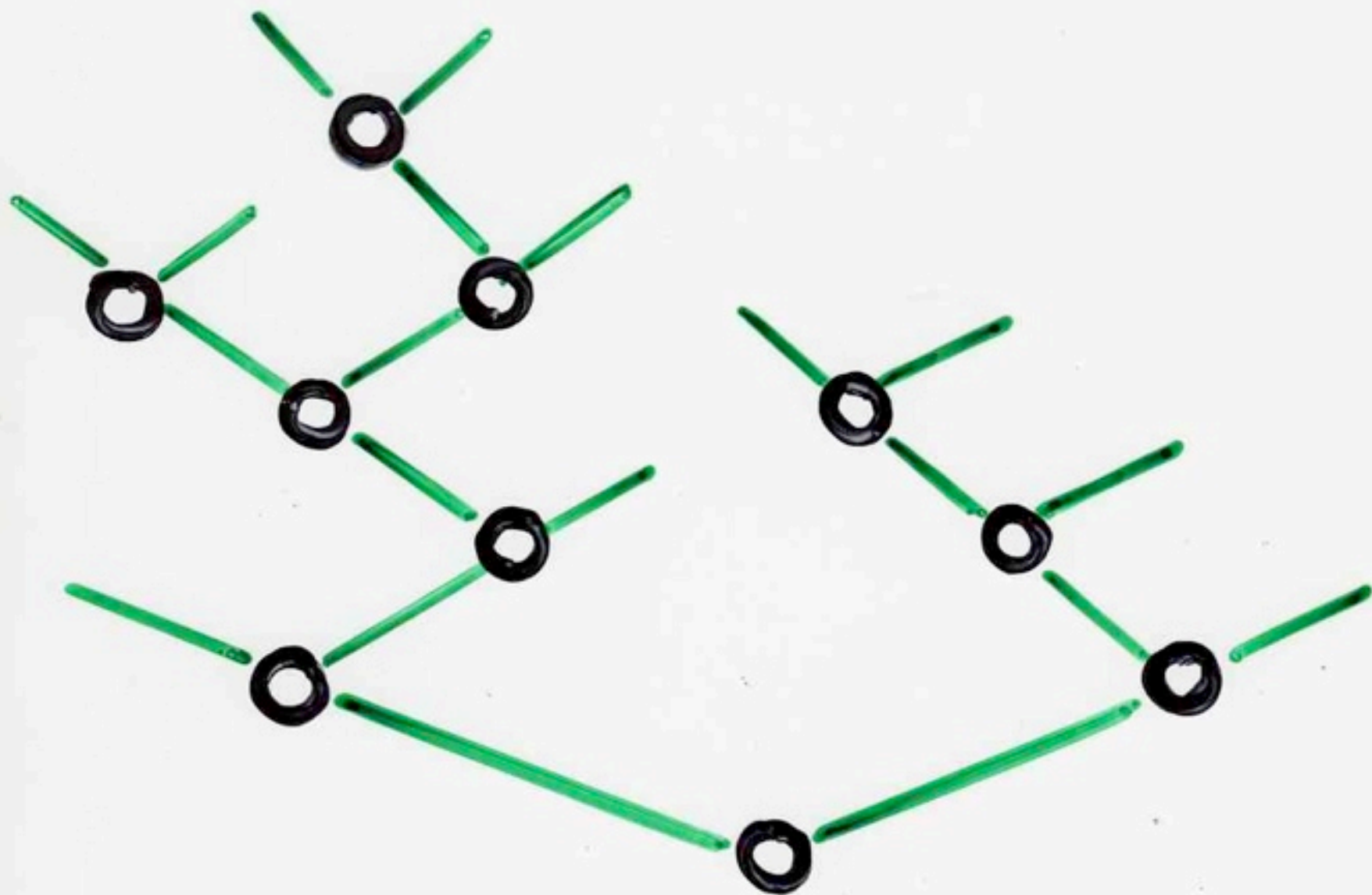


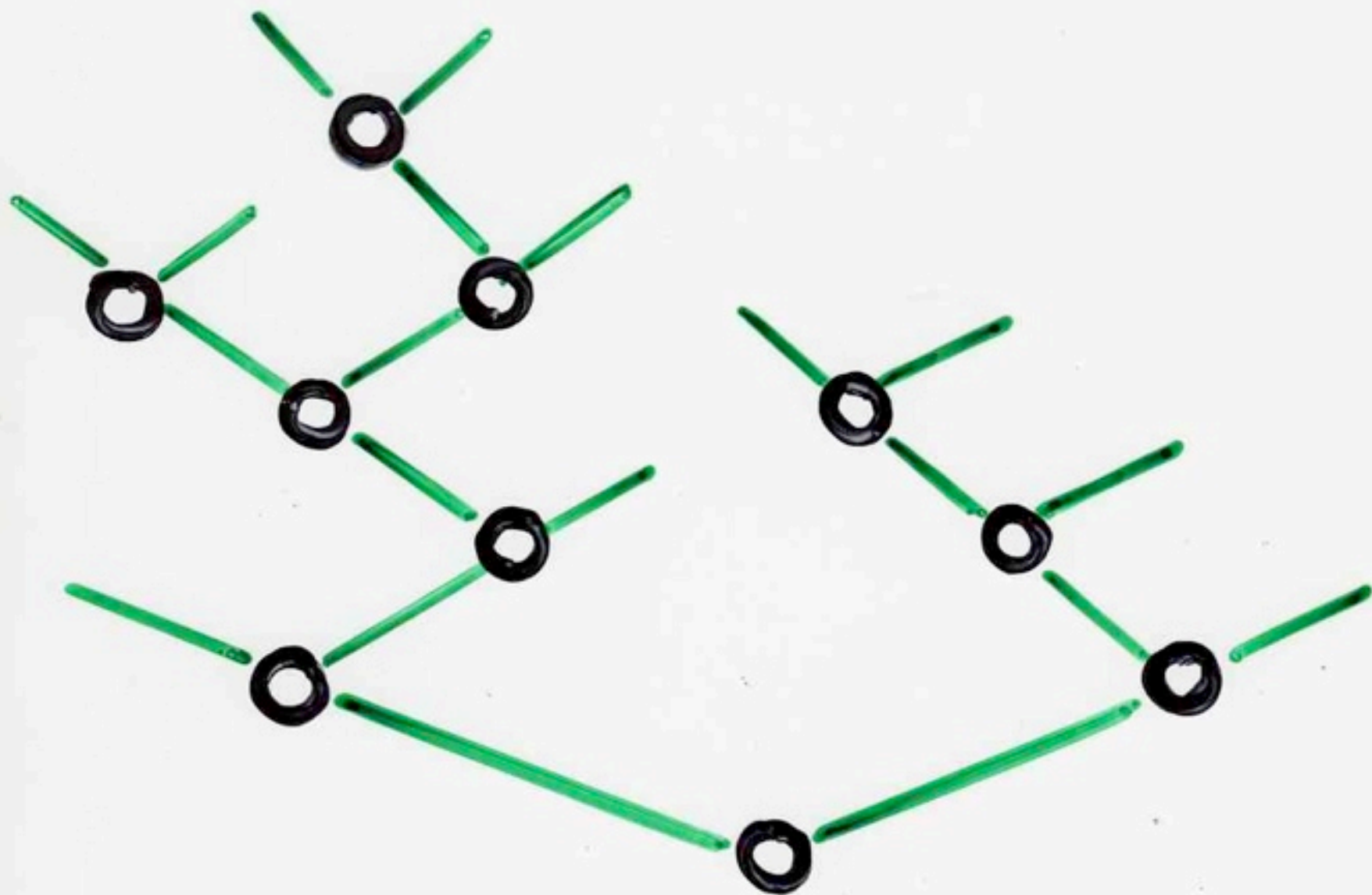


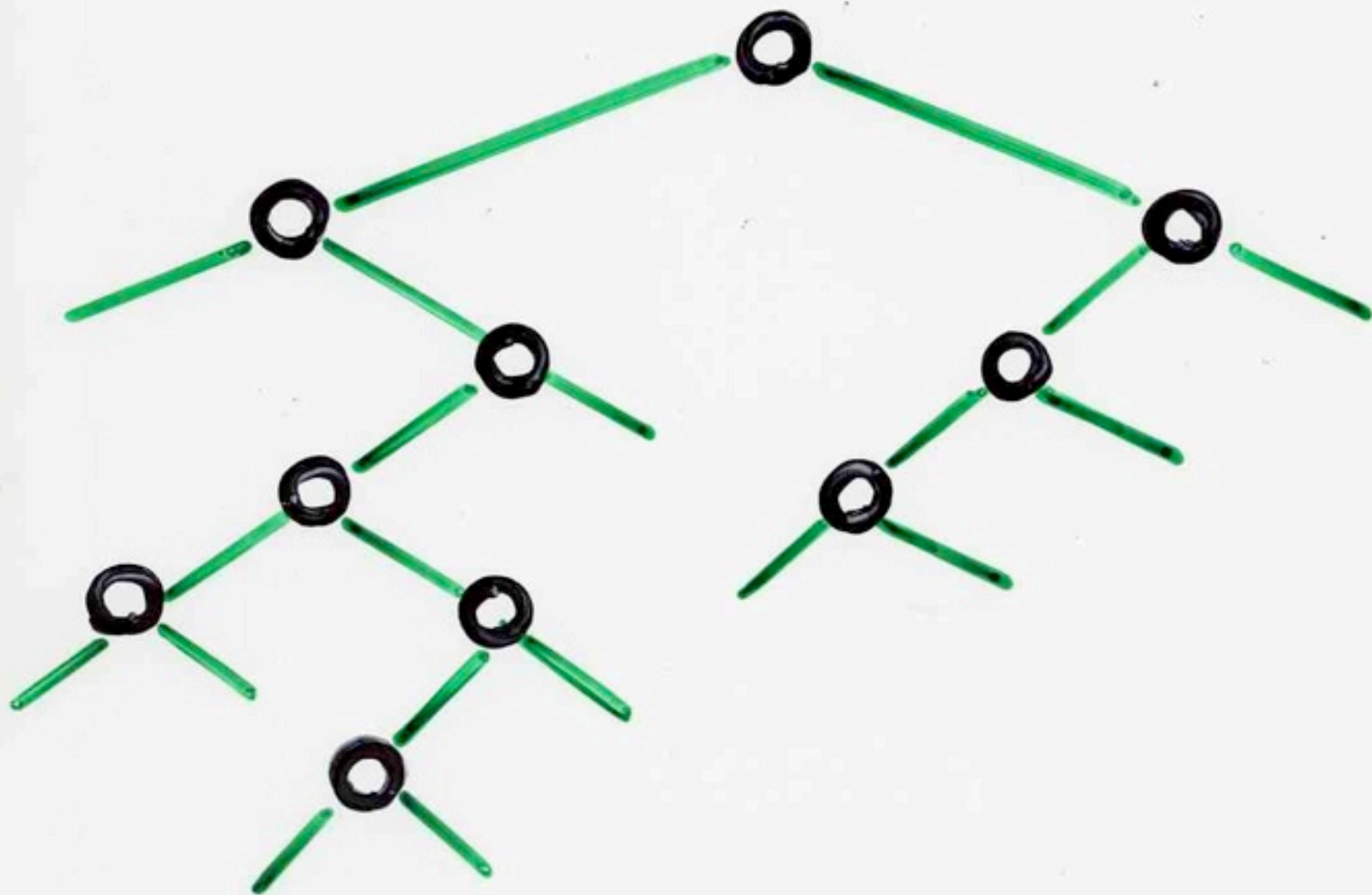


Un arbre binaire









Compter les arbres ...

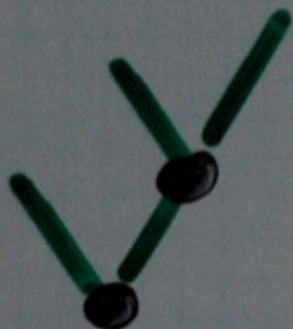
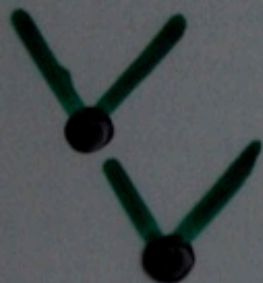
$C_n =$ nombre
d'arbres binaires
ayant n
sommets internes

(et donc $n+1$ feuilles)

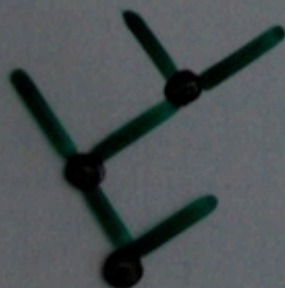
nombre de Catalan



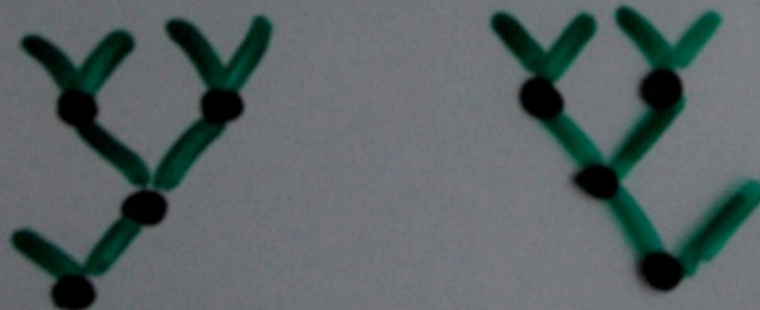
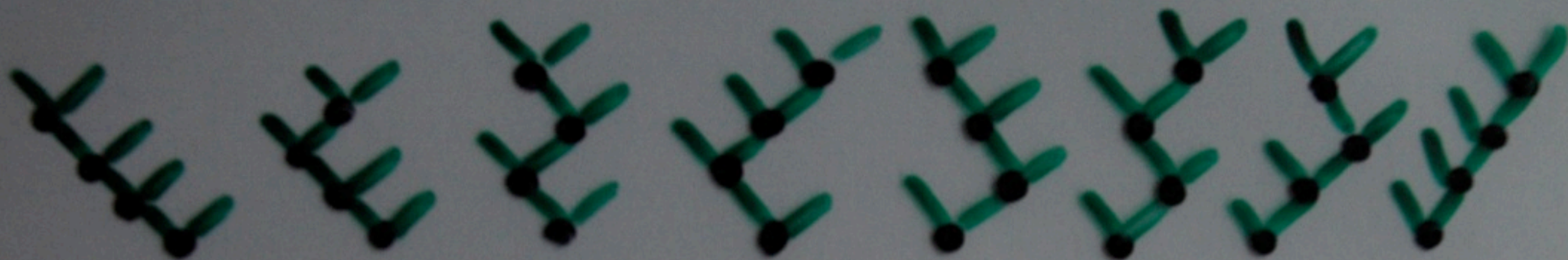
$$C_1 = 1$$



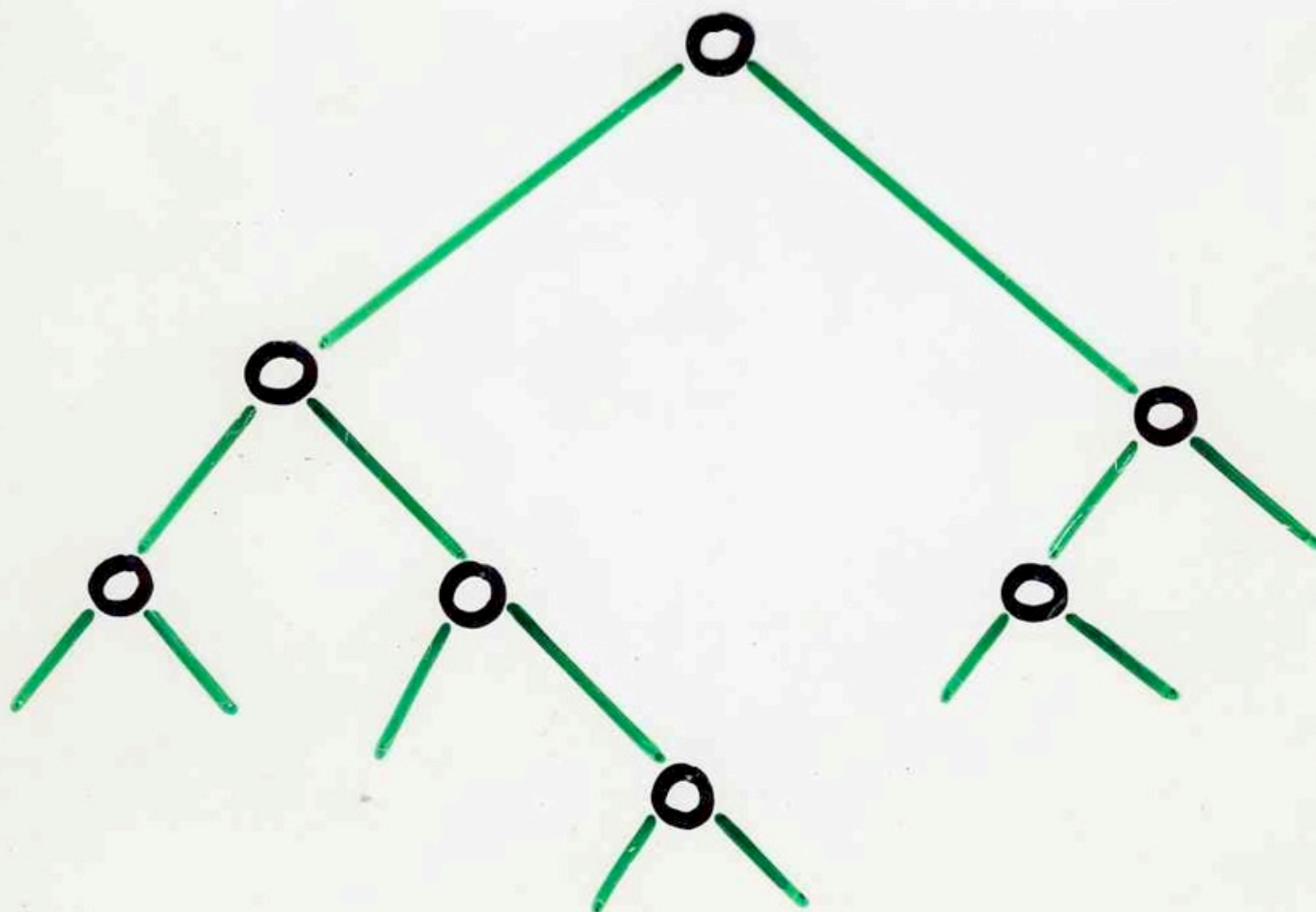
$$C_2 = 2$$



$$C_3 = 5$$



14



recurrence

$$C_{n+1} = \sum_{i+j=n} C_i C_j$$

$$C_0 = 1$$

1 1 2 5 14 42

nombres de Catalan



C_0 C_1 C_2 C_3 C_4 C_5
1, 1, 2, 5, 14, 42, ...

$$C_6 = C_0 C_5 + C_1 C_4 + C_2 C_3 + C_3 C_2 + C_4 C_1 + C_5 C_0$$

132 $1 \times 42 + 1 \times 14 + 2 \times 5 + 5 \times 2 + 14 \times 1 + 42 \times 1$

Une formule
pour les nombres de Catalan

Arbre

=



Arbre

Arbre

y

=

1

+

t

y

y

y

=

1

+

t (y)²

$$y = 1 + ty^2$$

equation algébrique

$$y = \frac{1 - (1 - 4t)^{1/2}}{2t}$$

$$(1+u)^m =$$

$$1 + \frac{m}{1!} u + \frac{m(m-1)}{2!} u^2 + \frac{m(m-1)(m-2)}{3!} u^3 +$$

+ ...

$$m = \frac{1}{2}$$

$$u = -4t$$

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

$$C_4 = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8}{1 \times 2 \times 3 \times 4 \times 1 \times 2 \times 3 \times 4 \times 5}$$

$$= 14$$

algèbre des polynômes

$$(1+t)^2 = 1 + 2t + t^2$$

$$(1+t)^3 = 1 + 3t + 3t^2 + t^3$$

$$1 + 1t + 2t^2 + 5t^3 + 14t^4 + 42t^5$$

polynôme

$$1 + 1t + 2t^2 + 5t^3 + 14t^4 + 42t^5$$

+ ...

série génératrice
des nombres C_n

série formelle

$$y = 1 + 1t + 2t^2 + 5t^3 + 14t^4 + \dots + C_n t^n + \dots$$

série génératrice des
nombres de Catalan C_n

$$t + t + t + \dots t + \dots$$

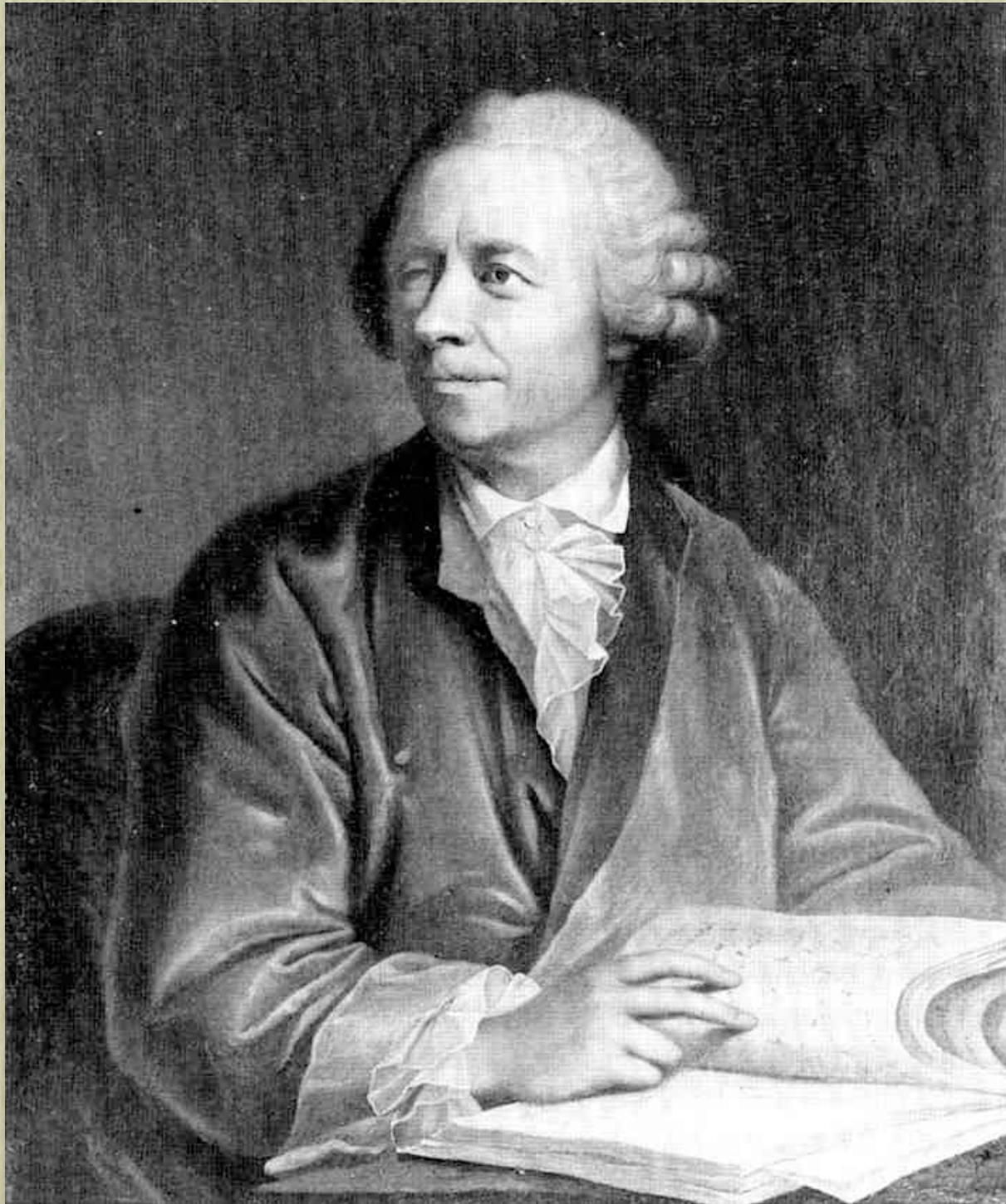
$$1 + 1 + 1 + \dots$$

~~$$t + t + t + \dots + t + \dots$$~~

~~$$1 + 1 + 1 + \dots$$~~

Une lettre oubliée d'Euler
à Christian Goldbach

Berlin, 4 Septembre 1751



Leonhard
Euler
49 ans



Leonh. Euler 300

300. Geburtstag

300^{ème} anniversaire

300^º anniversario

300th anniversary



entre los años 5 y 10
Londres. Las
matrices quedat

11. $(a+b-1)^2 + (a+c-1)^2 + (b+c-1)^2 + 1$
 Data resoluta.

$$I, a+b+c+d = 2 \cdot 10^4$$

$$aa+bb+cc+dd = (a+b-1)^2 + (a+c-1)^2 + (b+c-1)^2 + 1$$

$aa+bb+cc+dd = (a+b-1)^2 + (a+c-1)^2 + (b+c-1)^2$
 Solit. $aa+bb+cc+dd-1$ in 3 quadrata resolubet.

$8n+3$ is a quadratic residue

$$8n+3 = (a+b-1)^2 + (a+c-1)^2 + (b+c-1)^2$$

Successive Integers. Let $a+b+c = 3$

$$8n+3 = (a+b-1)^2 + (a+c-1)^2 + (b+c-1)^2$$

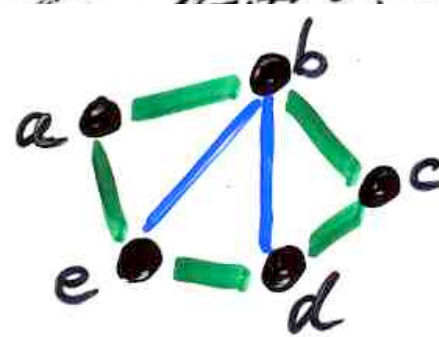
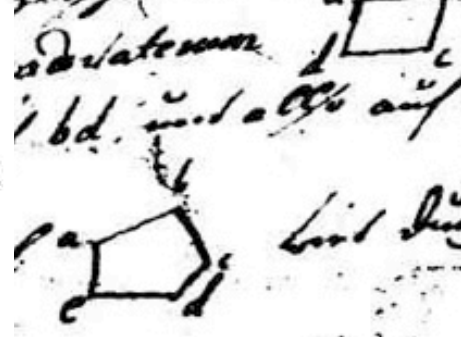
$8n+3 = (a+b-1) + (a+c-1)$
 $8n+3 = 2a+b+c-2$ Dargestellt als Summe
 $8n+3 = 2a+b+c-2$ von Theoremen (bisher) der Typen 8k+1
 $8n+3 = 2a+b+c-2$ und 8k+3.

$8n+3 = (a+b-1) + (a+c-1)$
 $8n+4 = a+b+c+d$ Ingesamt Las 1. Teil
 Das ist die 1. von Theorema 1. Teil
 Theorema 1. Teil

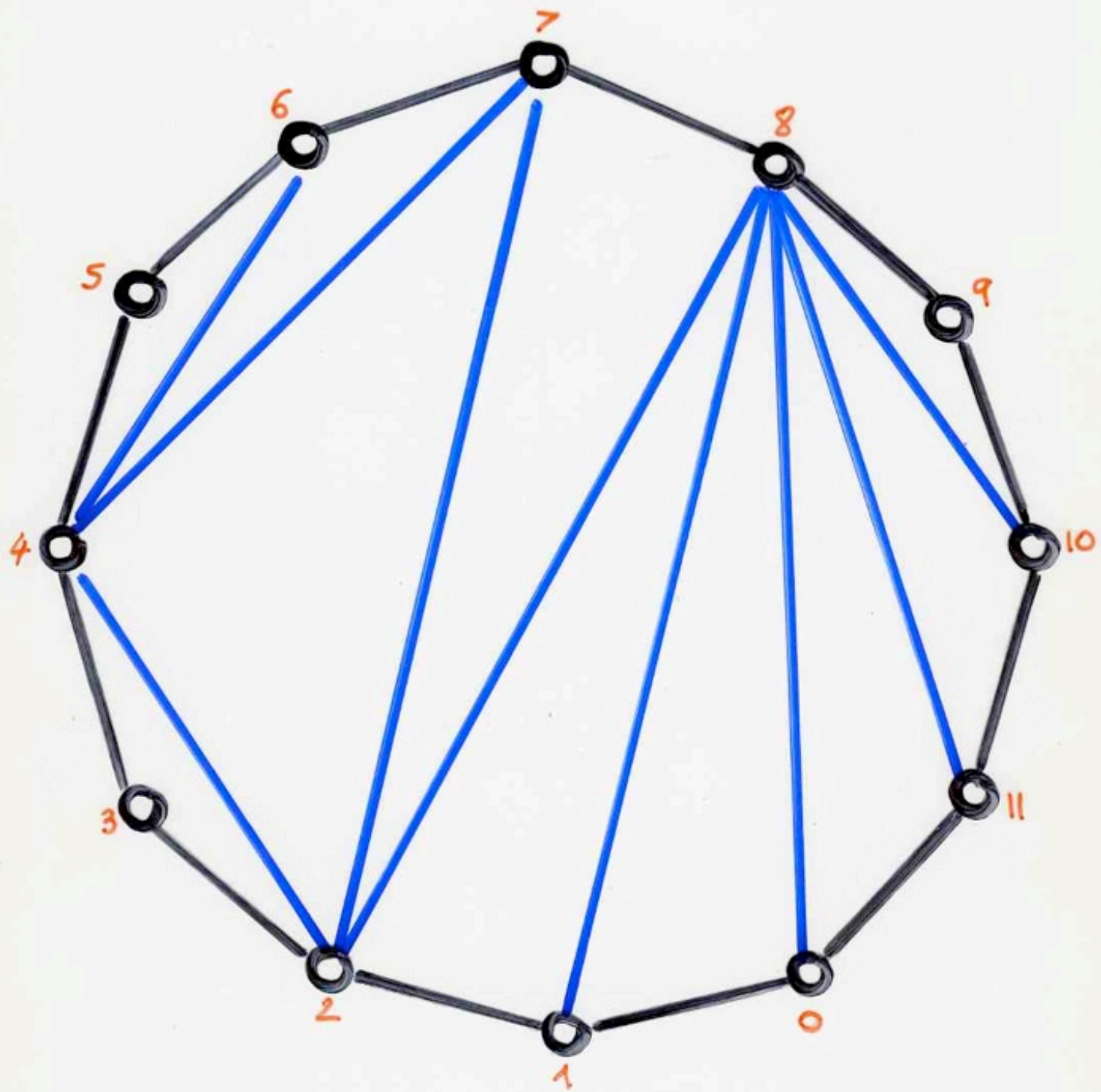
and Iron Phosphate Hematone fuge
Hemoglobin & fullness, looks

Geht man nun auf eine andere Seite, so erhält man ein
andere, nämlich die Seiten. Die Seiten sind die
Seiten des Polygons. Die Seiten sind die

policy taken was given
to the President then Lincoln's time.

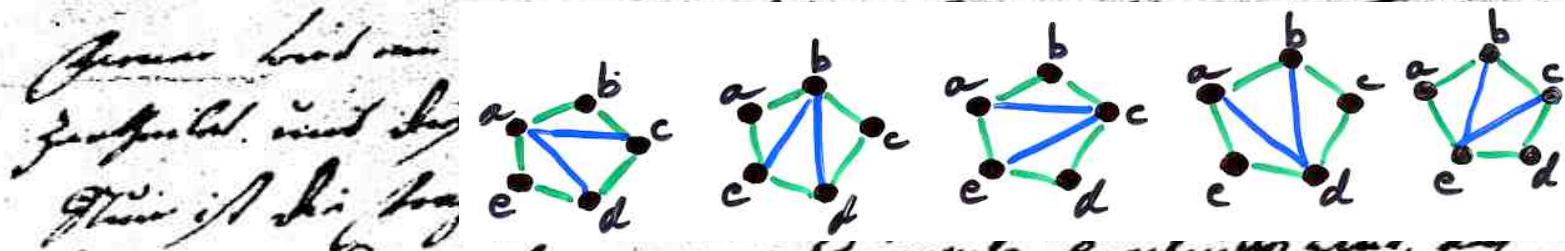


Des Diagonales
Triangula, etc.



grasping the answer
Euler

Geht, und steht hier auf 8 nicht liegendem haben gegeben, nämlich
 fünf der Diagonale: I. $\frac{a}{d}$; II. $\frac{b}{e}$; III. $\frac{c}{f}$; IV. $\frac{d}{a}$; V. $\frac{e}{b}$



Hier ist die Folge $n-3$ Diagonale in $n-2$ Triangula gegeben, man
 bei beliebig liegendem Daten, selbst gegeben kann.
 Aufgibt man die Aufgabe dieser liegendem Daten = x

so sieht man

Wenn $n = 1, 2, 5, 14, 42, 132, 429, 1430, \dots$

ist $x = 1, 2, 5, 14, 42, 132, 429, 1430, \dots$

Hieraus sieht man den Zusammenhang. In generaliter
 ist

$$x = \frac{2 \cdot 6 \cdot 10 \cdot 14 \cdot 18 \cdot 22 \cdot \dots \cdot (4n-10)}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot \dots \cdot (n-1)} = \frac{(2n)!}{(n+1)!n!}$$

$$5 = 2 \cdot \frac{10}{4}, 14 = 5 \cdot \frac{14}{3}, 42 = 14 \cdot \frac{18}{6}, 132 = 42 \cdot \frac{22}{7}$$

so sieht man $C_n = \frac{1}{n+1} \binom{2n}{n}$ und $n! = 1 \times 2 \times 3 \times \dots \times n$

vermuthet, dass die Reihe ist unendl. Ist die Reihe unendl. so ist die Summe die Proportion

$$\frac{1 - 2a - \sqrt{1 - 4a^2}}{2a^2}$$

$$1 + 2a + 5a^2 + 14a^3 + 42a^4 + 132a^5 + \text{etc}$$

gemessen. Ist

$$1 + 2a + 5a^2 + 14a^3 + 42a^4 + 132a^5 + \text{etc} = \frac{1 - 2a - \sqrt{1 - 4a^2}}{2a^2}$$

$$\text{alle. wenn } a = \frac{1}{4} \text{ ist } 1 + \frac{2}{4} + \frac{5}{16} + \frac{14}{64} + \frac{42}{256} + \text{etc} = 1$$

Die hier erwähnte Reihe ist für die Bestimmung der unendlichen Reihe der Catalan'schen Zahlen, welche die Anzahl der verschiedenen Arten, eine Parabel in n Theile zu zerlegen, darstellt. Die Summe dieser Reihe ist $a = \frac{1}{4}$.

von Joseph Euler

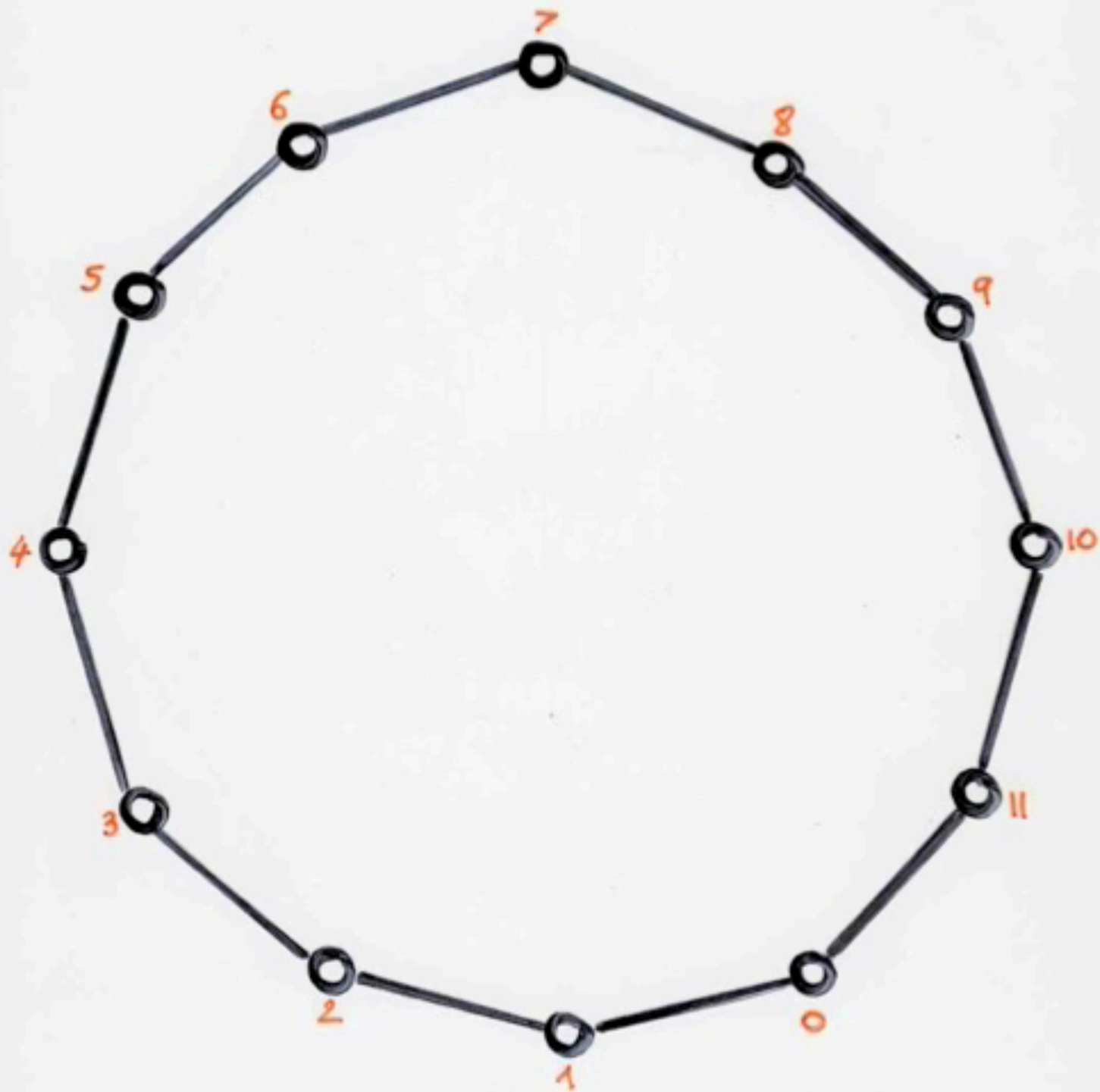
Wien 24^{te} Sept
1751.

4 Sept 1751
Berlin

geforschtes
Euler

$$2(2n+1)C_n = (n+2)C_{n+1}$$

des triangulations
aux arbres binaires



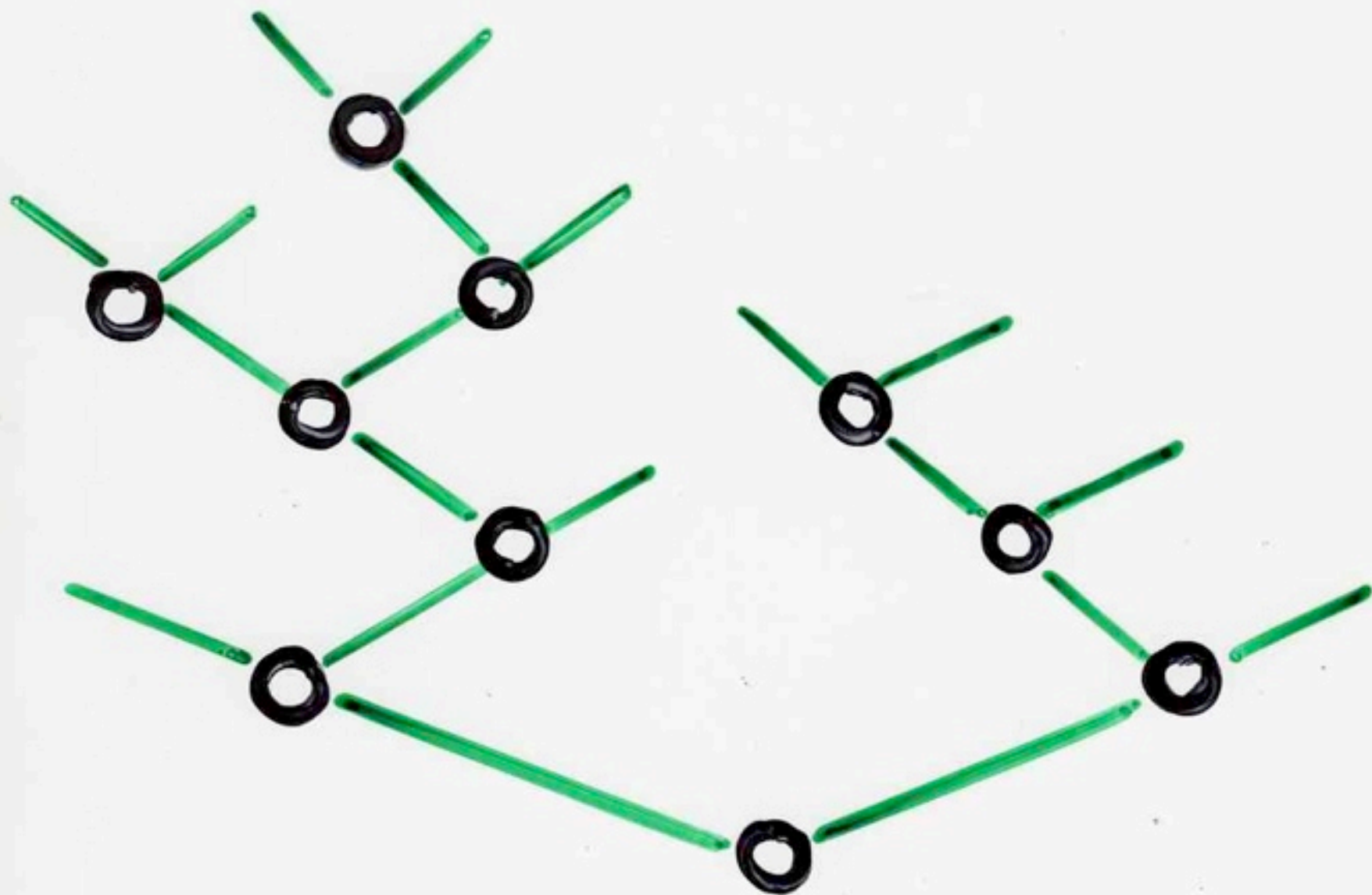
Vous venez de voir
une bijection combinatoire

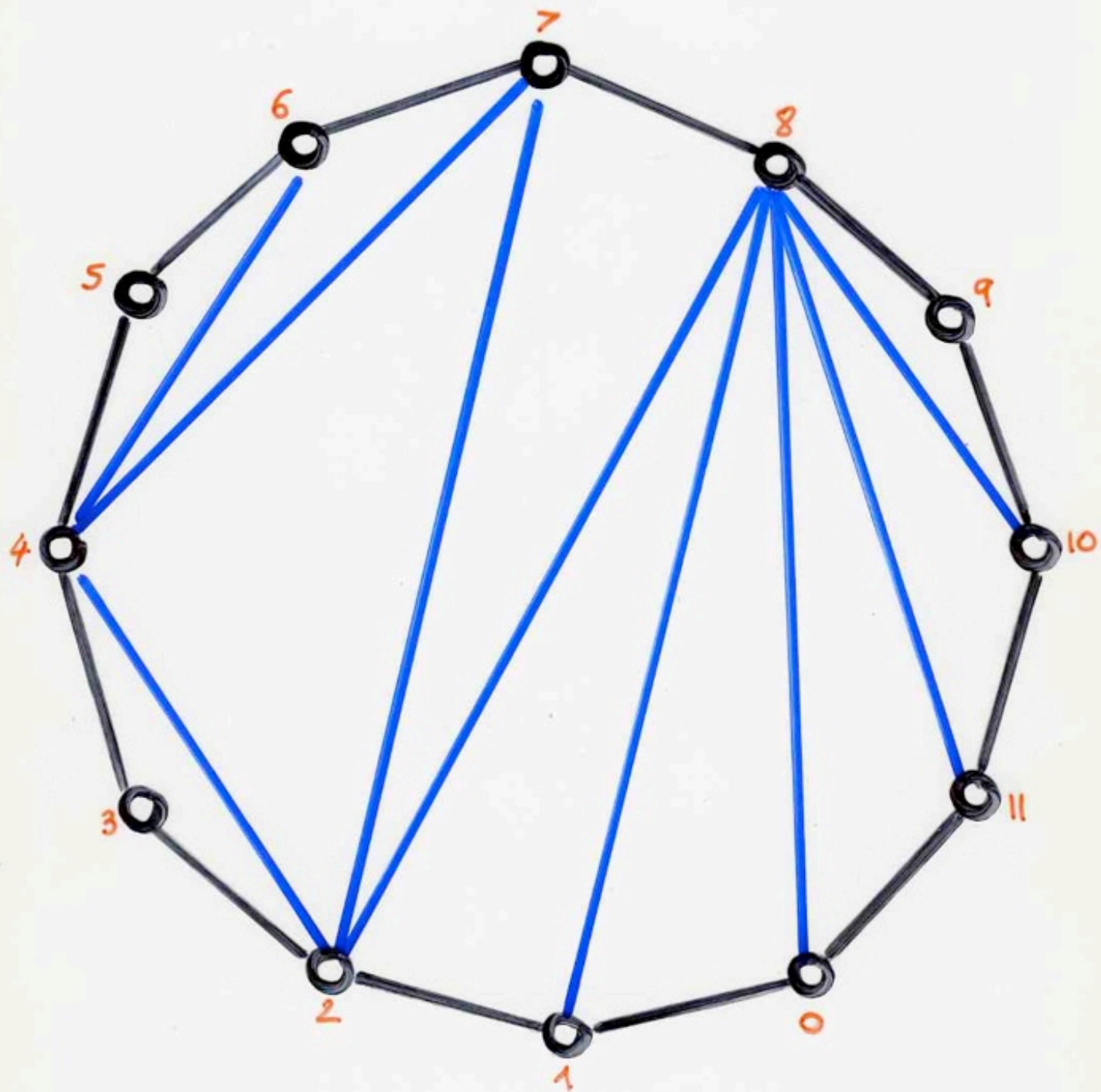
Une triangulation d'un polygone
régulier à $10+2$ côtés est
devenue un arbre binaire ayant
 10 branchements (ou "sommets internes")

Inversement, la donnée de cet arbre
binaire permet de retrouver la
triangulation de départ.

Cette construction permet de démontrer
que le nombre de triangulations
d'un polygone régulier à $n+2$ côtés
est le même que le nombre
d'arbres linéaires ayant n
branchements.

non seulement démontrer ...
mais plus... comprendre pourquoi

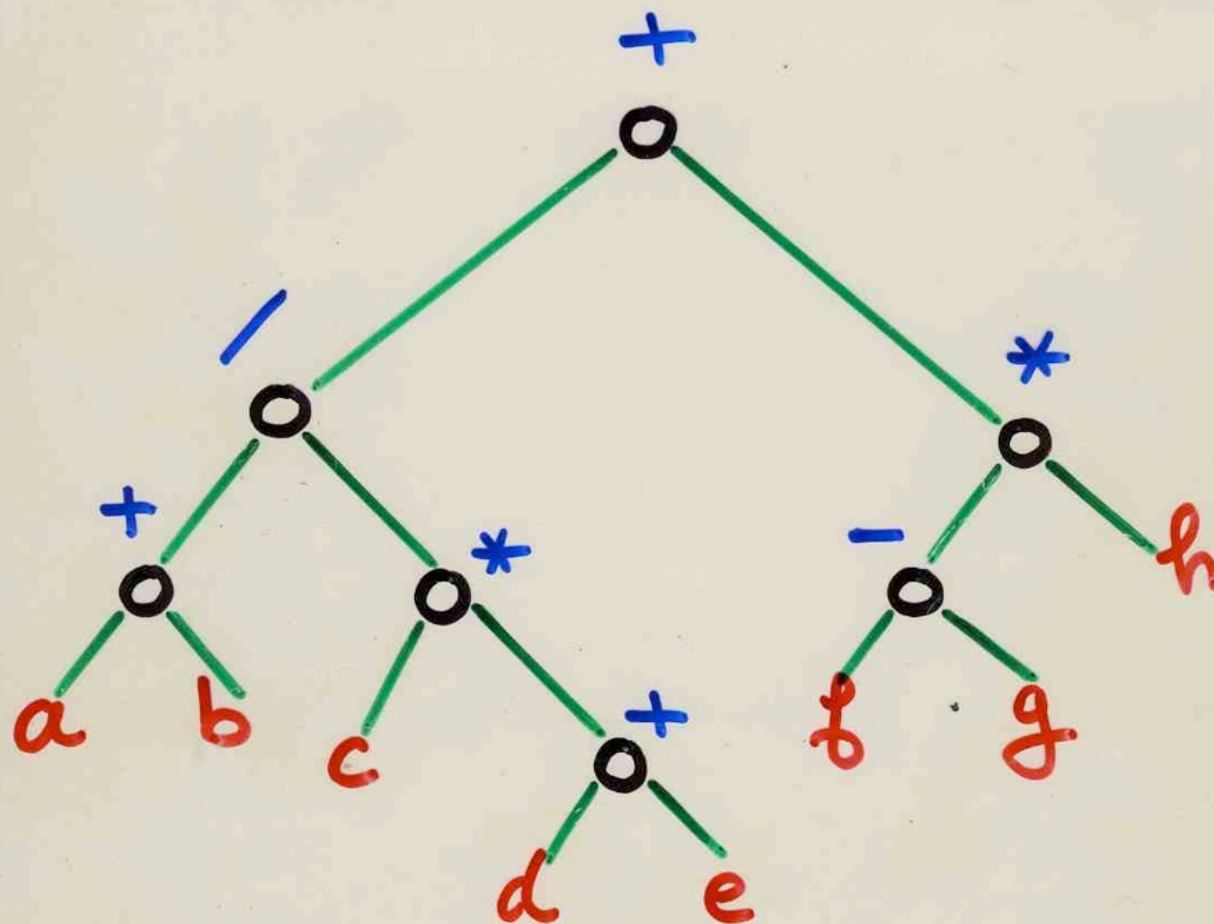




des arbres dans les ordinateurs

calcul d'une expression arithmétique

$$\frac{(a+b)}{c(d+e)} + (f-g)h$$



$$\frac{(a+b)}{c(d+e)} + (f-g)h$$

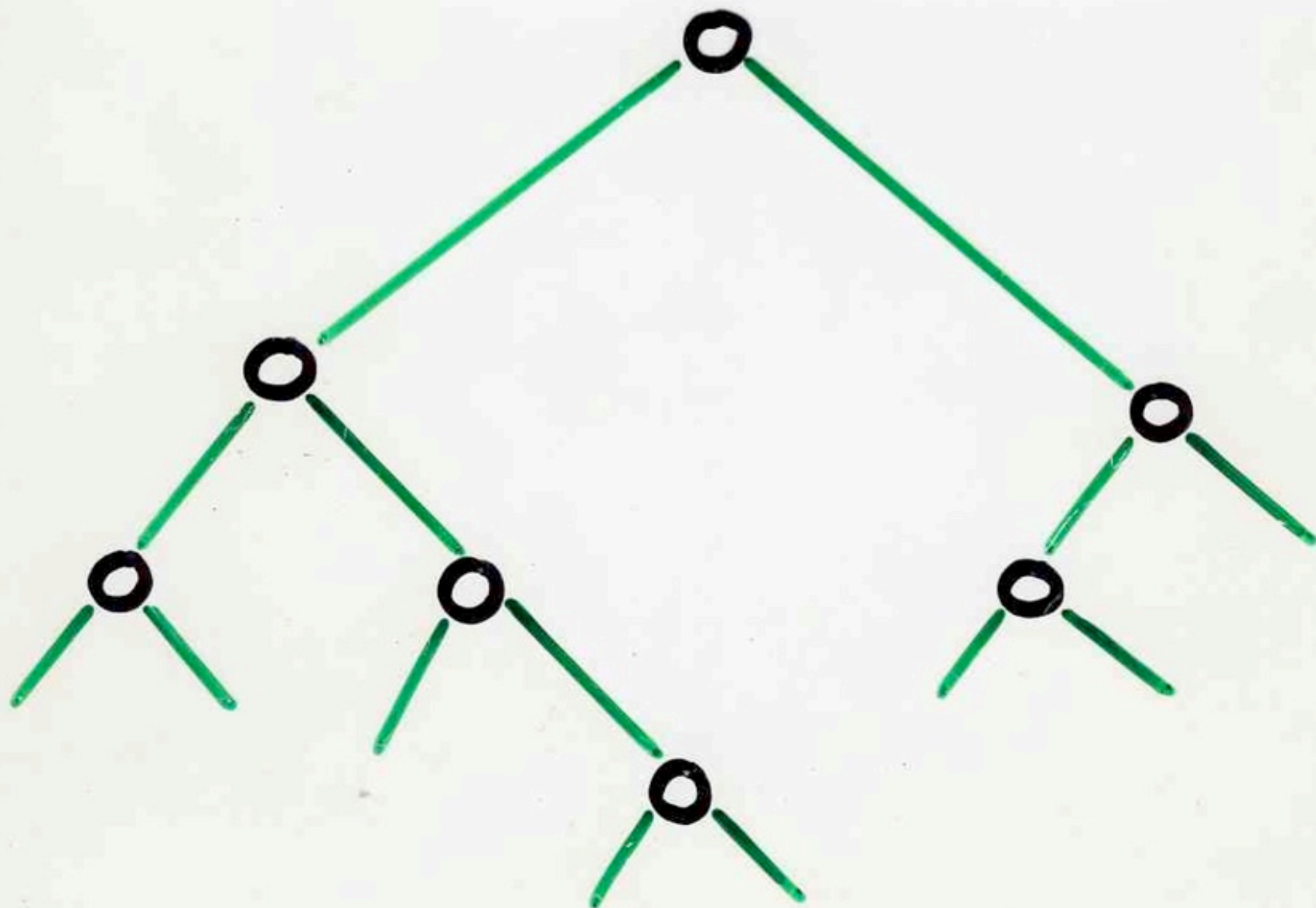
minimum number

of

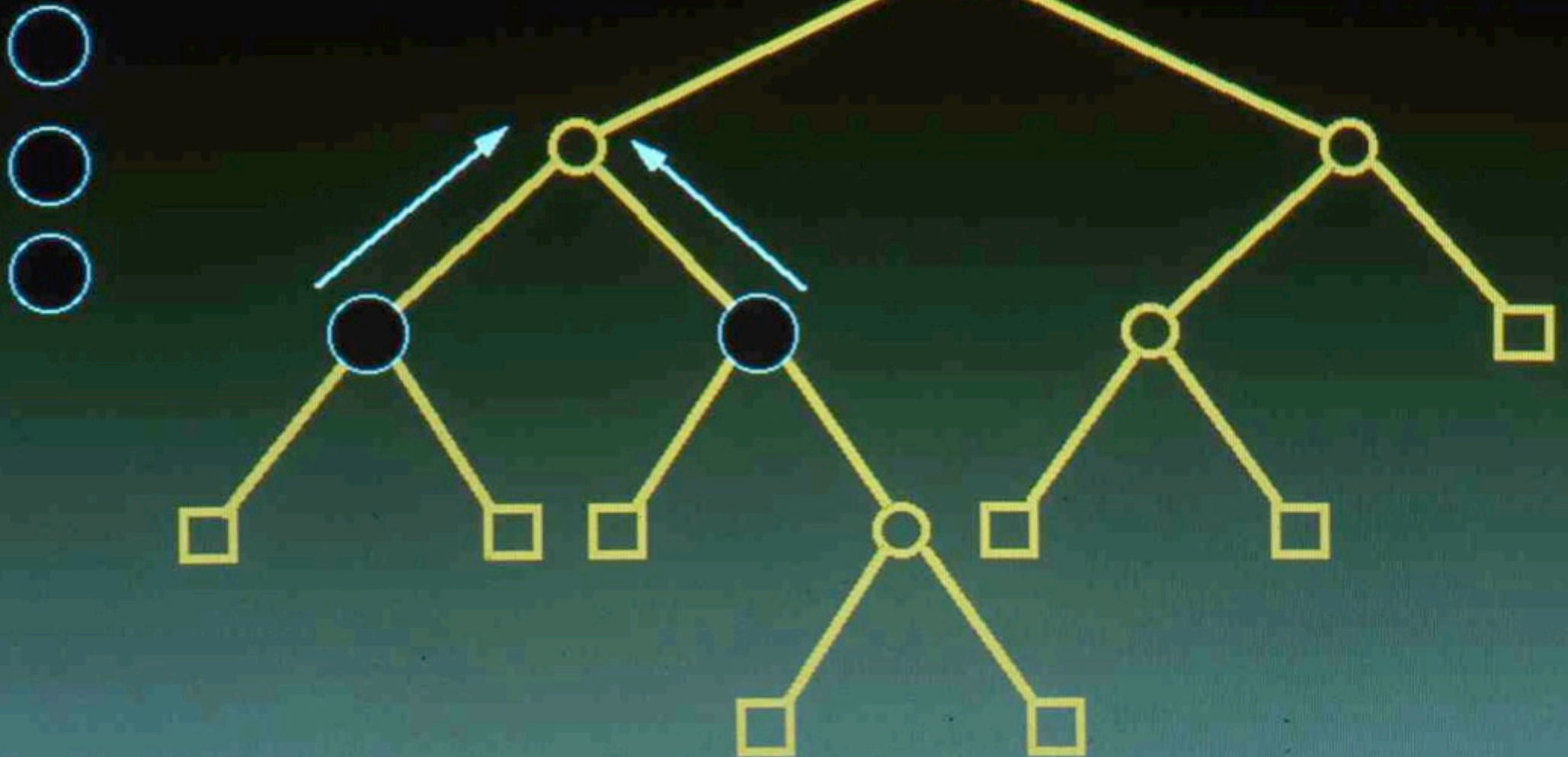
registers

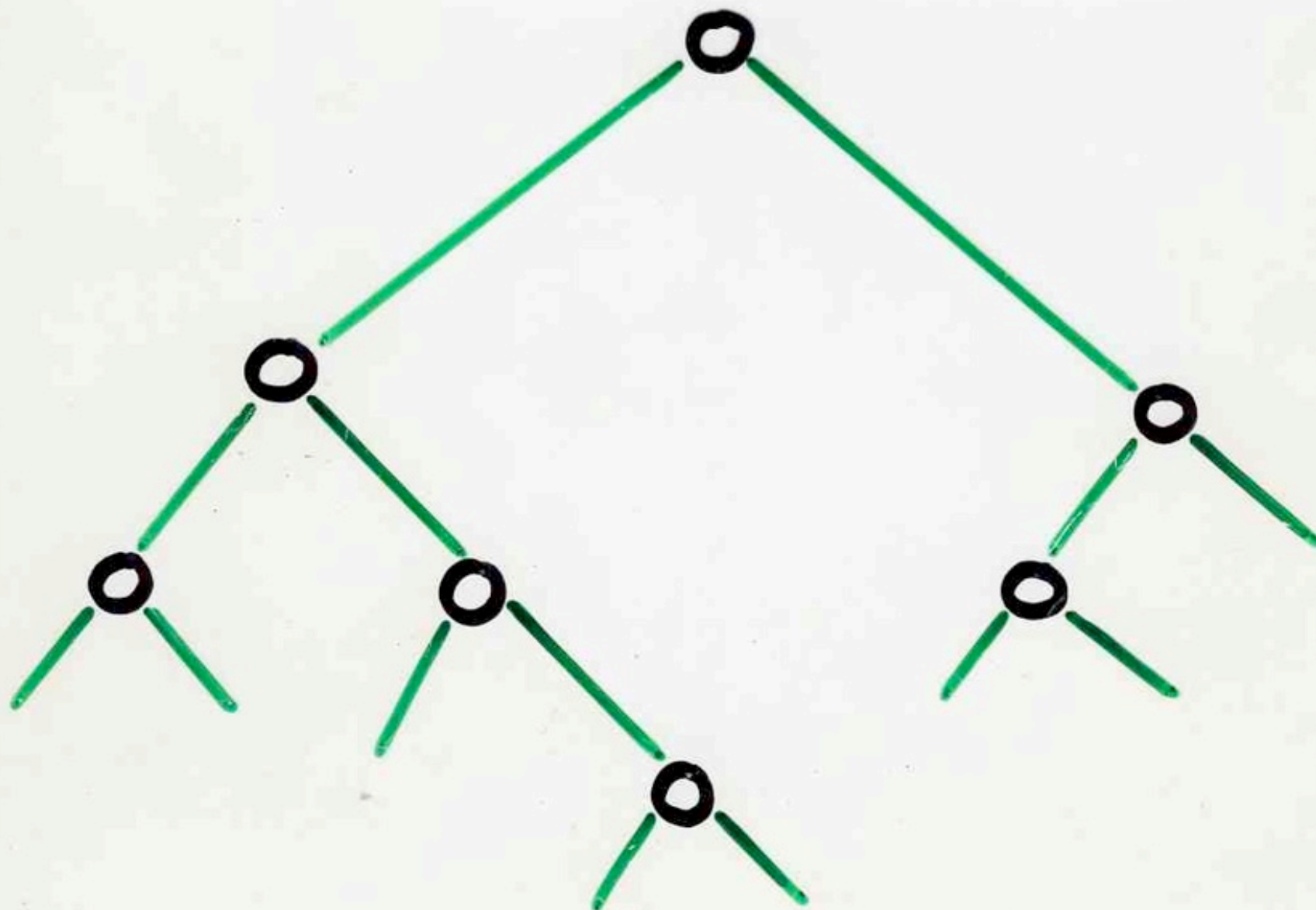
needed to compute an

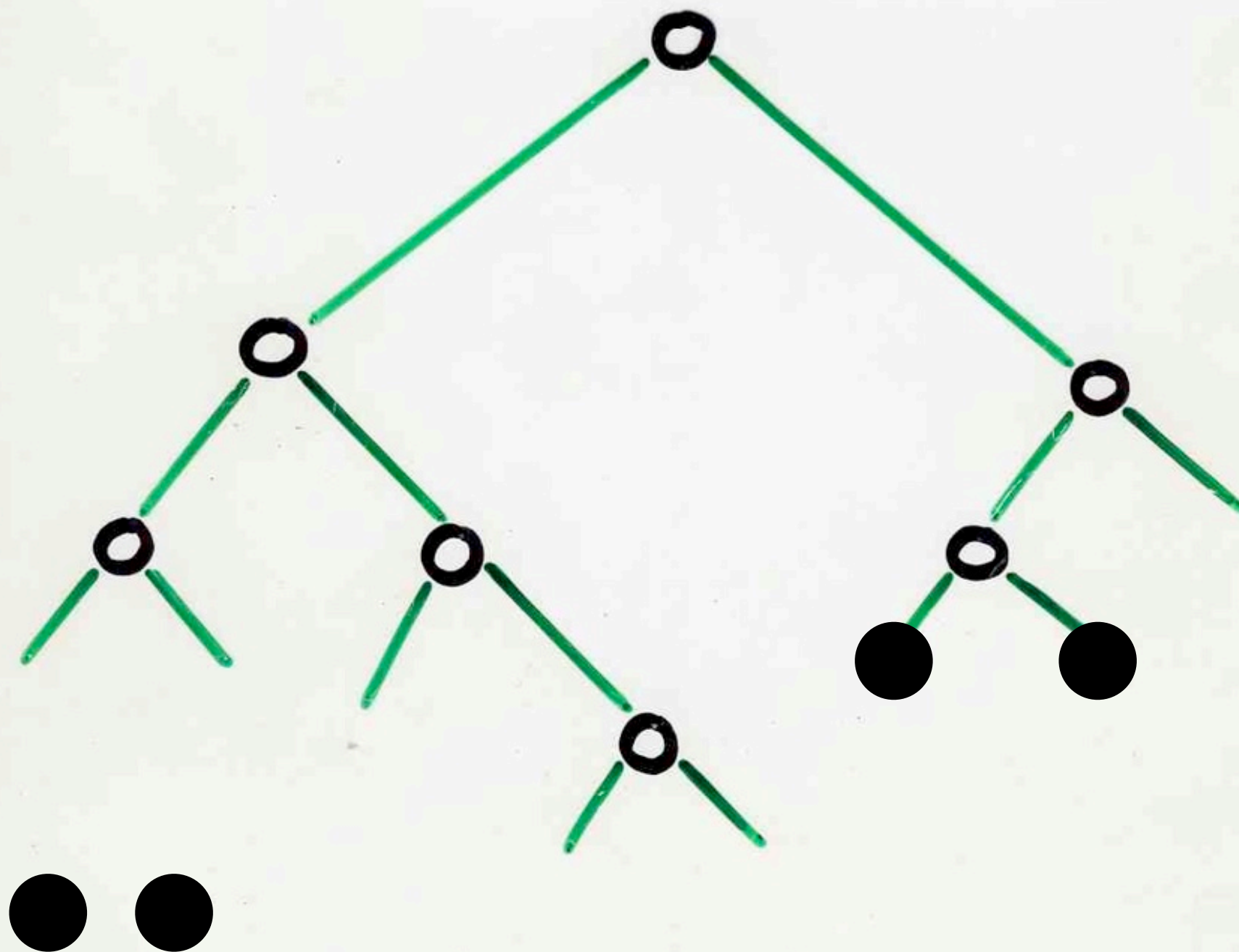
arithmetical expression

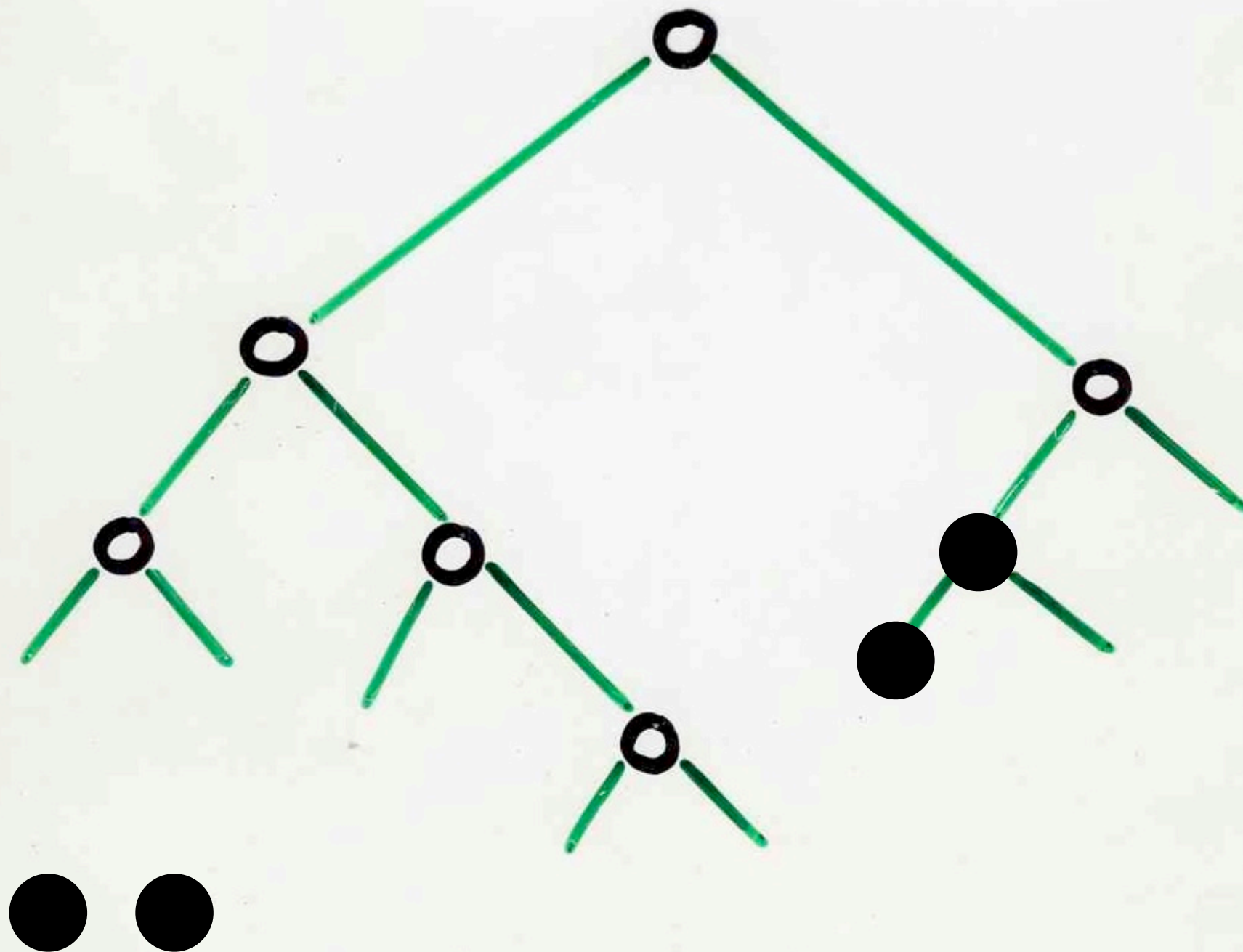


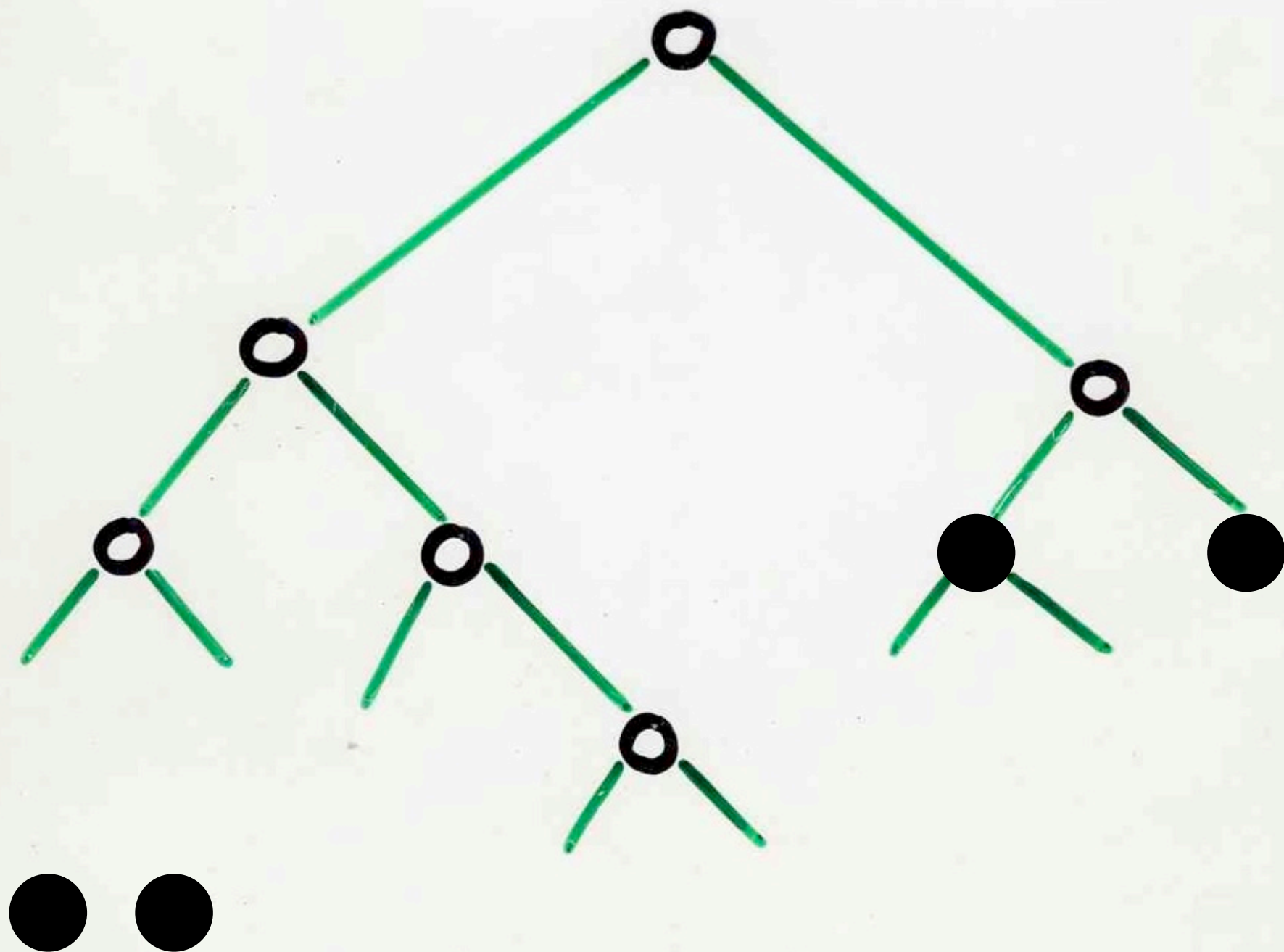
Pebbles problem

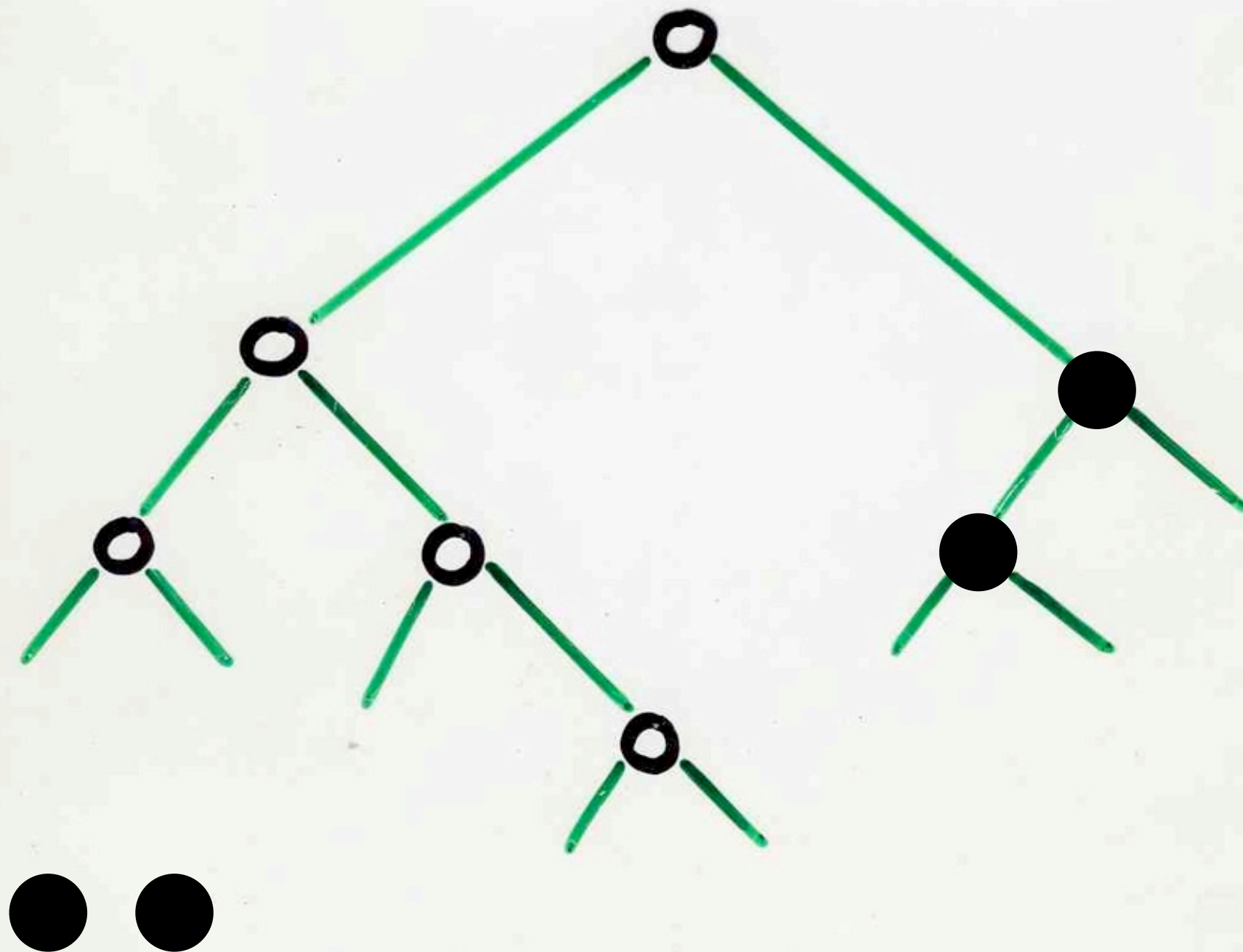


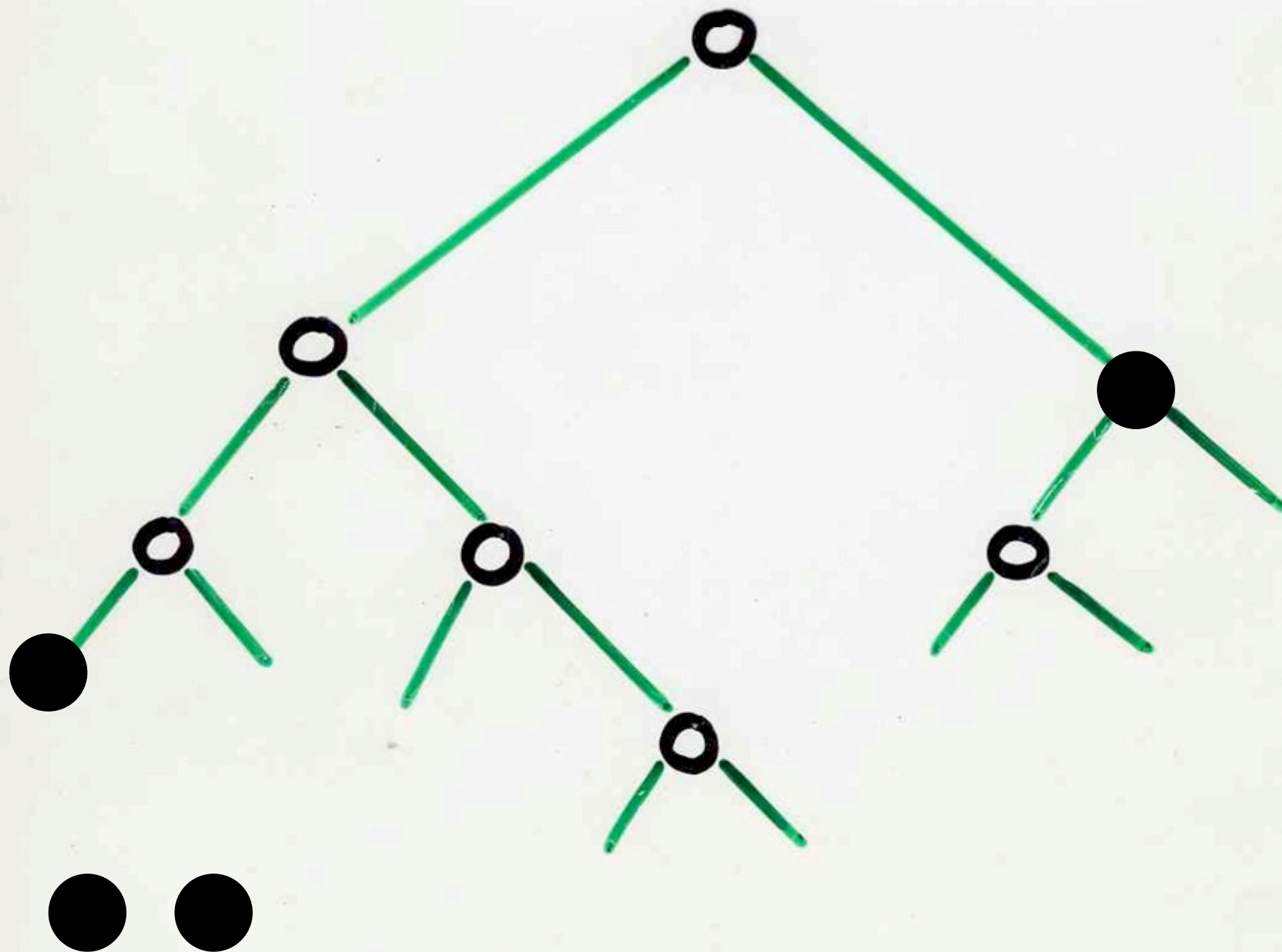


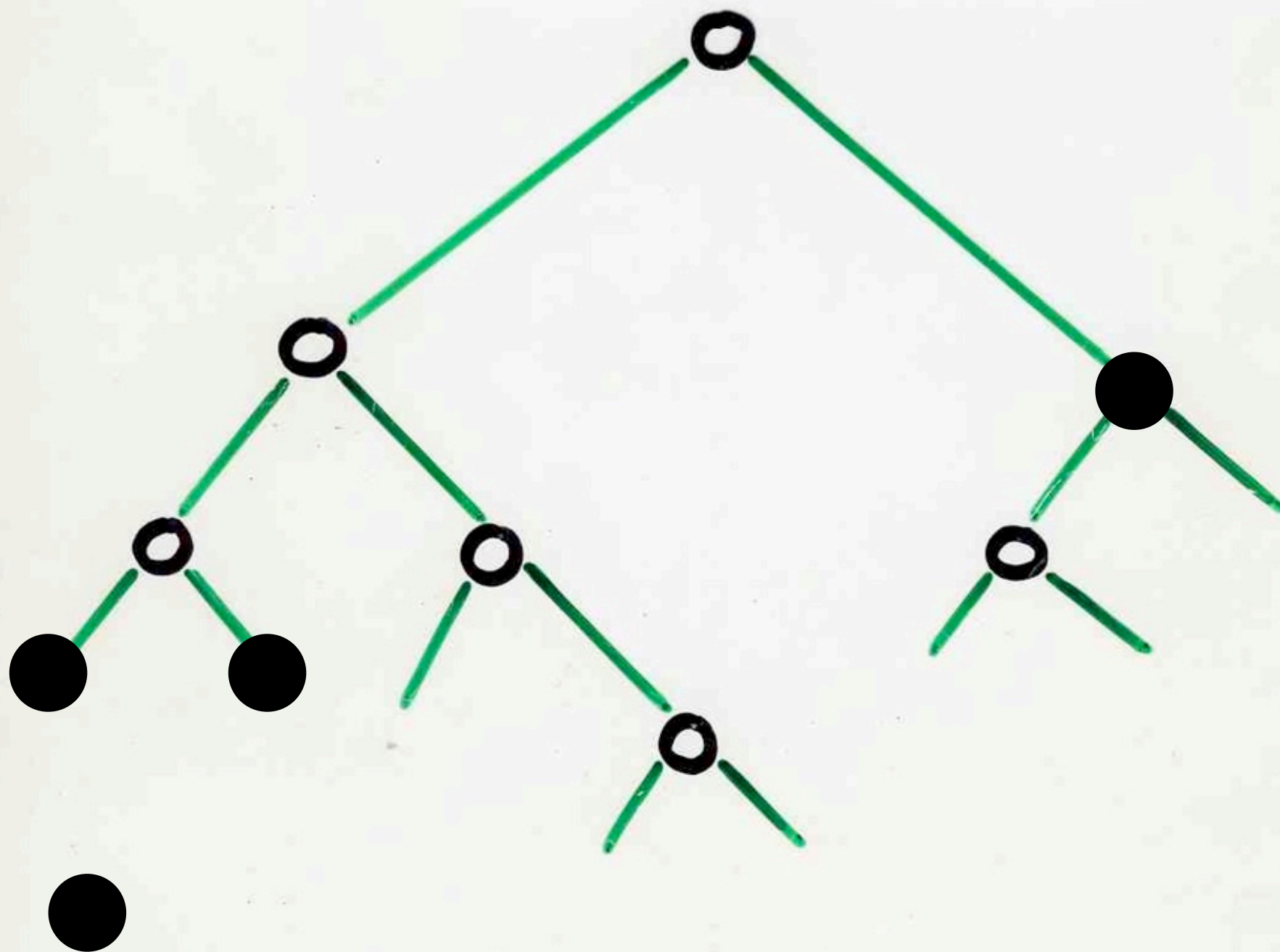


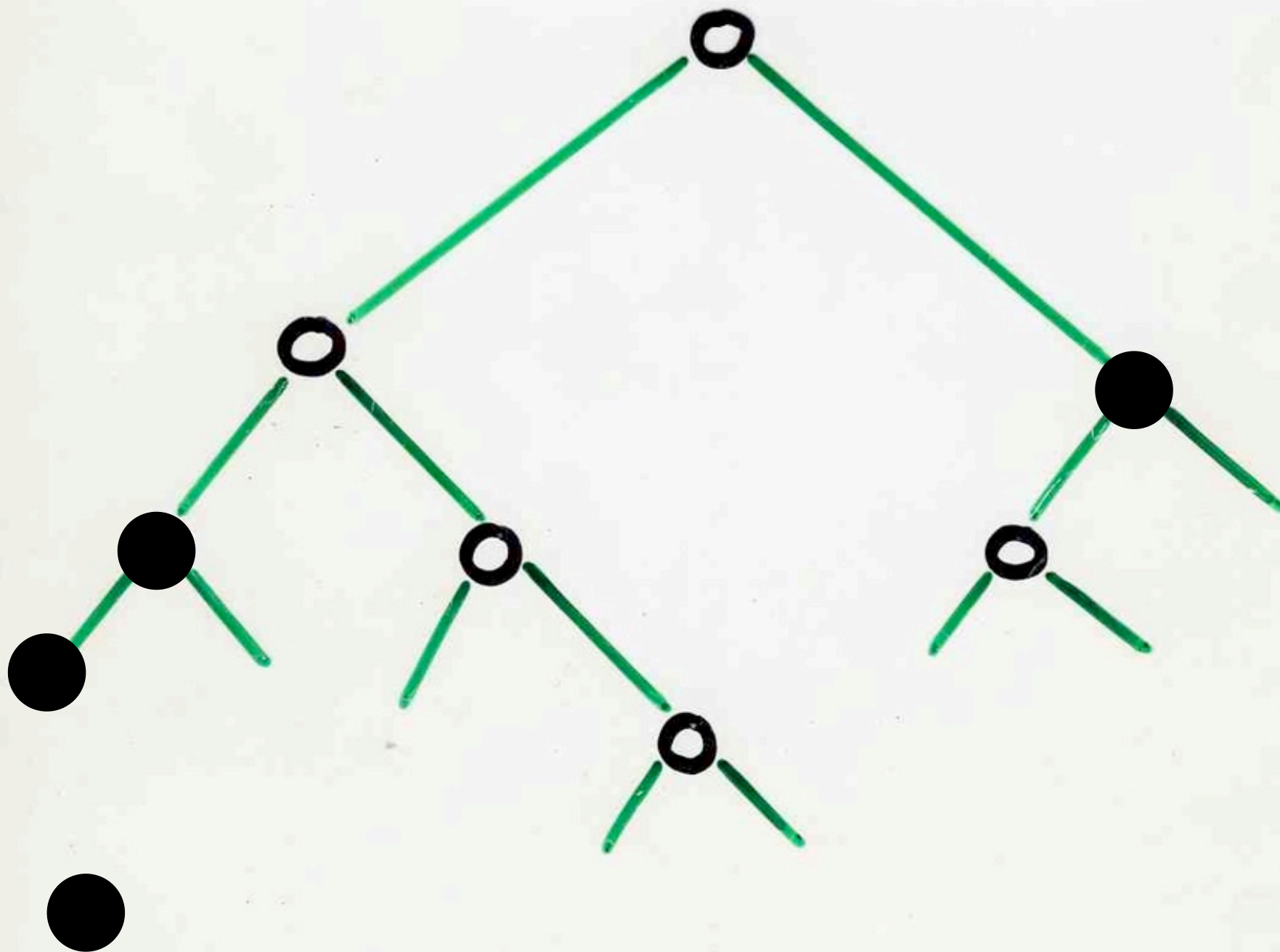


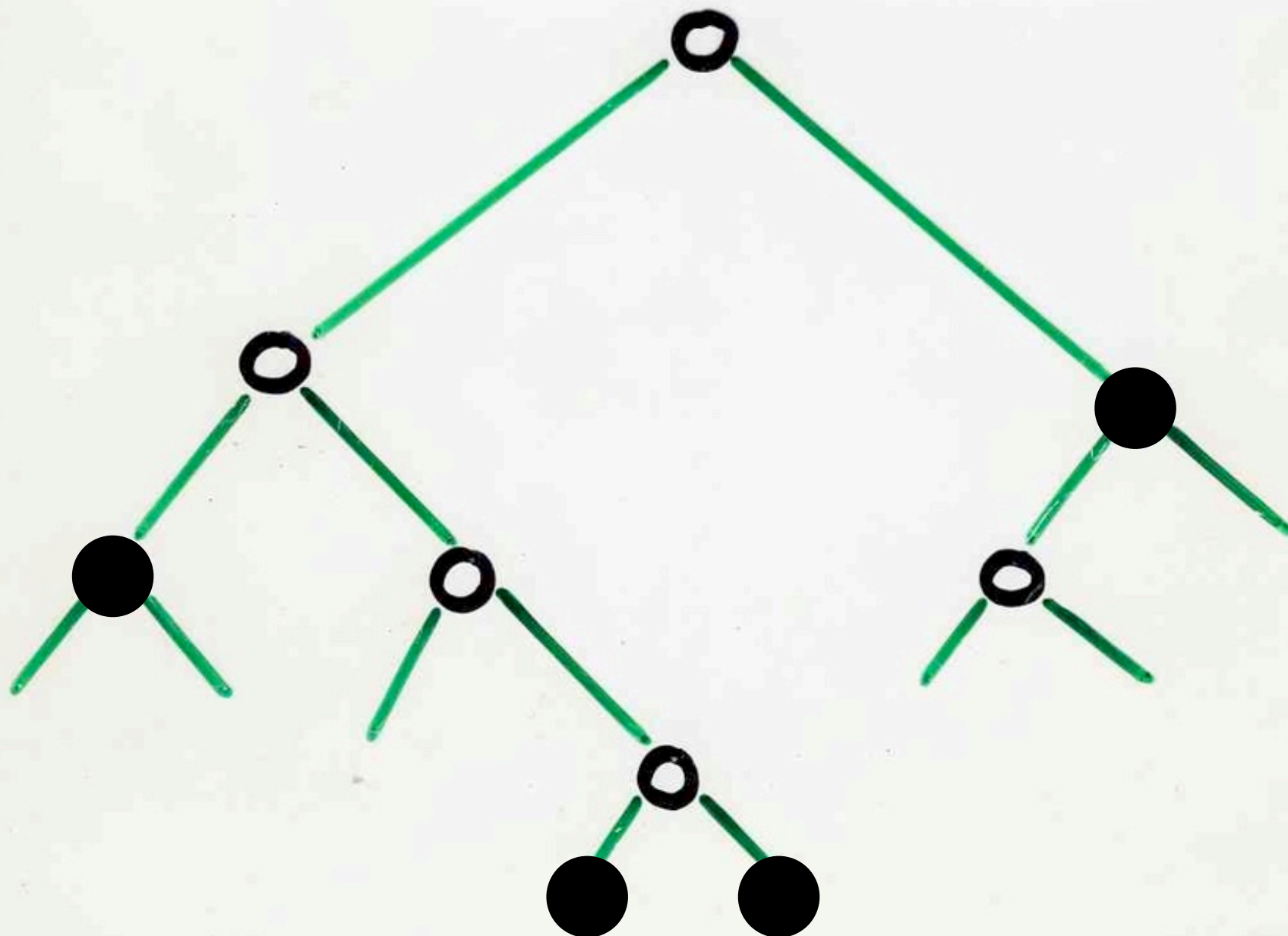


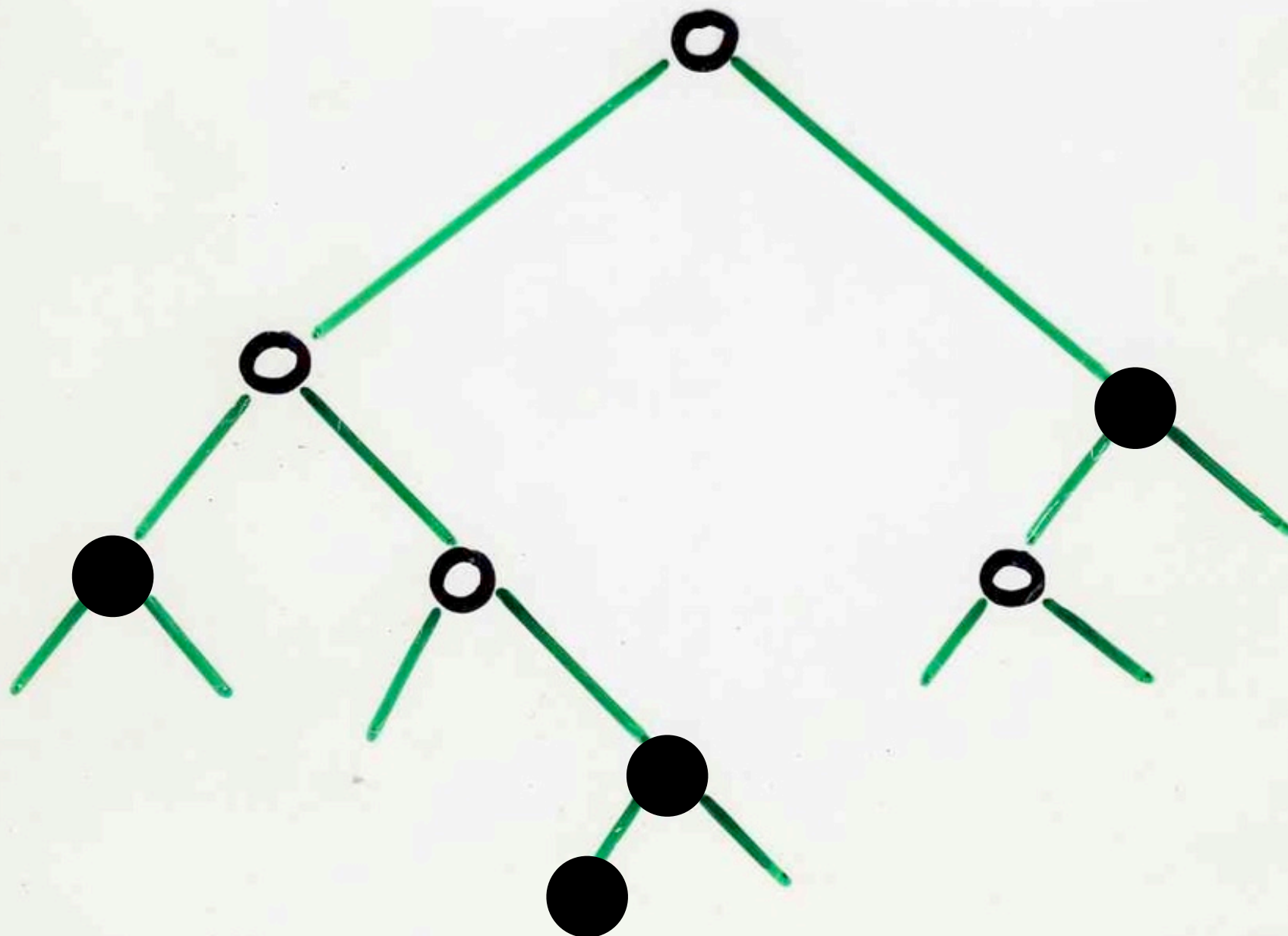


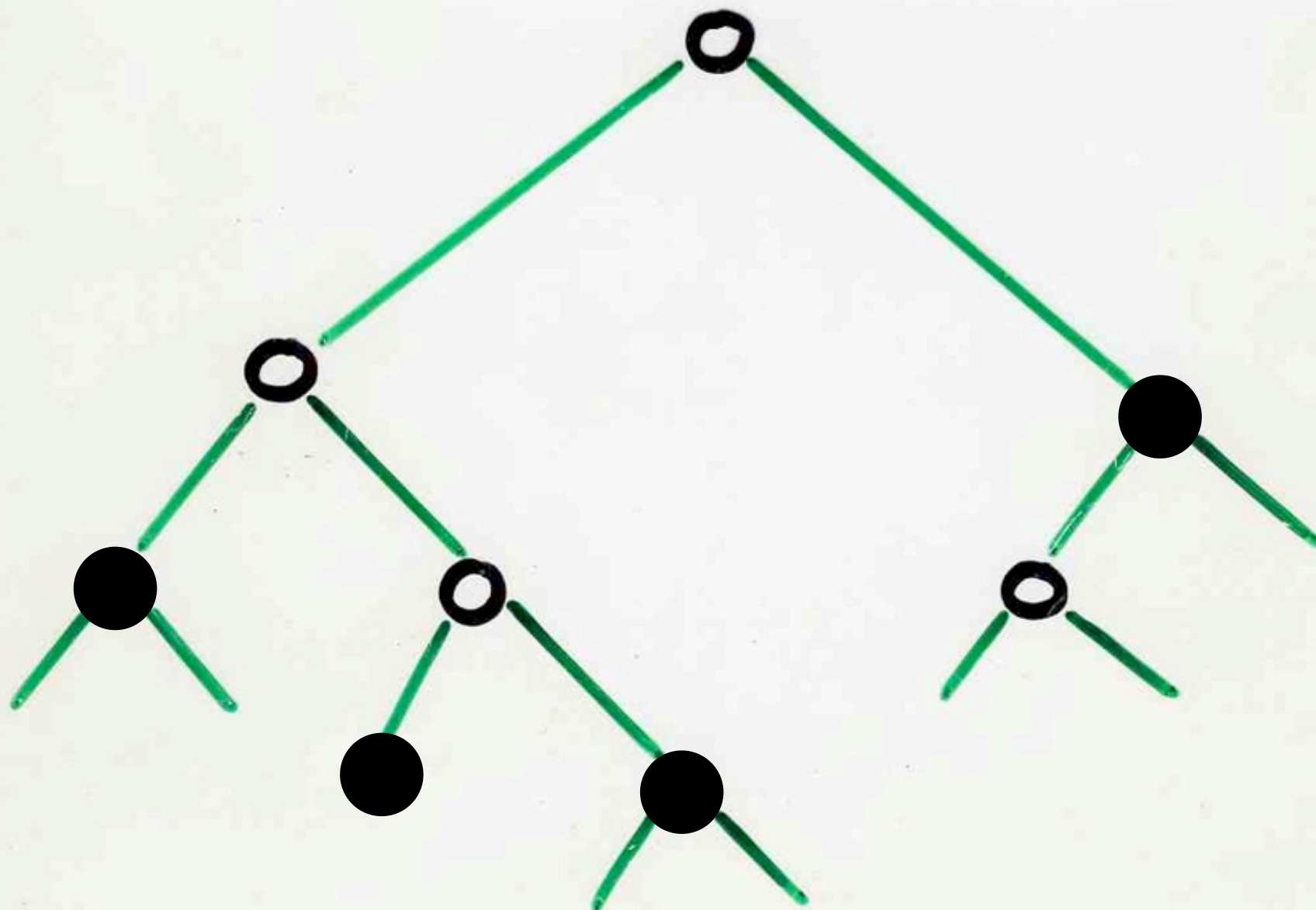


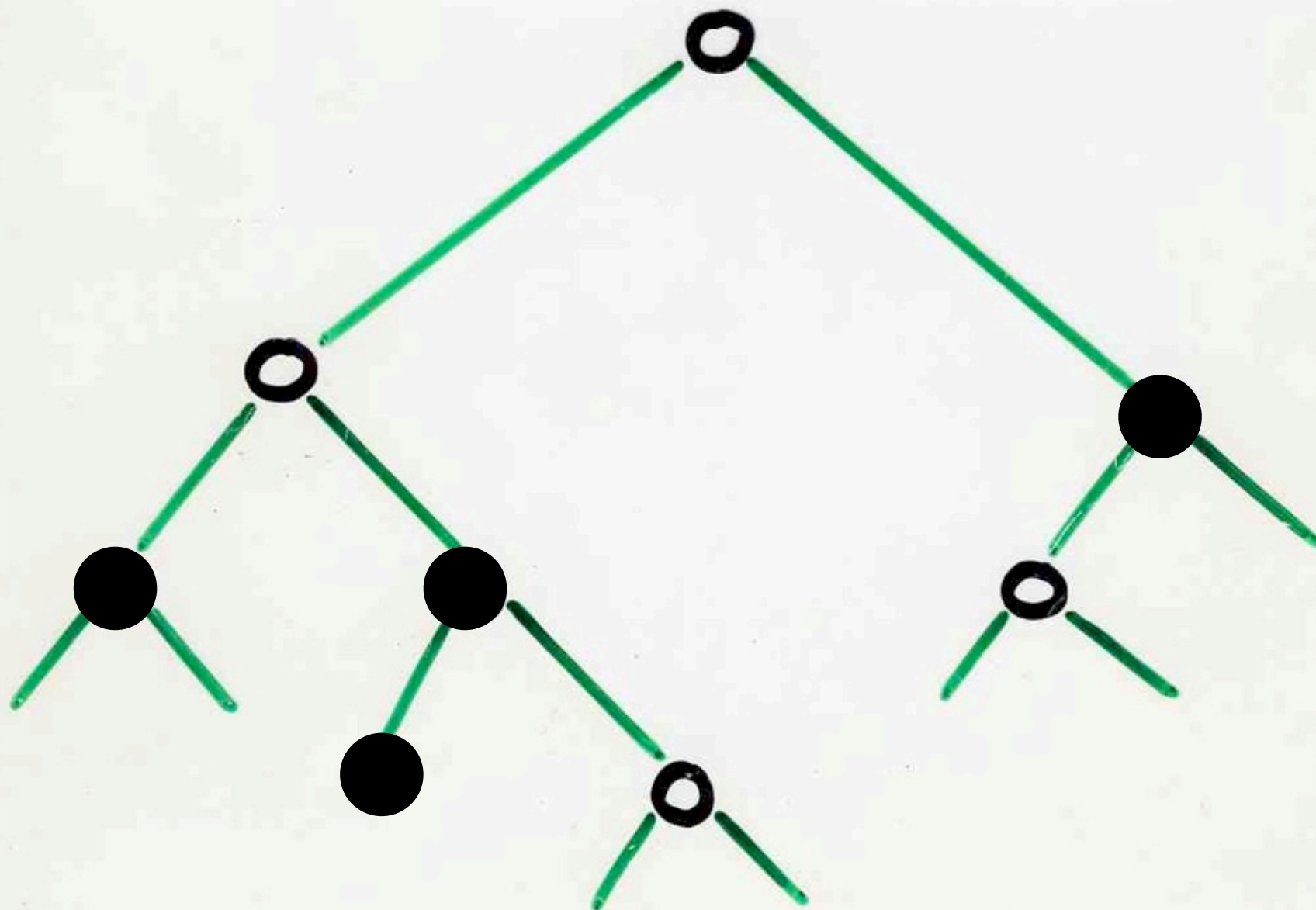


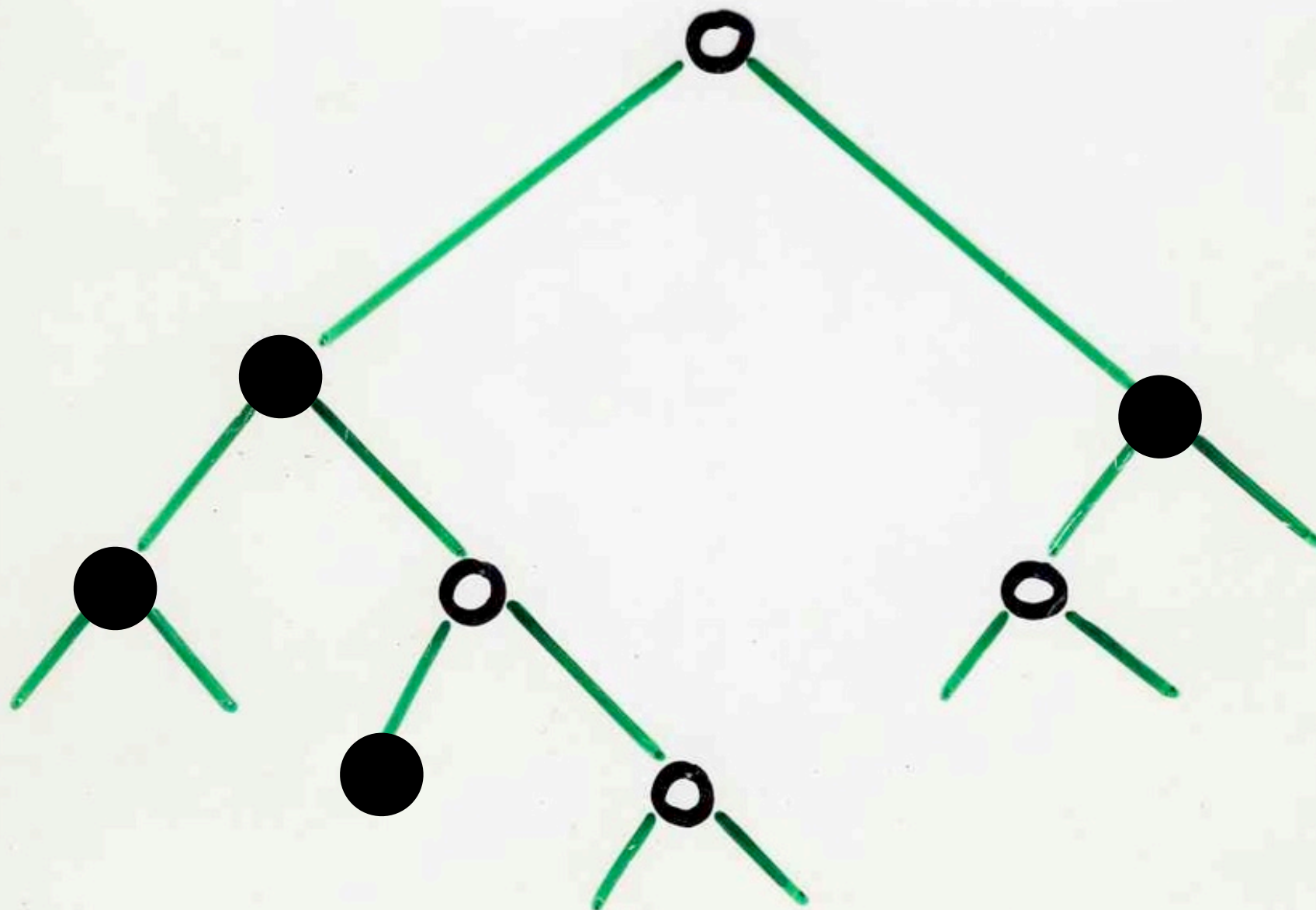


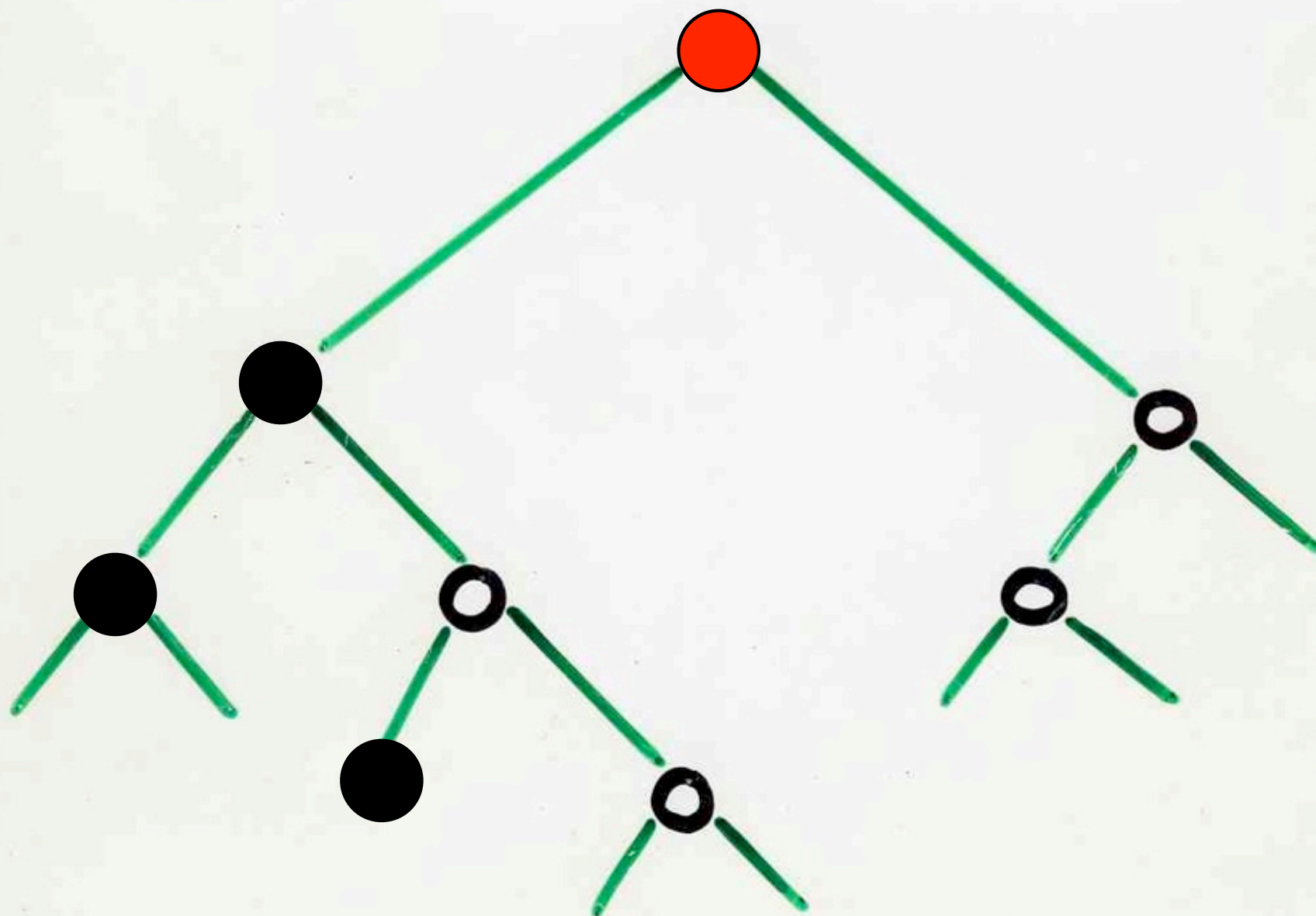


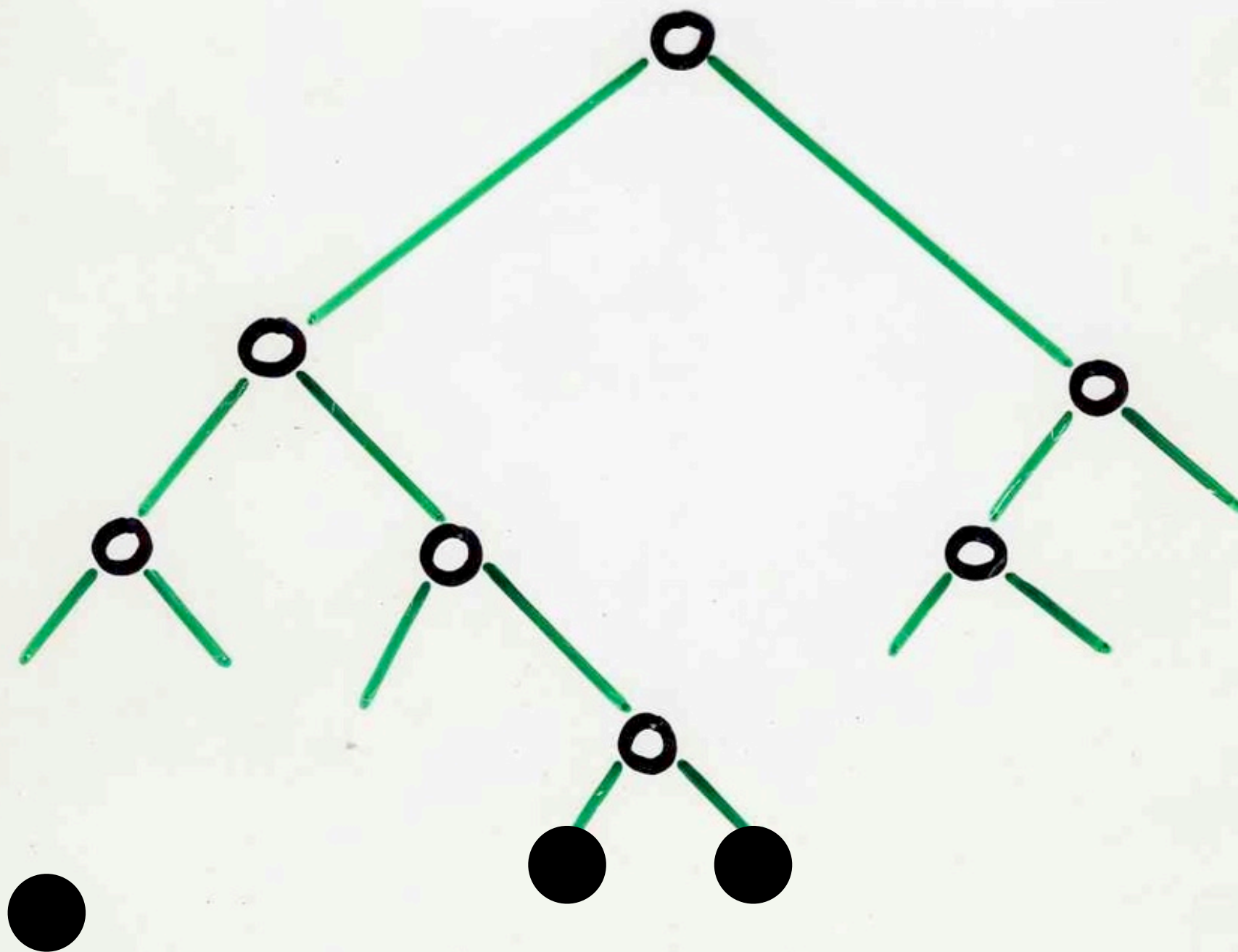


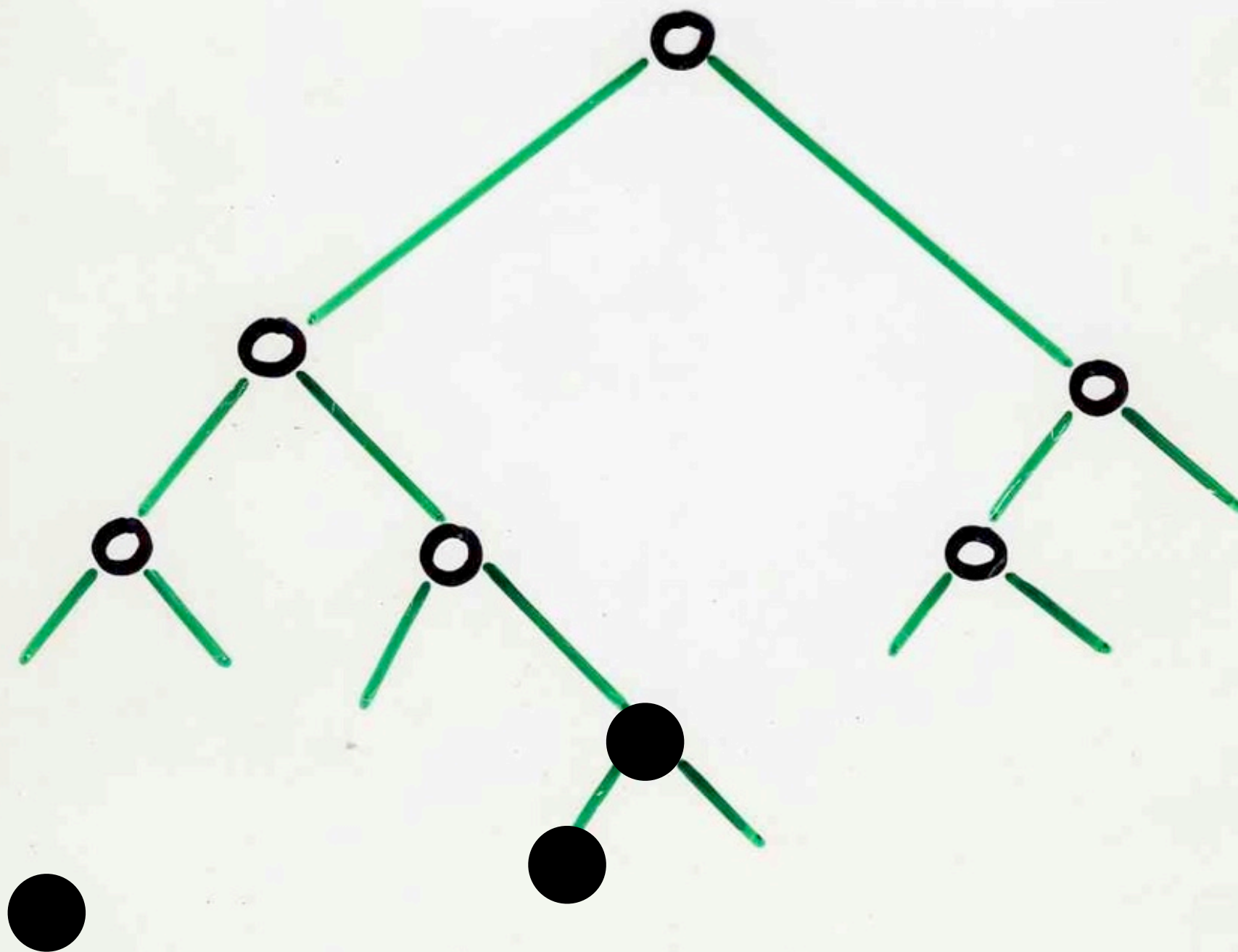


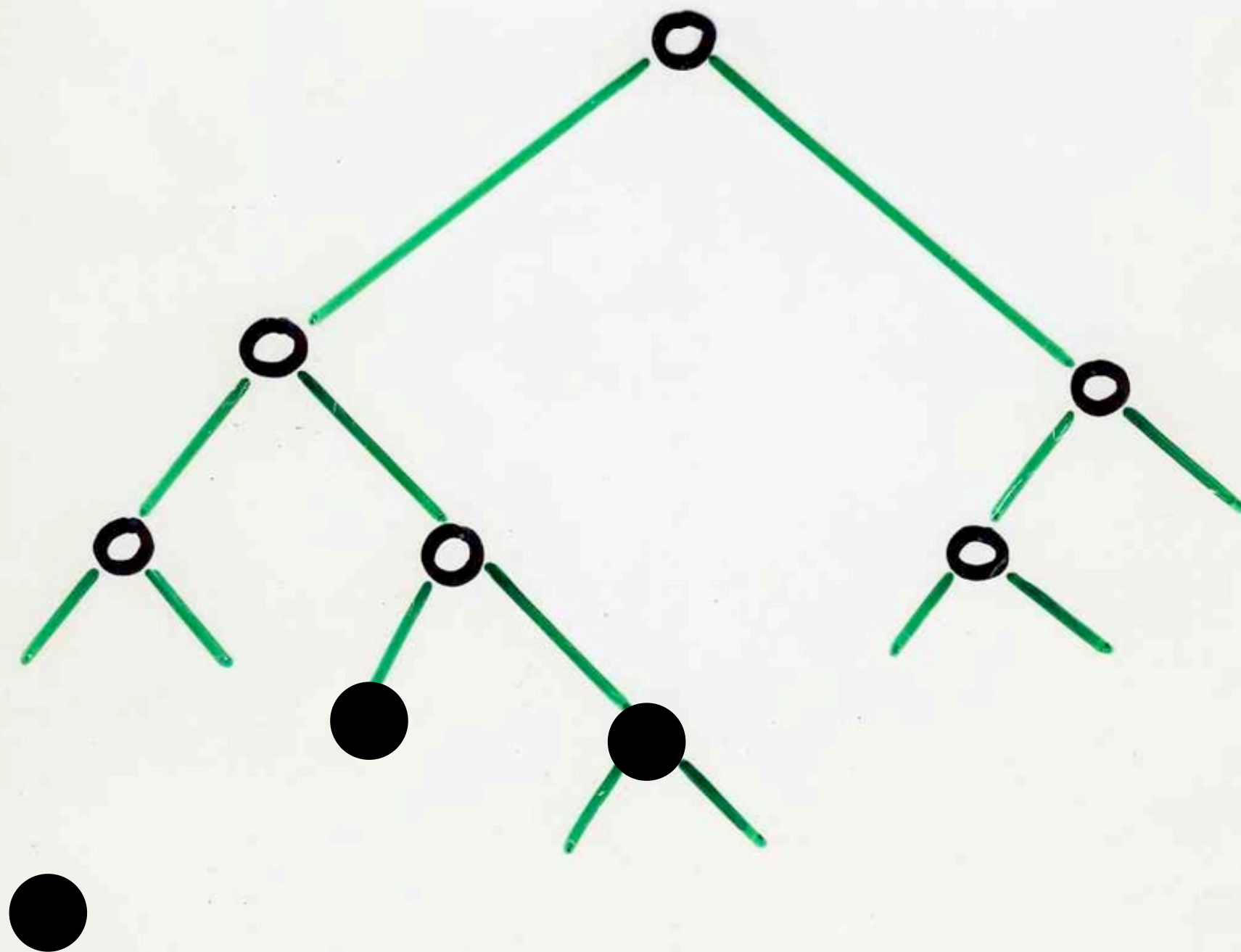


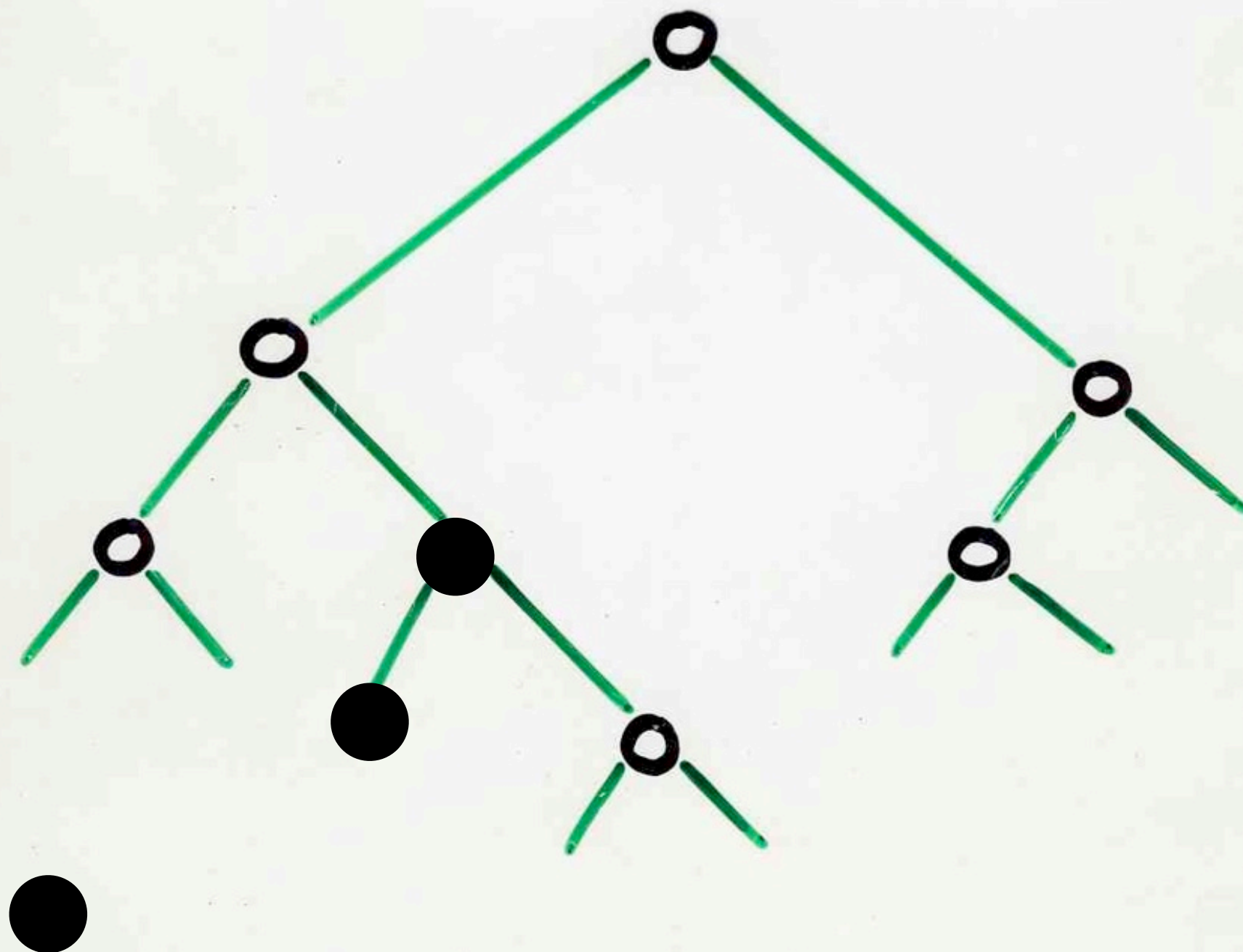


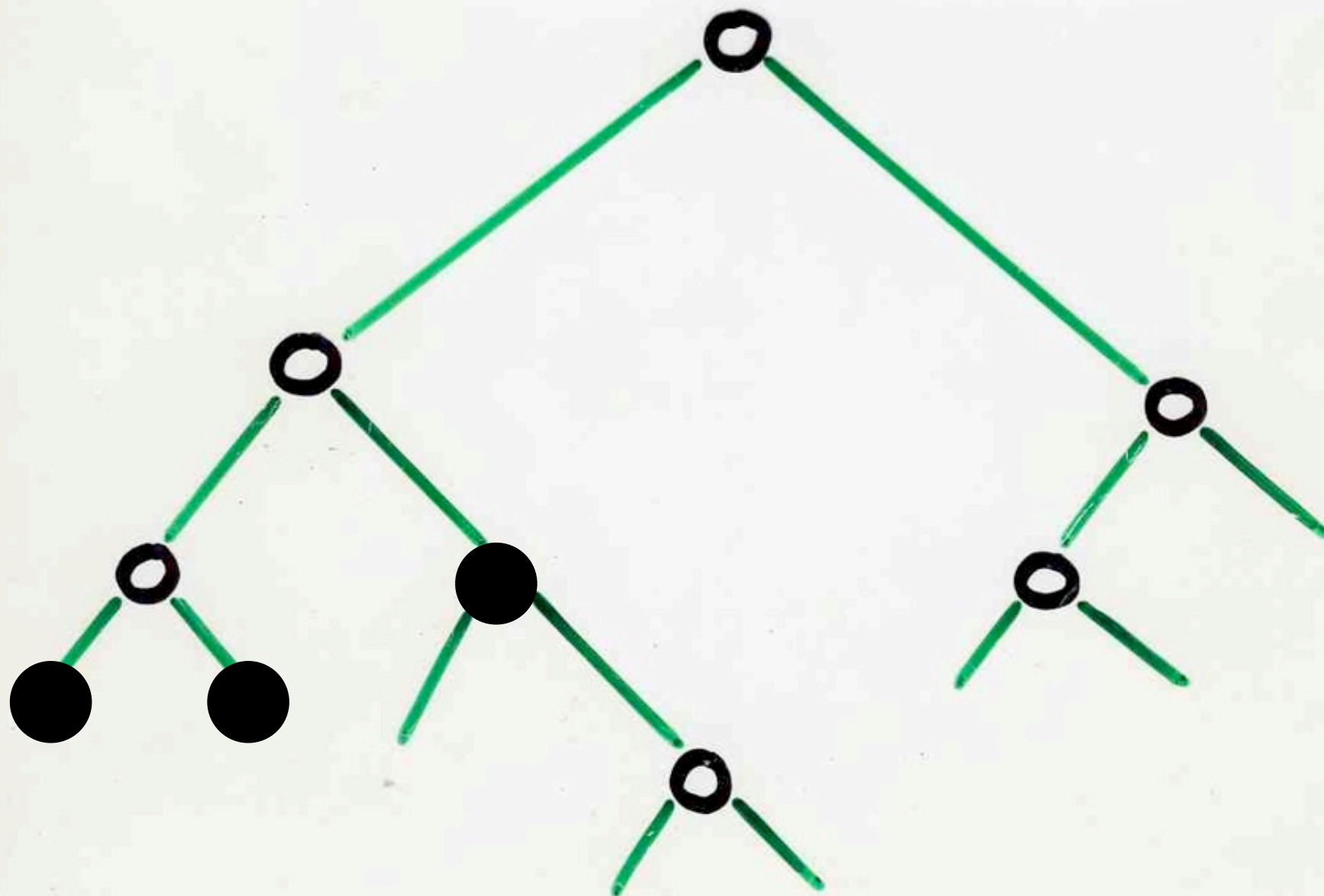


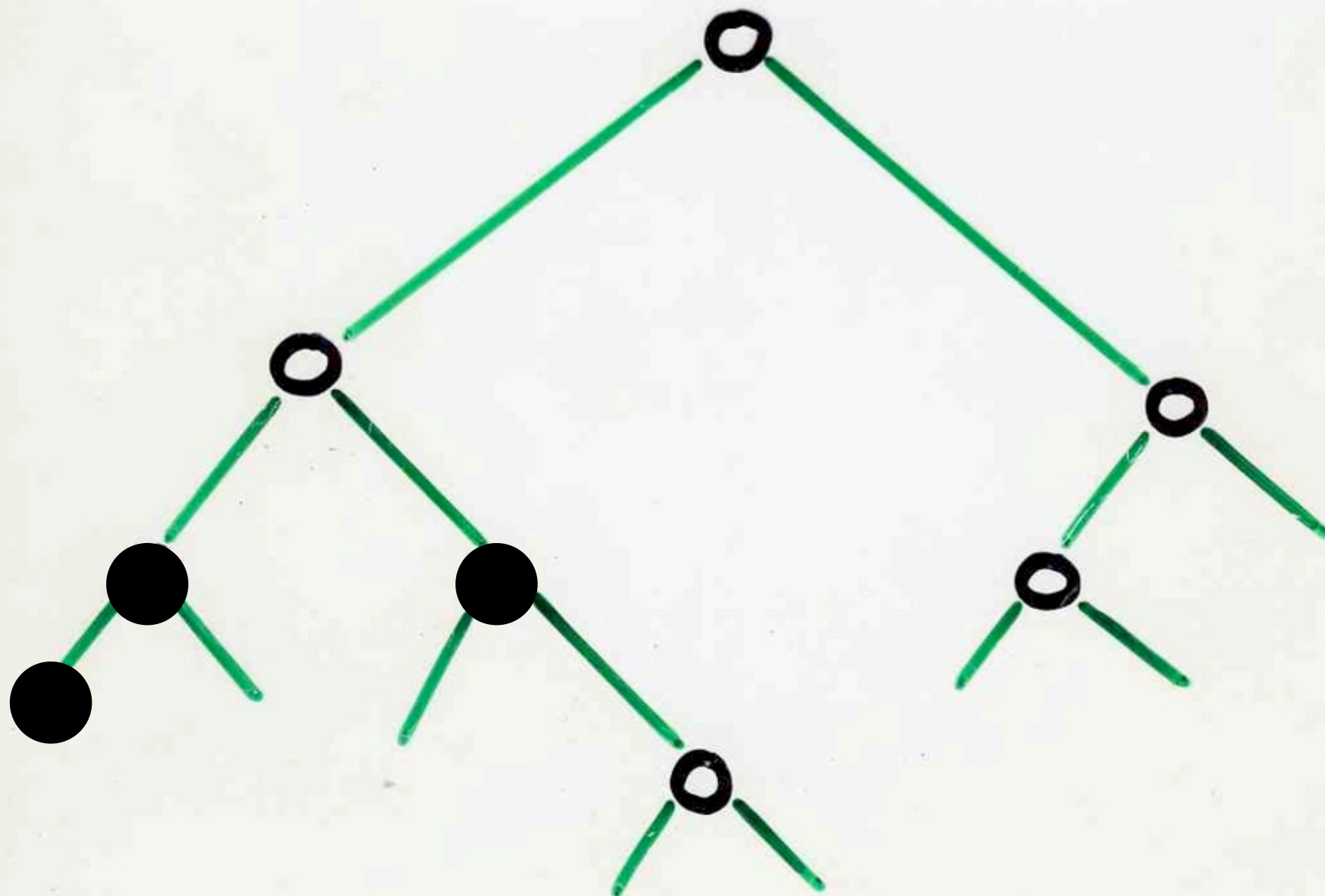


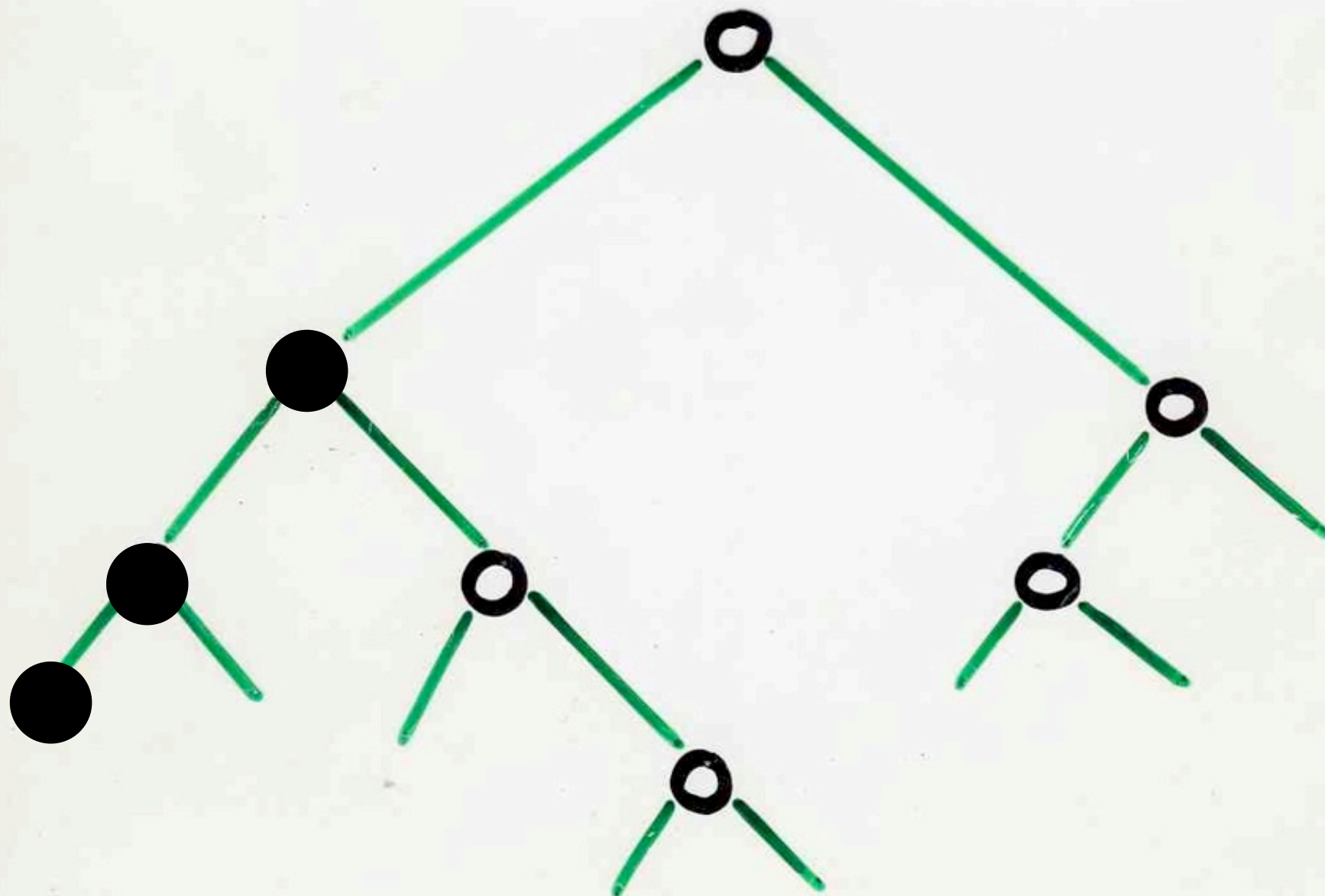


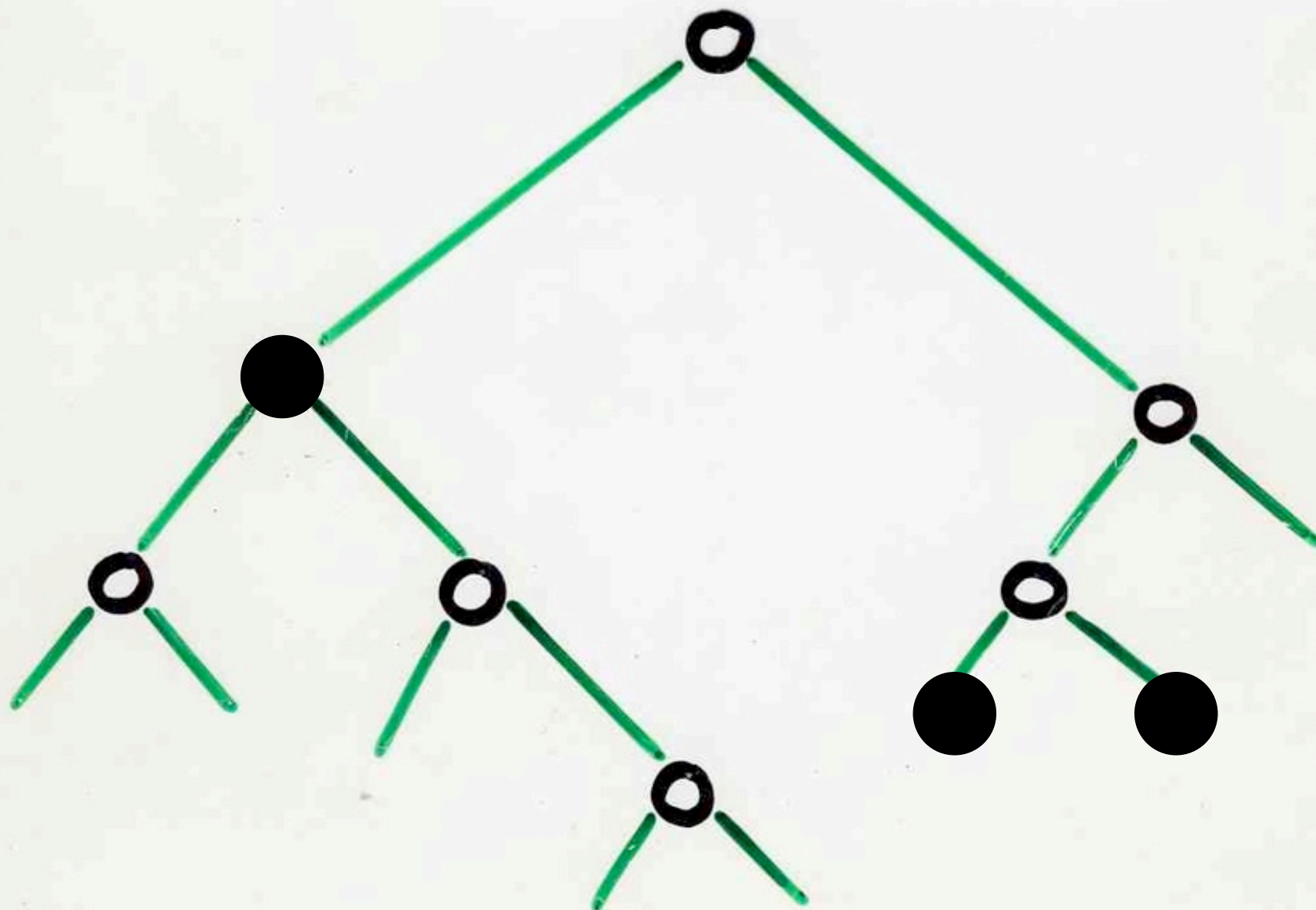


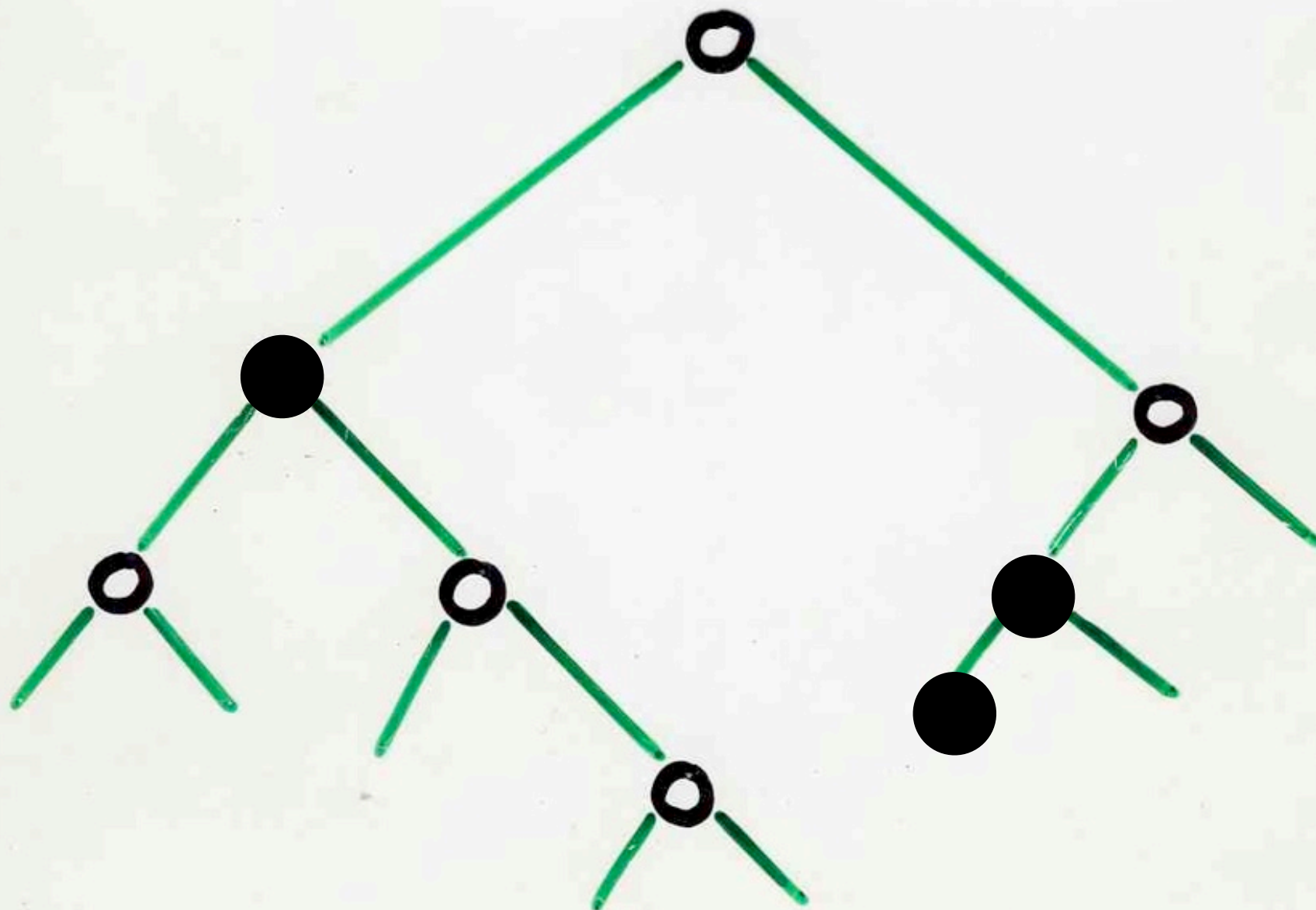


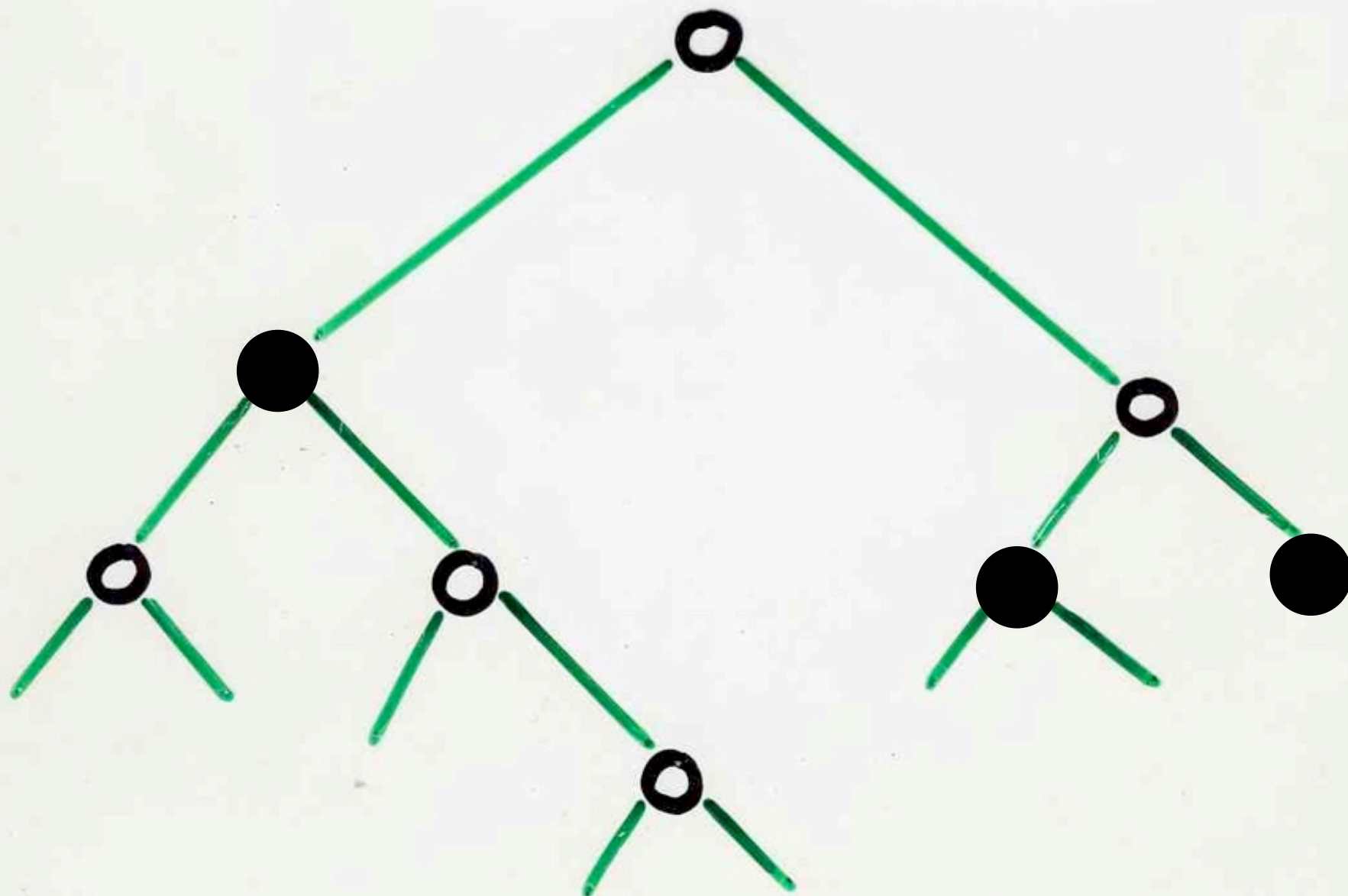


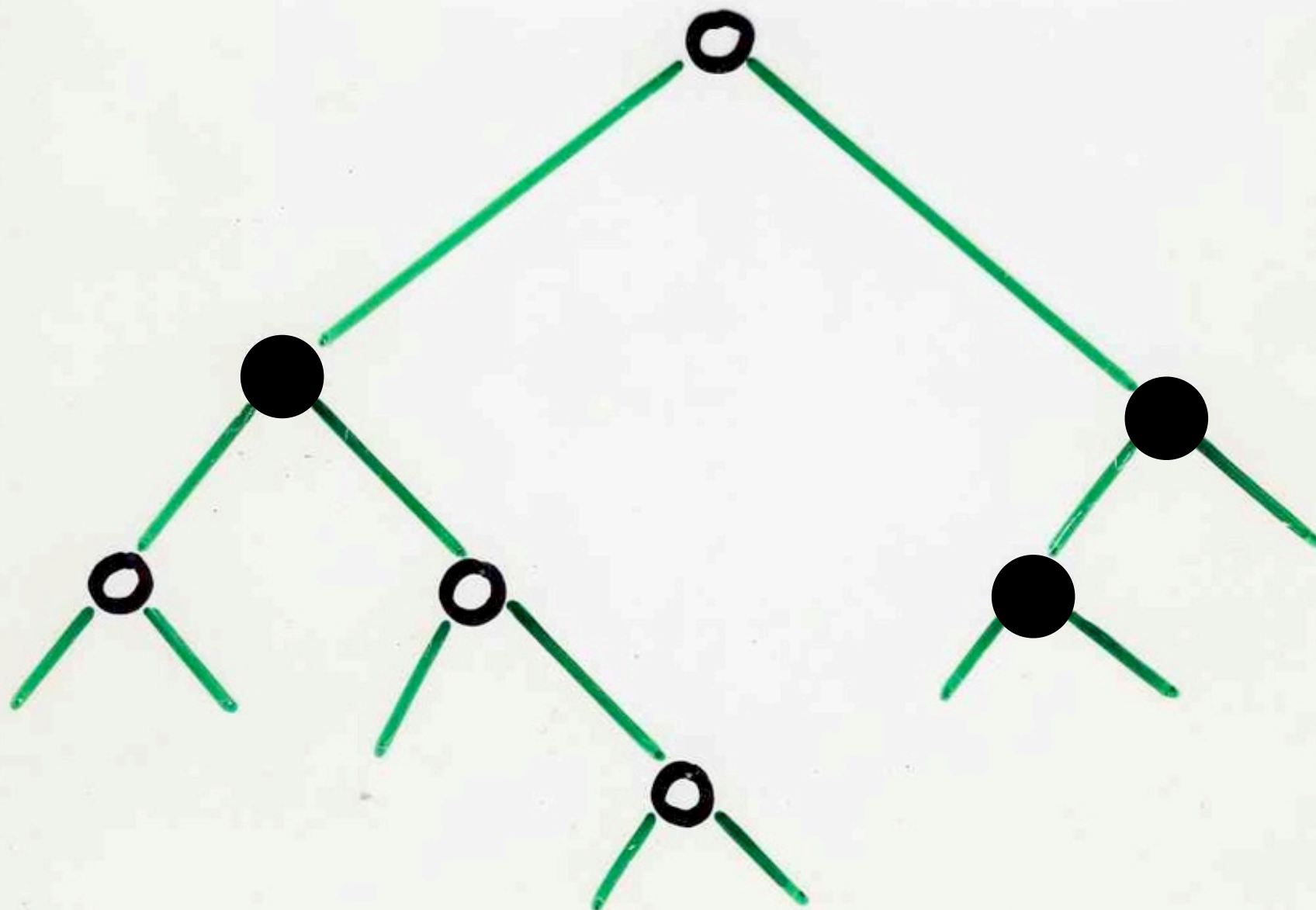


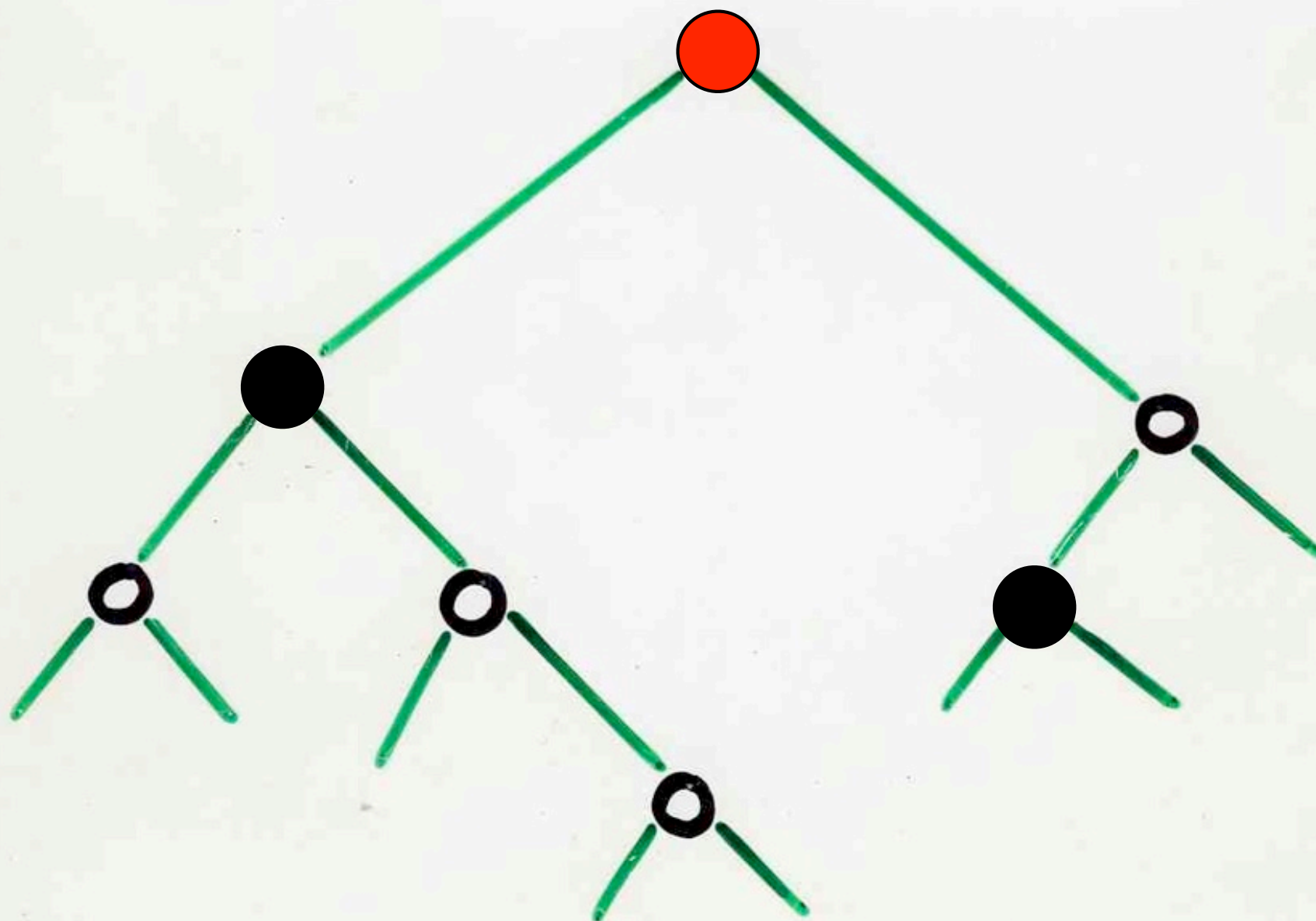






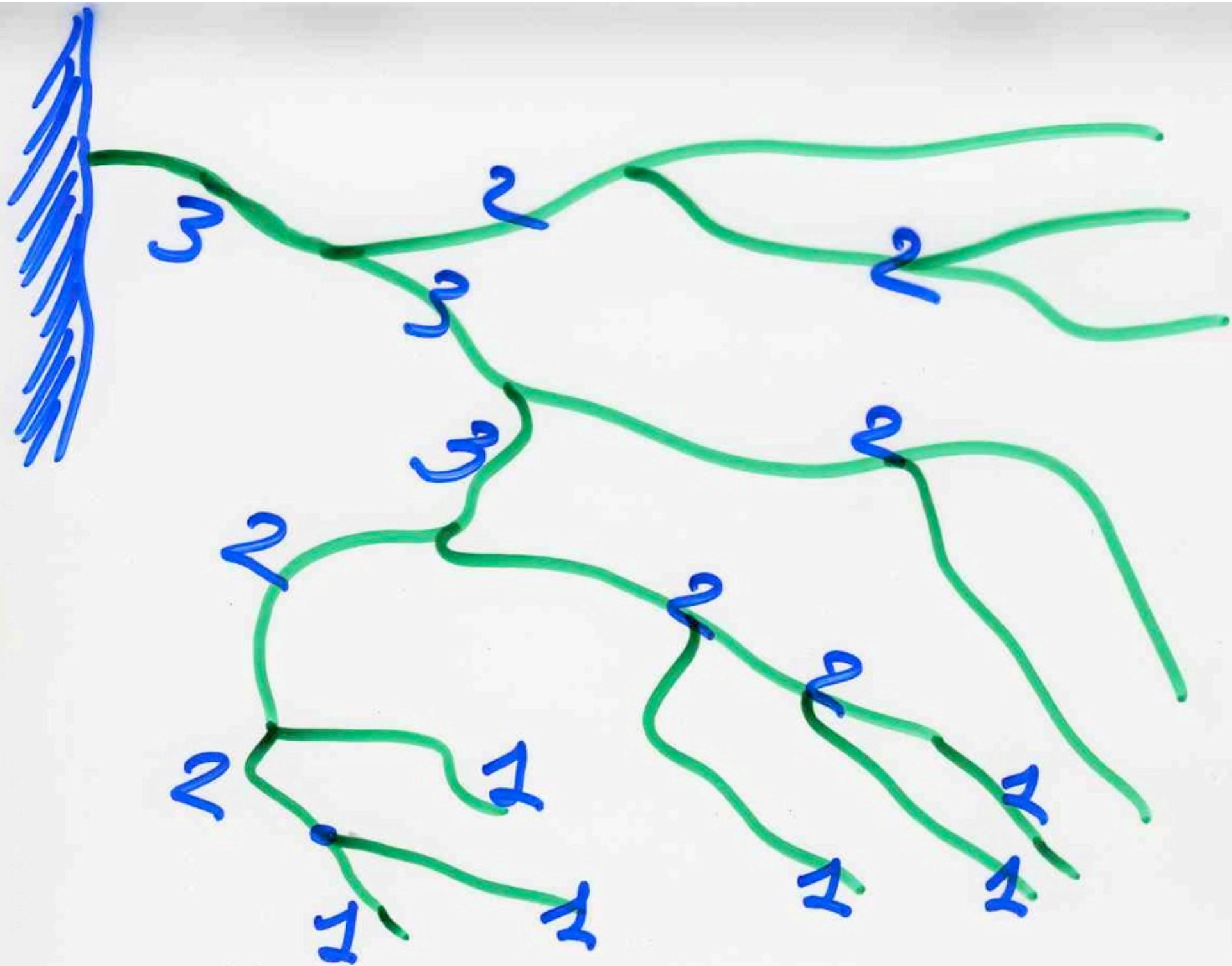


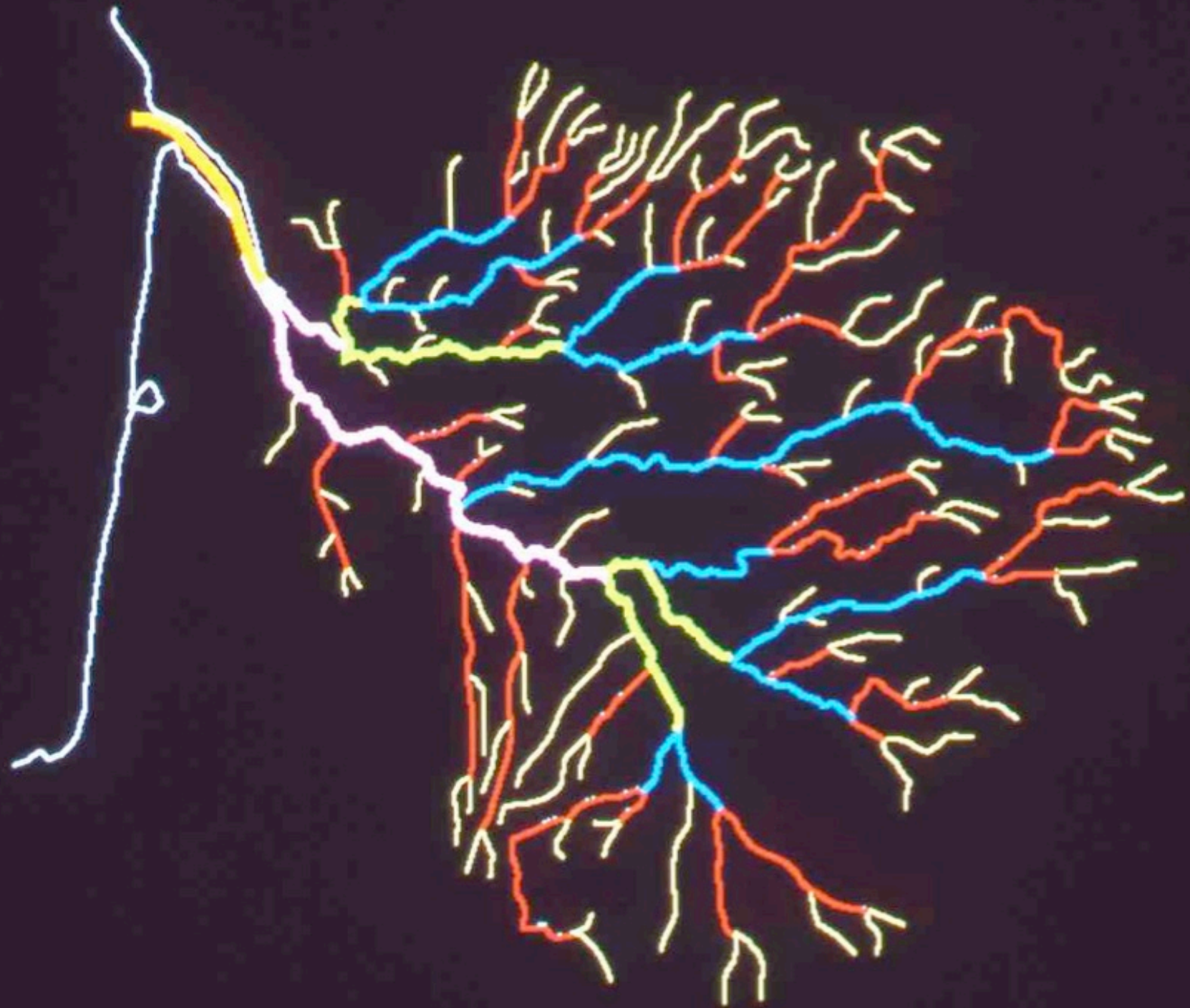


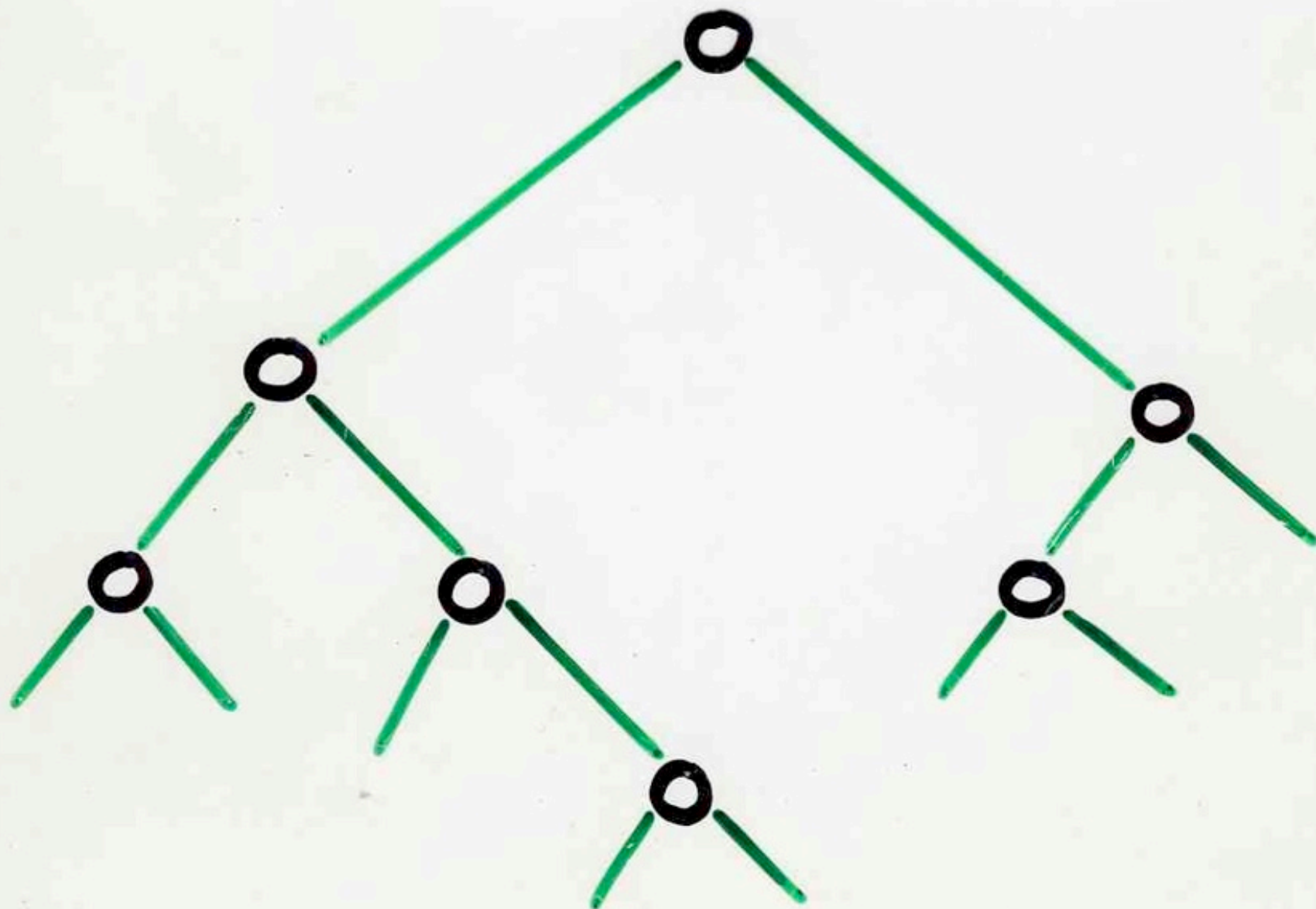


Des fleuves, des rivières

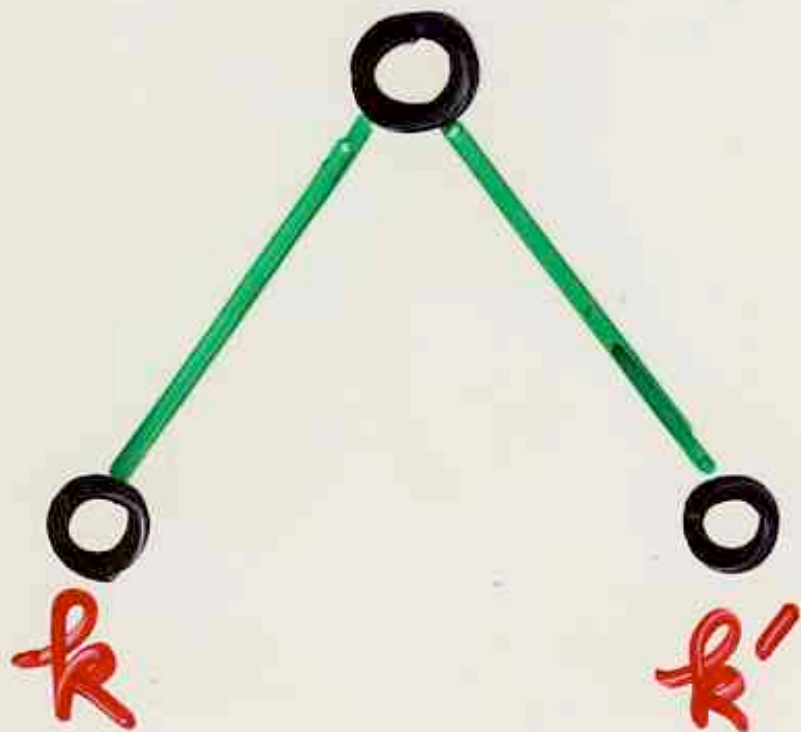




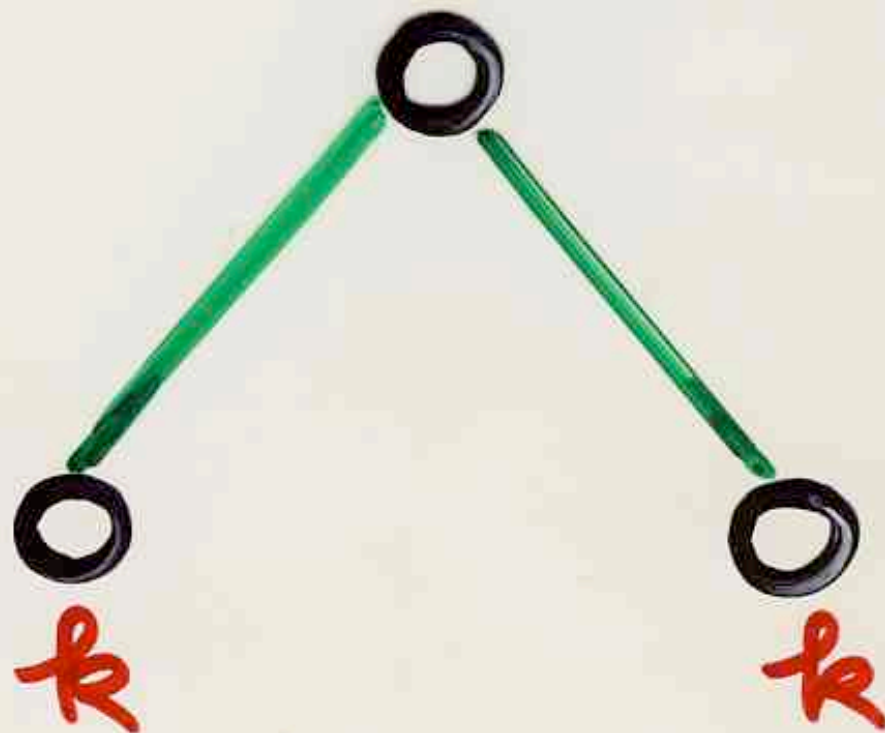




$\max(k, k')$



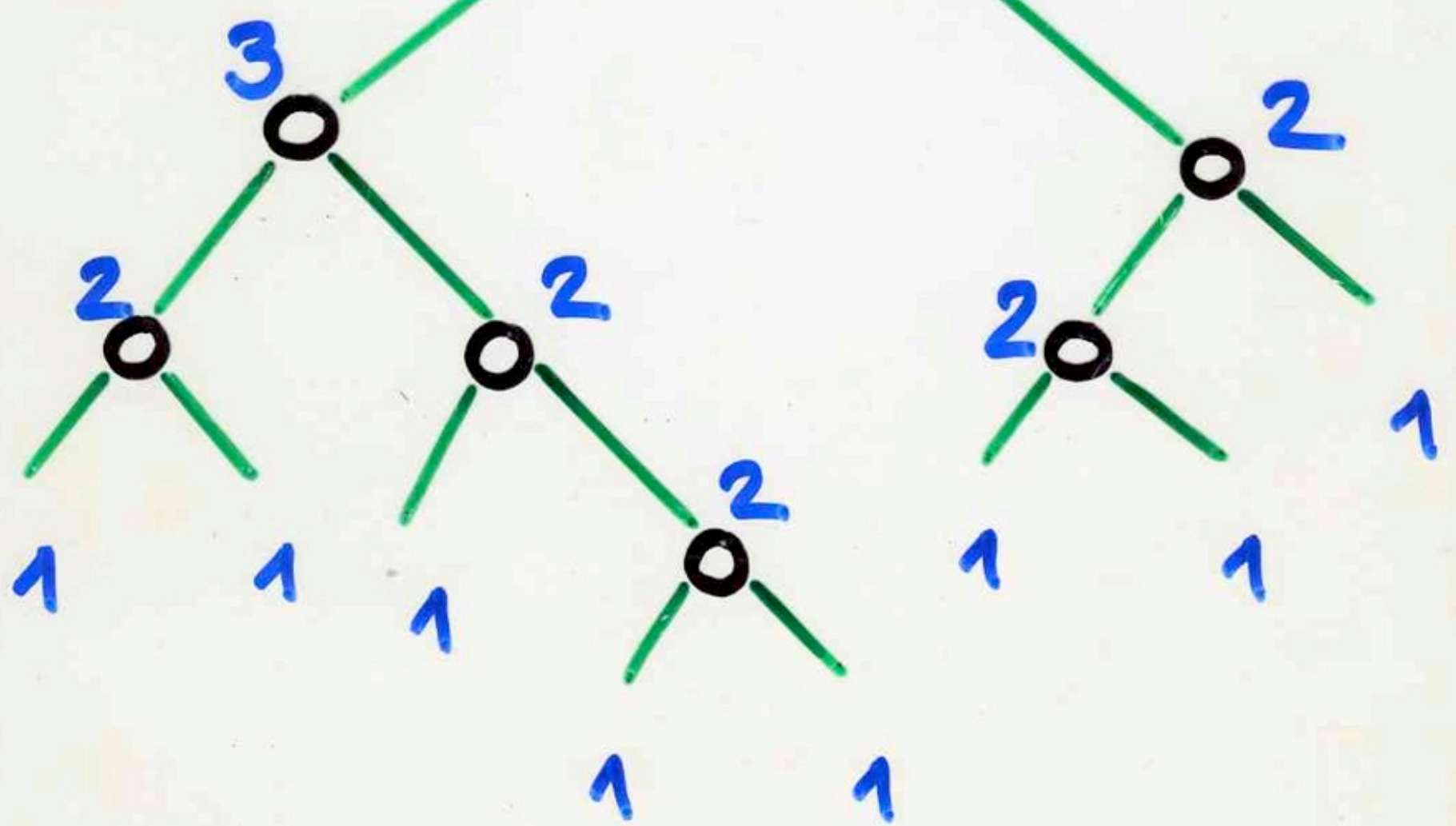
$k+1$



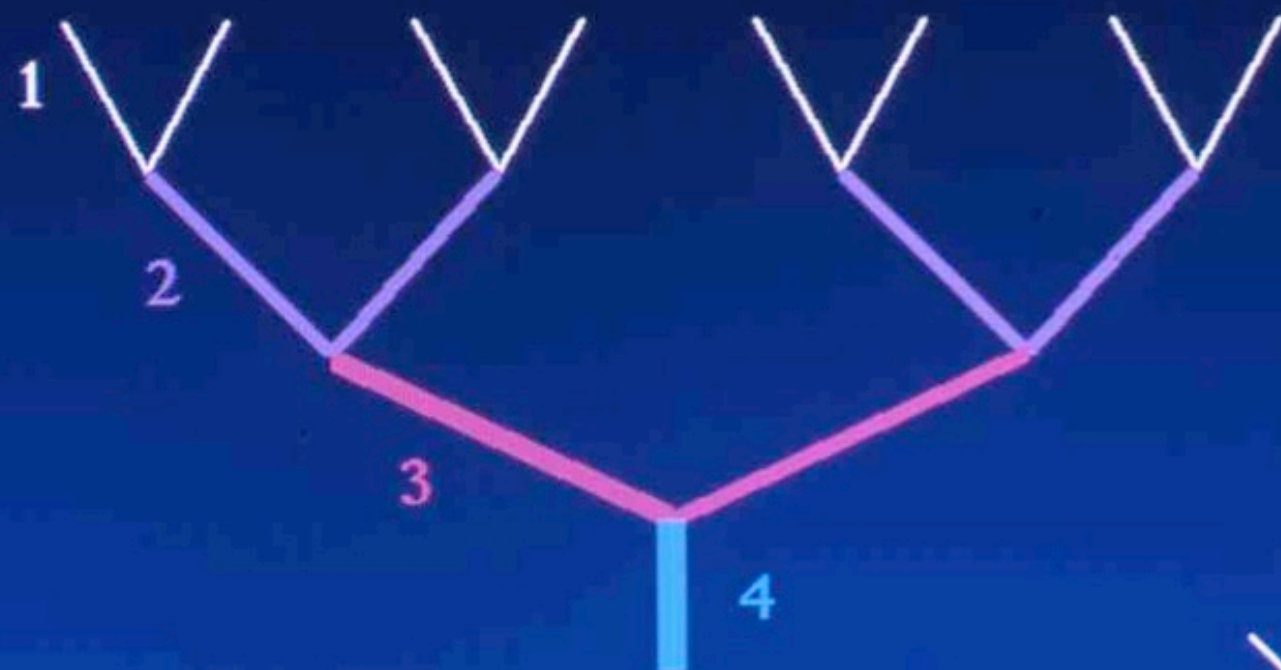


$= St(B)$

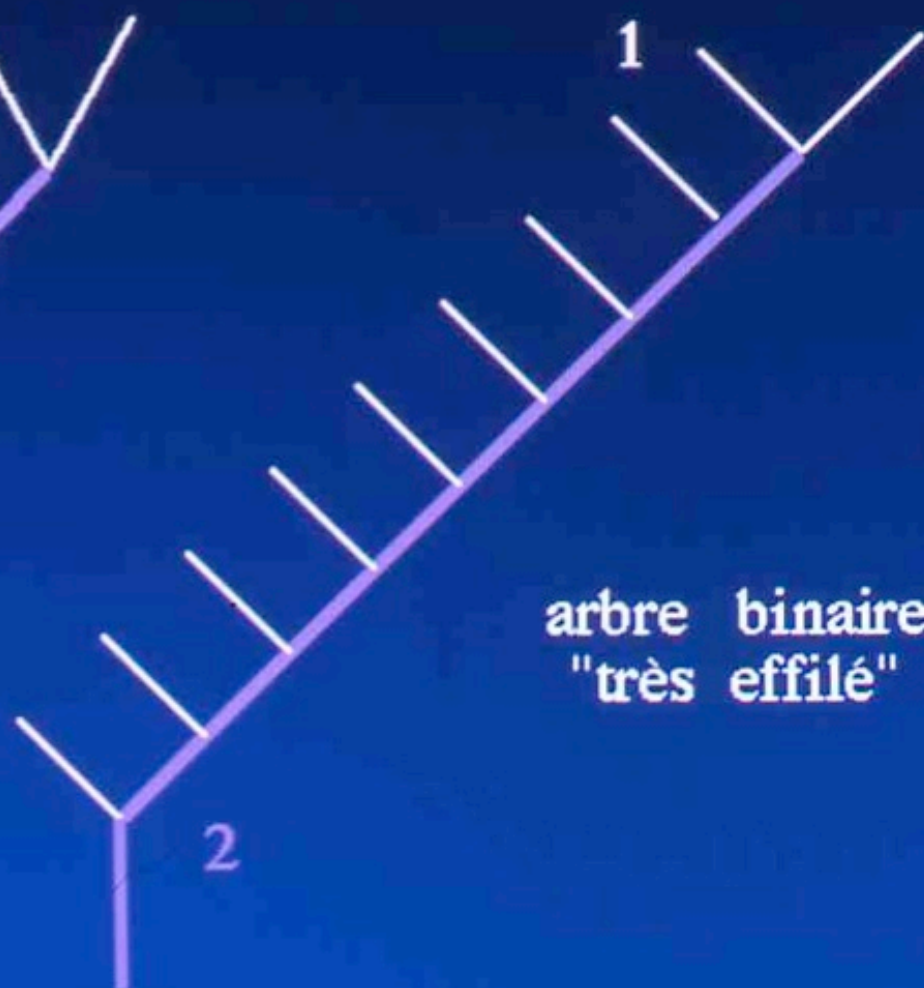
nombre de
Strahler



synthèse d'images d'arbres

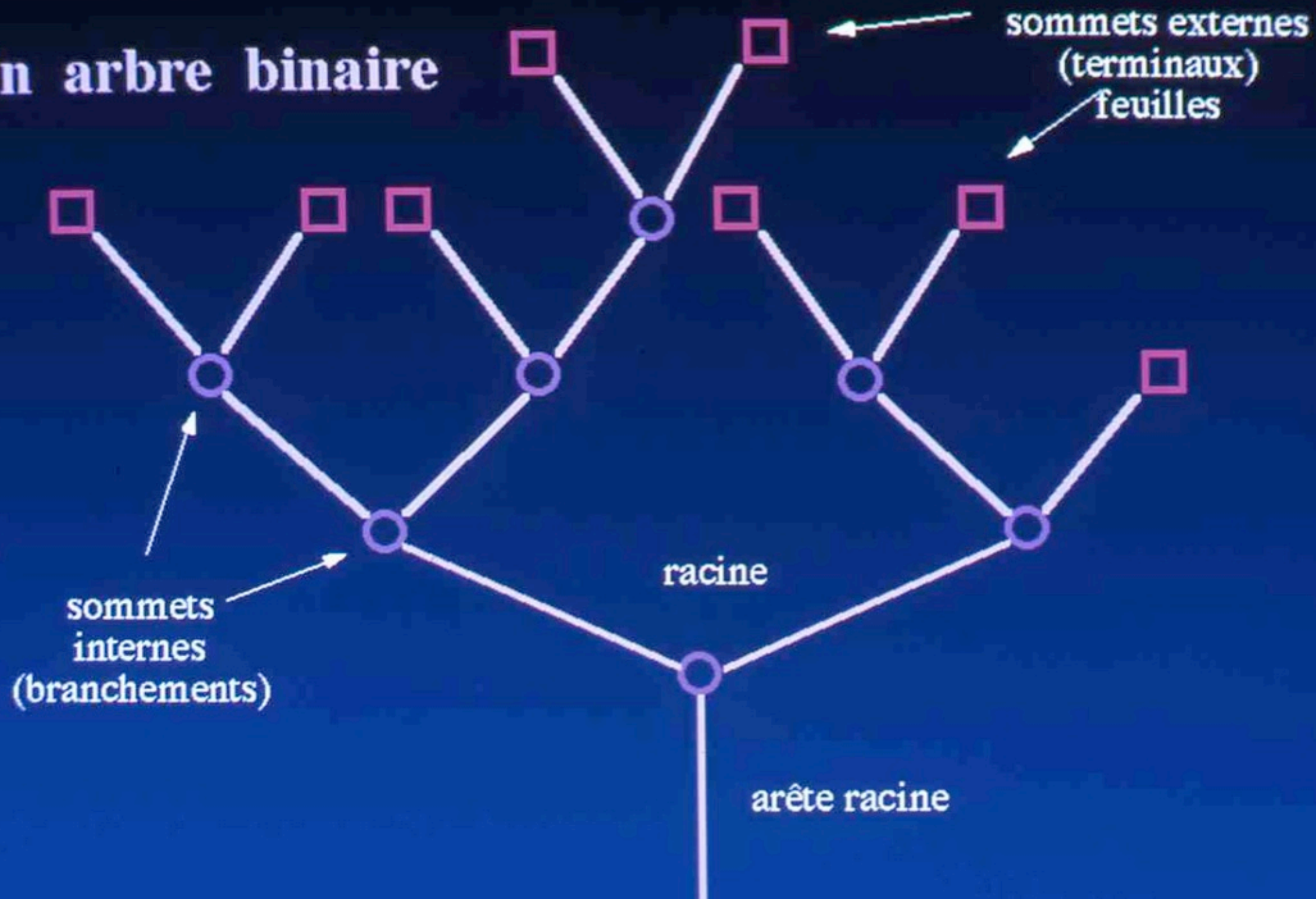


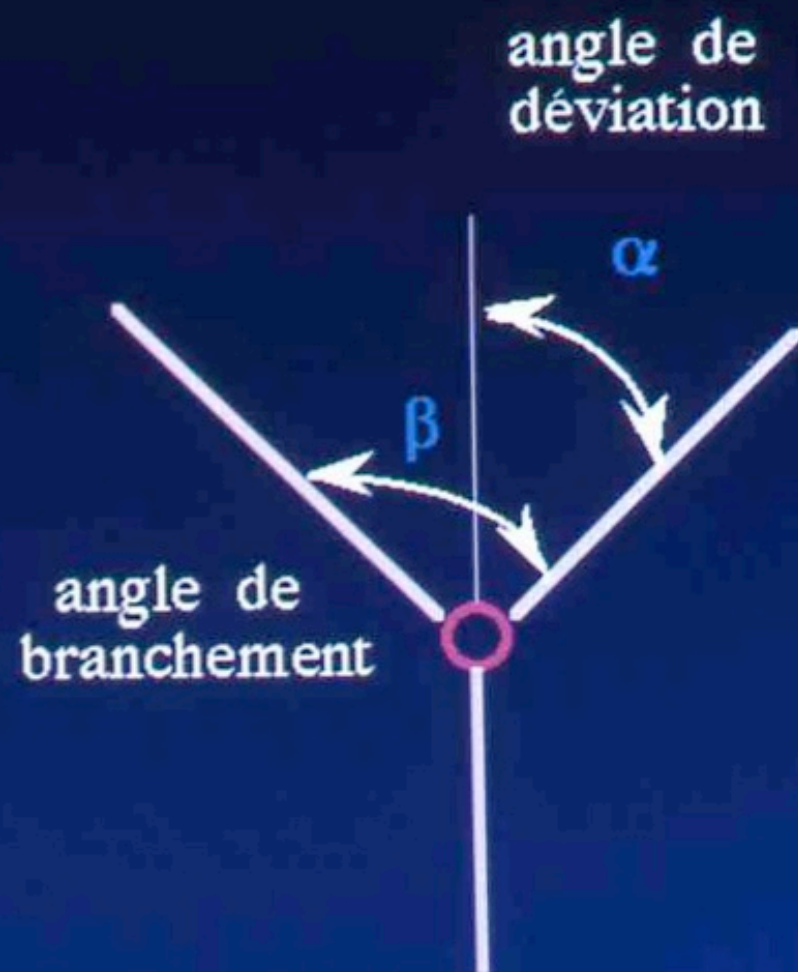
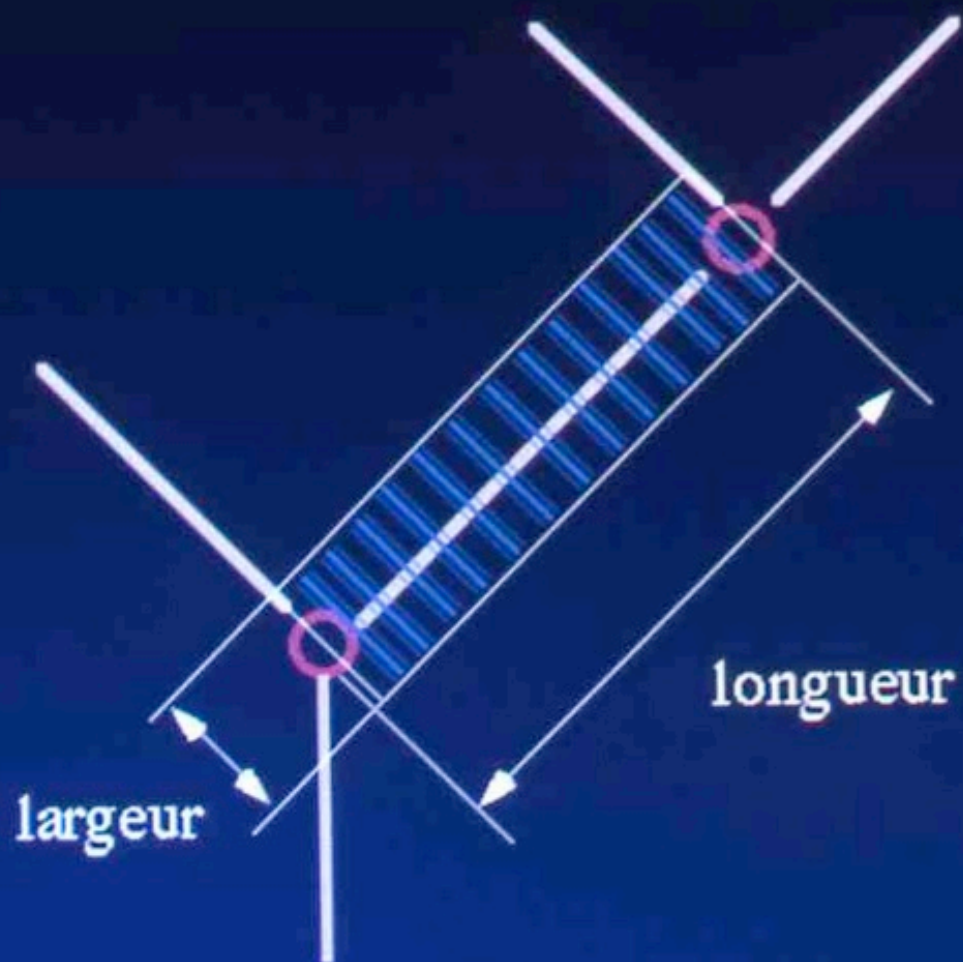
arbre binaire parfait



arbre binaire
"très effilé"

Un arbre binaire





Génération de l'arbre combinatoire

choix de la matrice de ramification

Génération de l'arbre géométrique

- largeur $w(k)$
 - longueur $L(k)$
- } fonction de l'ordre
- angle de déviation
 - angle de branchement
- } fonction du biordre

dessin en 2D







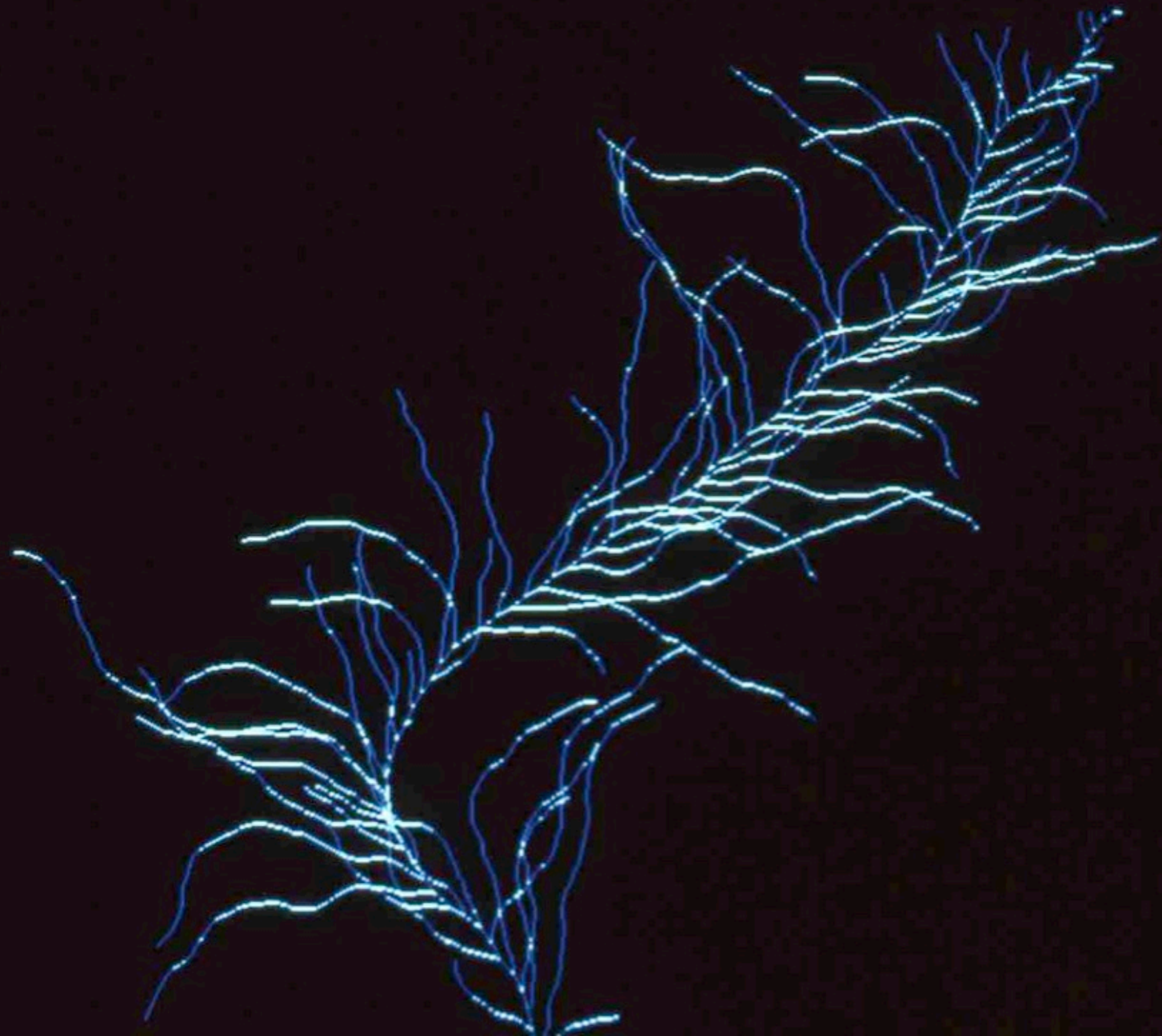




A\$A

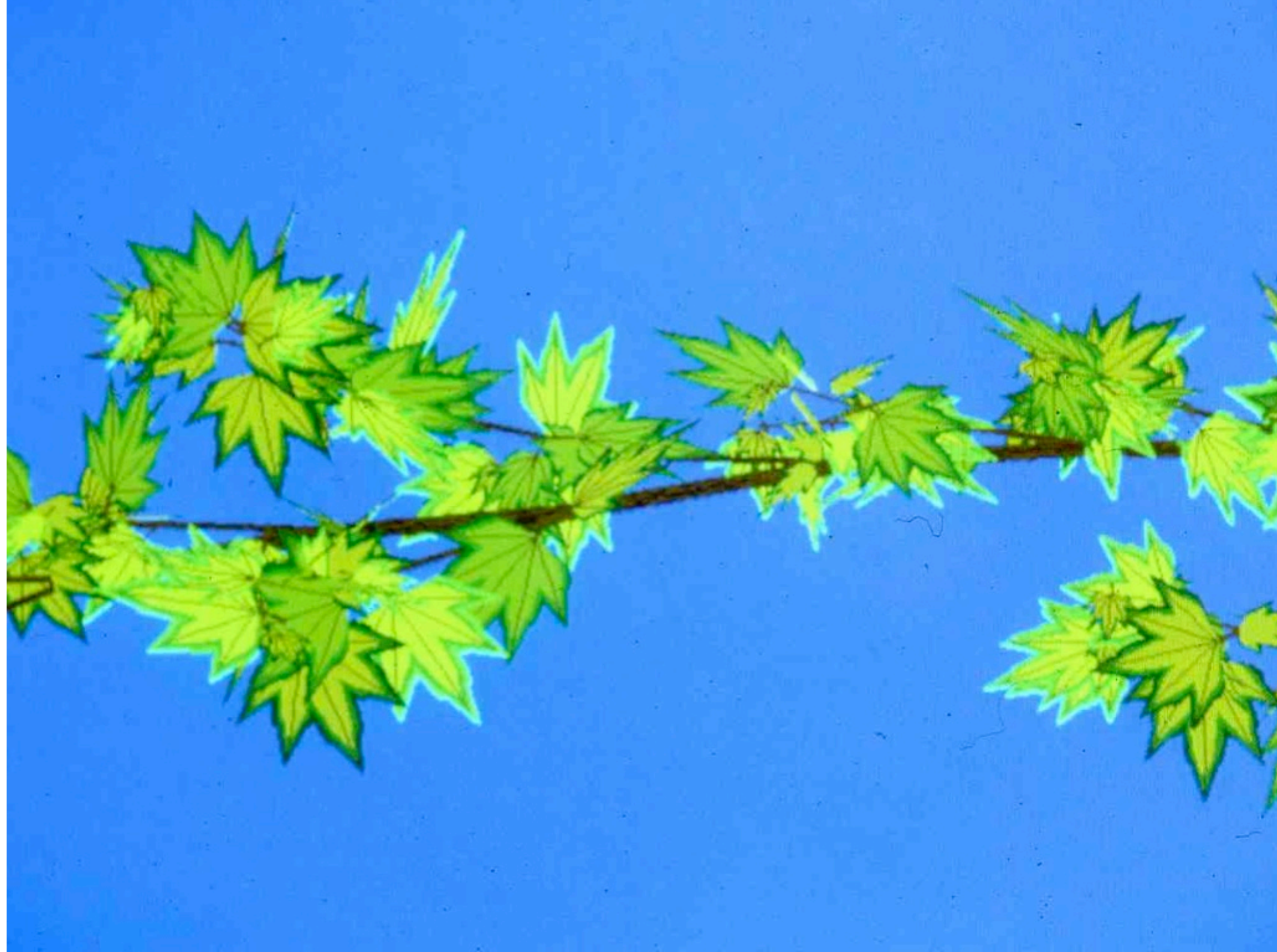














S'il existe une quelconque beauté dans
ces images de synthèse d'arbres, elle
n'est que le pâle reflet de l'extraordinaire
beauté des mathématiques se cachant
derrière les algorithmes générant ces
images.

$S_{n,k}$ = nombre d'arbres binaires
ayant n sommets internes
et nombre de Strahler
égal à k

$$S_k = \sum_{n \geq 0} S_{n,k} t^n$$

$$S_1 = 1$$

$$S_2 = \frac{t}{1 - 2t}$$

$$S_3 = \frac{t^3}{1 - 6t + 10t^2 - 4t^3}$$

$$S_1 = 1$$

$$S_2 = \frac{t}{1 - 2t}$$

$$S_3 = \frac{t^3}{1 - 6t + 10t^2 - 4t^3}$$

$$S_4 = \frac{t^7}{1 - 14t + 78t^2 - 220t^3 + 330t^4 - 252t^5 + 84t^6 - 8t^7}$$

1, 3, 21, 987, ...

1, 2, 3, 5, 8, 13, 21, 34, 55,

89, 144, 233, 377, 610, 987,

① 2, ③ 5, 8, 13, ②1 34, 55,

89, 144, 233, 377, 610, ⑨87,

① 2, ③ 5, 8, 13, ② 34, 55,
89, 144, 233, 377, 610, ⑨ 987,

Fibonacci ! F_{15}

F_{2^k-1}

Pascal Triangle

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

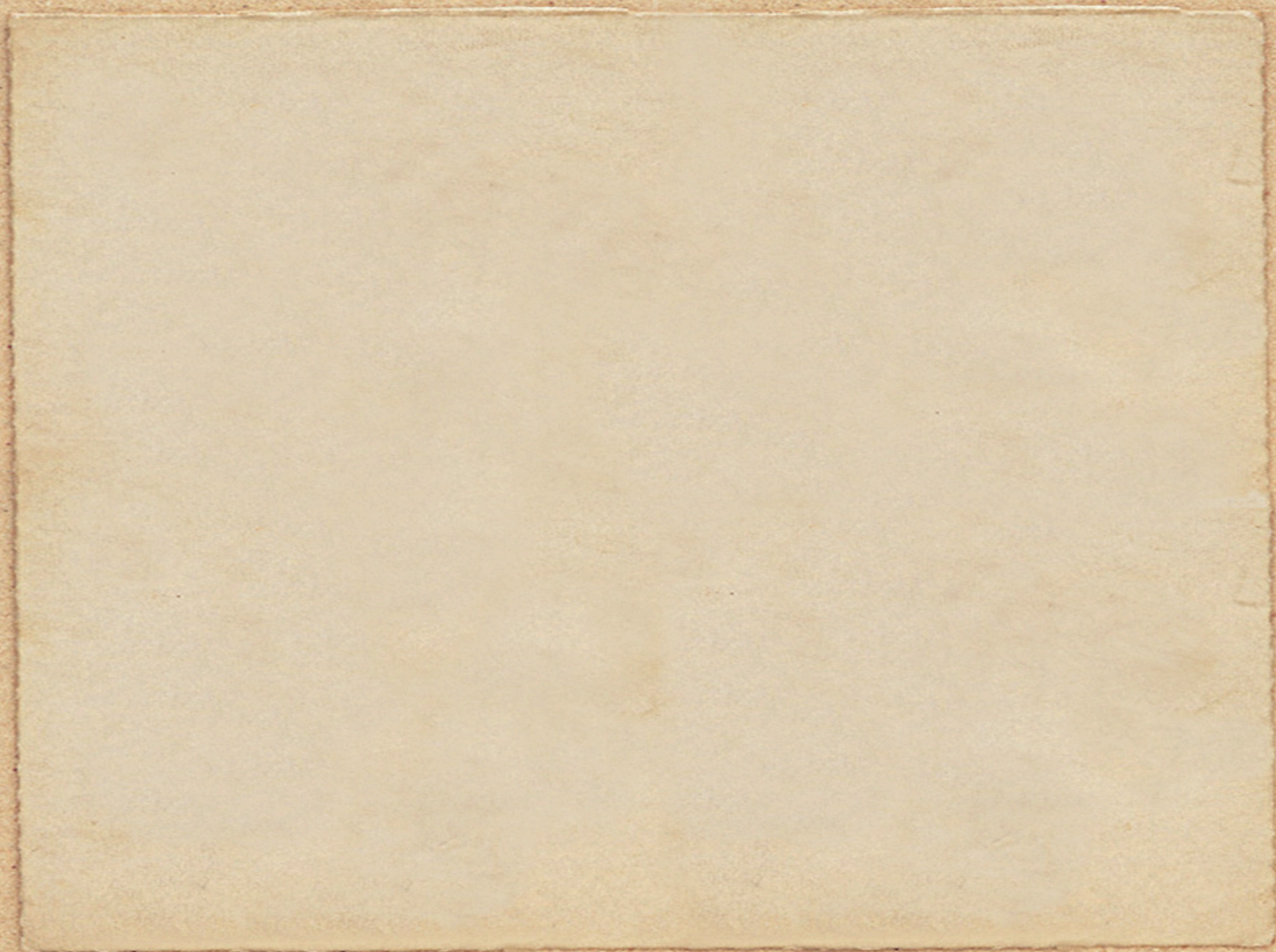
1										
1	1									
1	2	1								
1	3	3	1							
1	4	6	4	1						
1	5	10	10	5	1					
1	6	15	20	15	6	1				
1	7	21	35	35	21	7	1			
1	8	28	56	70	56	28	8	1		
1	9	36	84	126	126	84	36	9	1	
1	10	45	120	210	252	210	120	45	10	
1	11	55	165	330	462	462	330	165	55	
1	12	66	220	495	792	924	792	495	220	
1	13	78	286	715	1287	1716	1716	1287	715	
1	14	91	364	1001	2002	3003	3432	3003	2002	
1	15	105	455	1365	3003	5005	6435	6435	5005	

Pascal Triangle

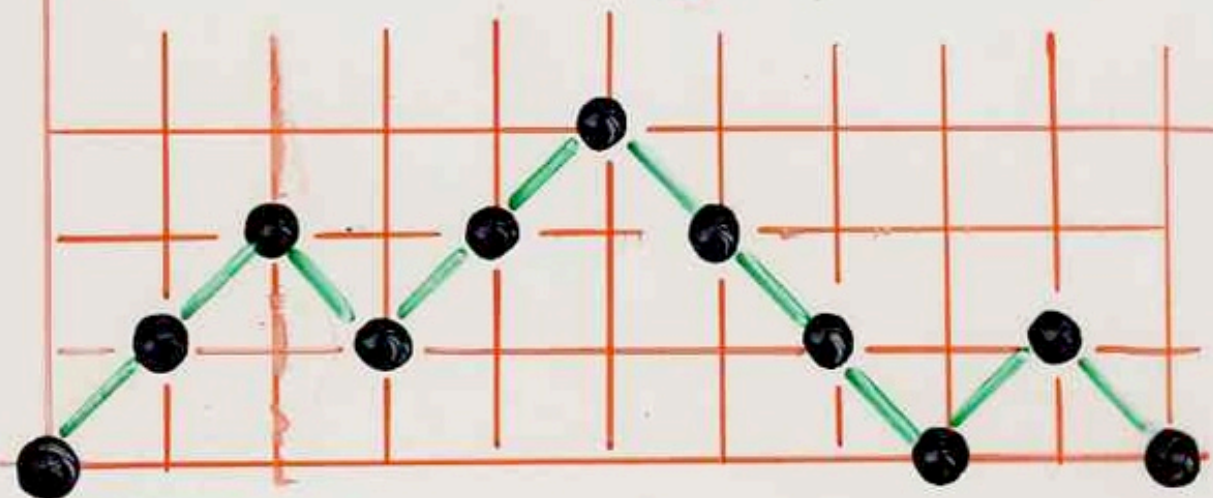
$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

1															
1	1														
1	2	1													
1	3	3	1												
1	4	6	4	1											
1	5	10	10	5	1										
1	6	15	20	15	6	1									
1	7	21	35	35	21	7	1								
1	8	28	56	70	56	28	8	1							
1	9	36	84	126	126	84	36	9	1						
1	10	45	120	210	252	210	120	45	10	1					
1	11	55	165	330	462	462	330	165	55	11	1				
1	12	66	220	495	792	924	792	495	220	66	12	1			
1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1		
1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1	
1	15	105	455	1365	3003	5005	6435	6435	5005	1365	455	105	15	1	





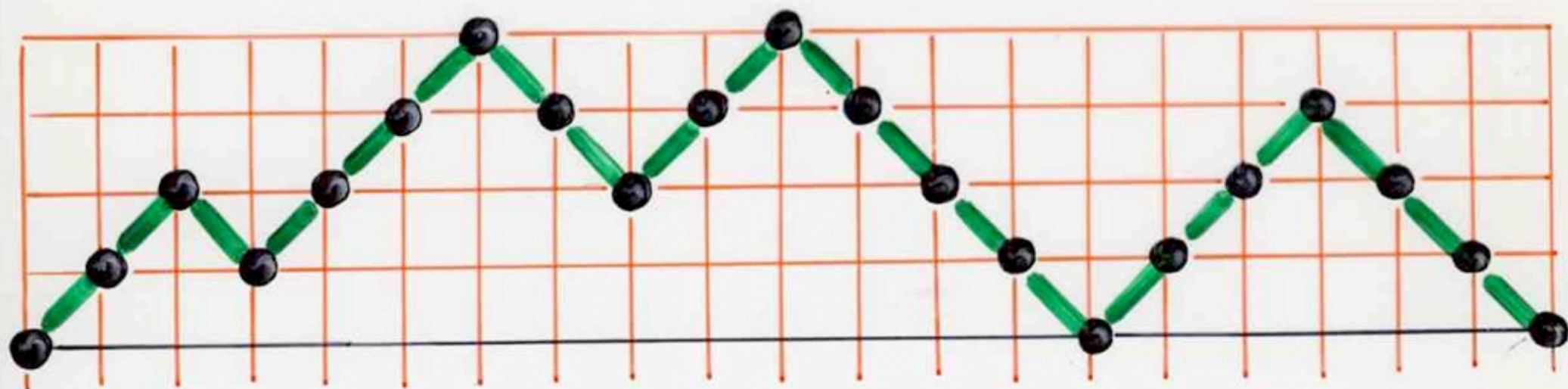
chemin
de
Dyck

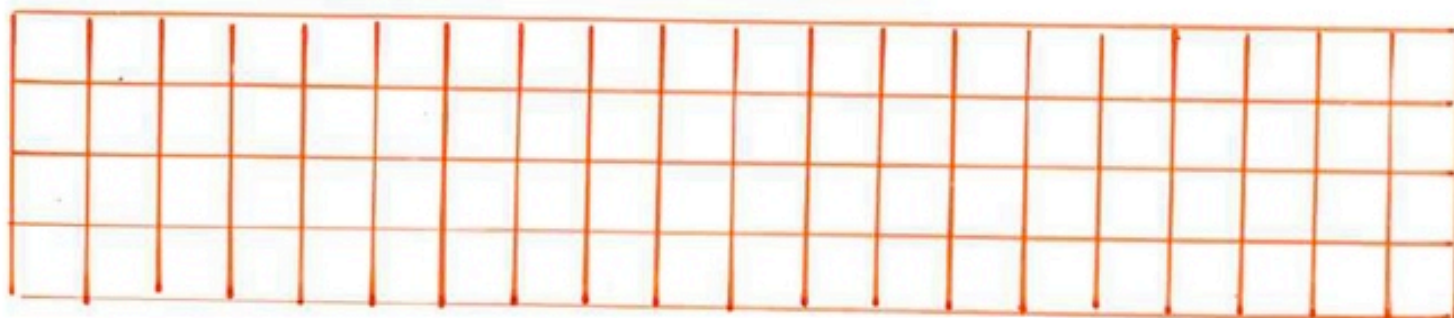
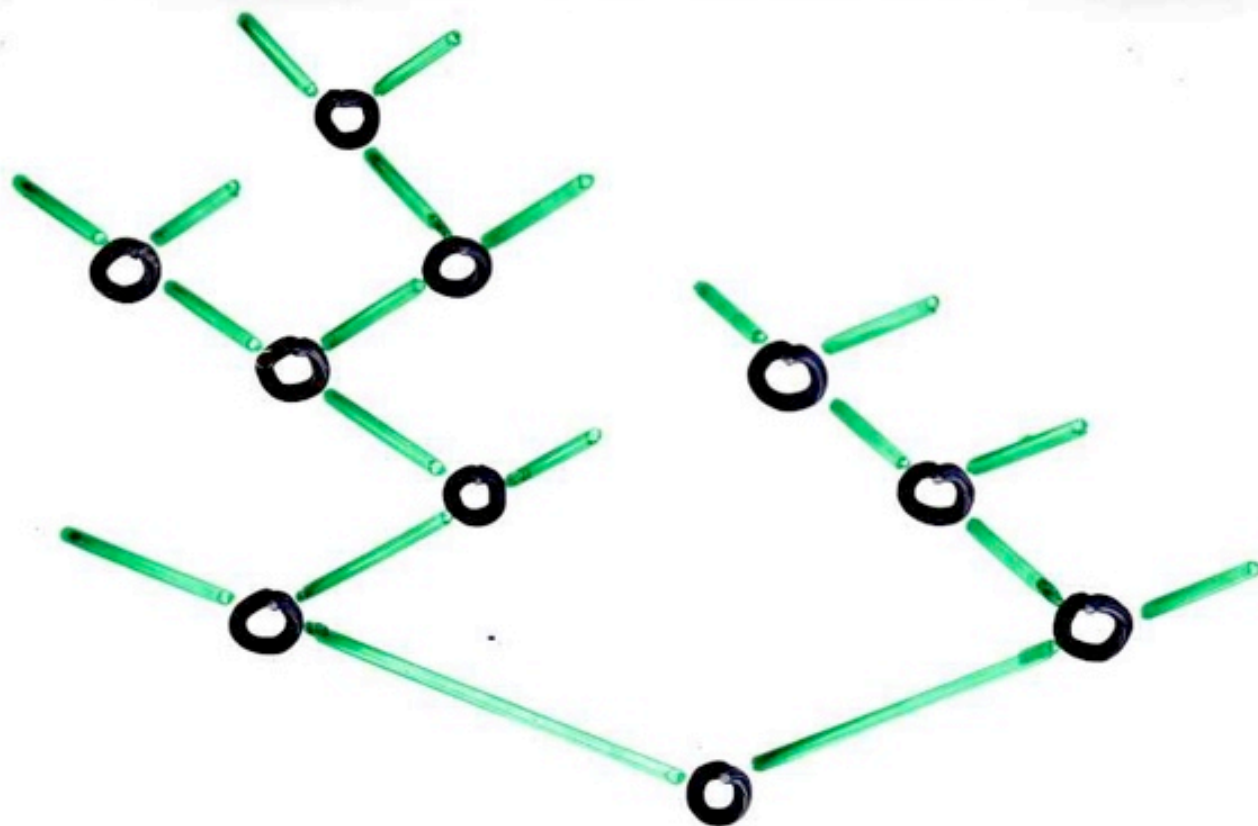


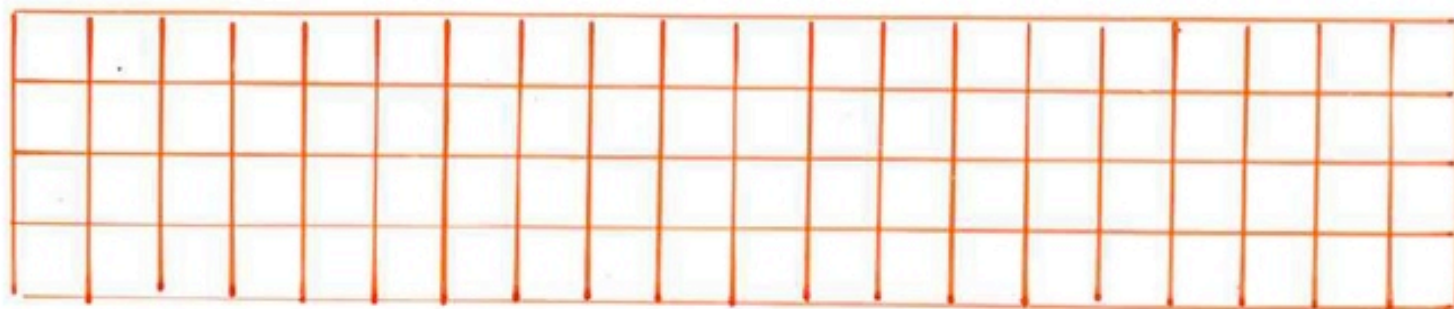
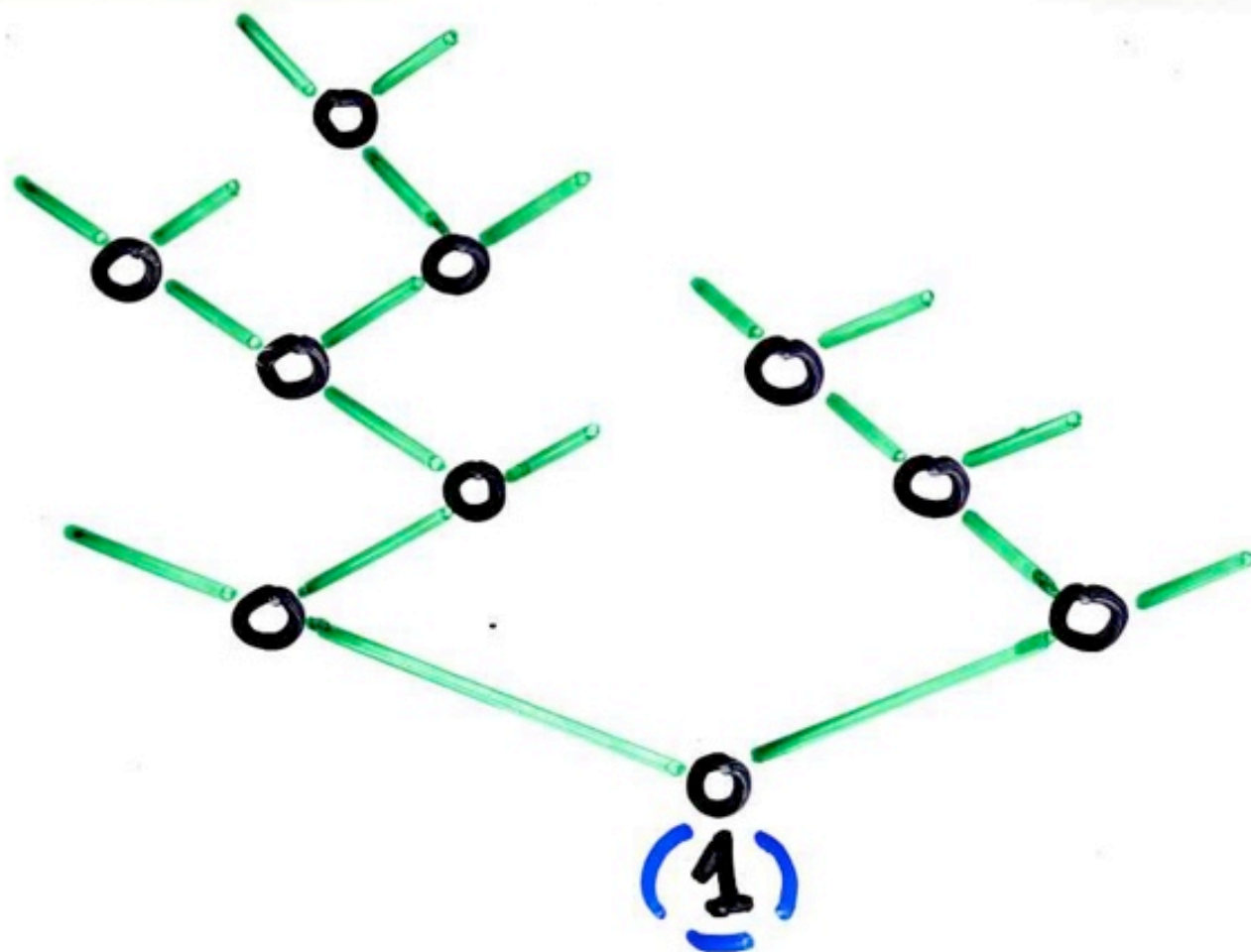
$$\frac{1}{n+1} \binom{2n}{n}$$

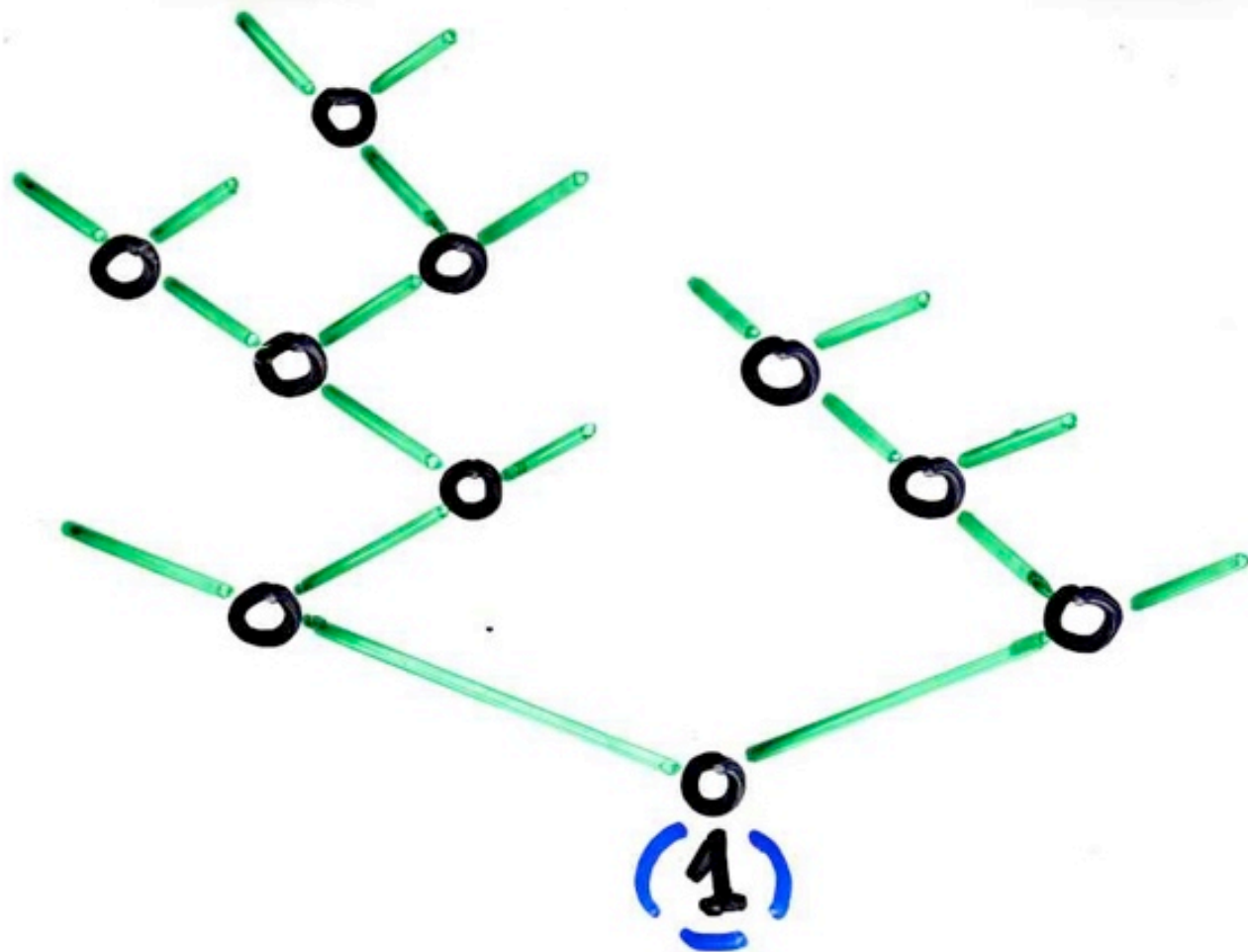
Catalan

des arbres binaires ...
aux chemins de Dyck

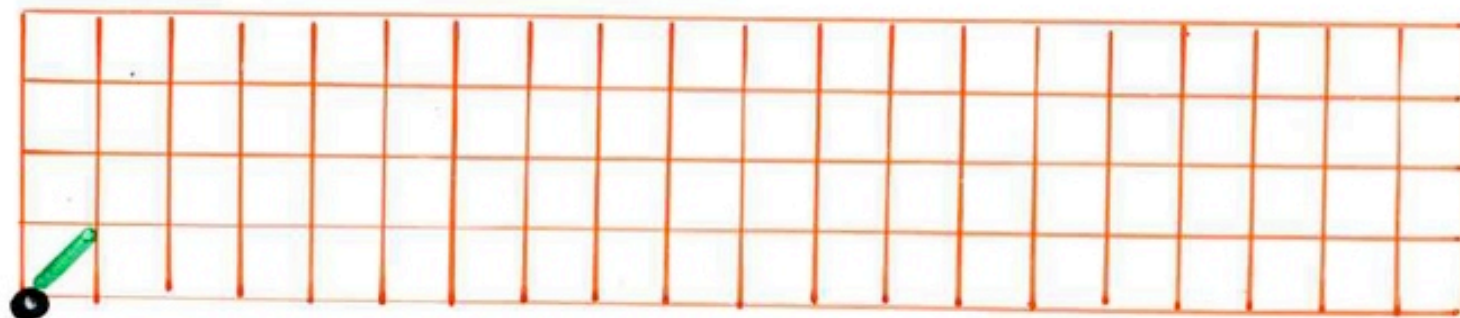
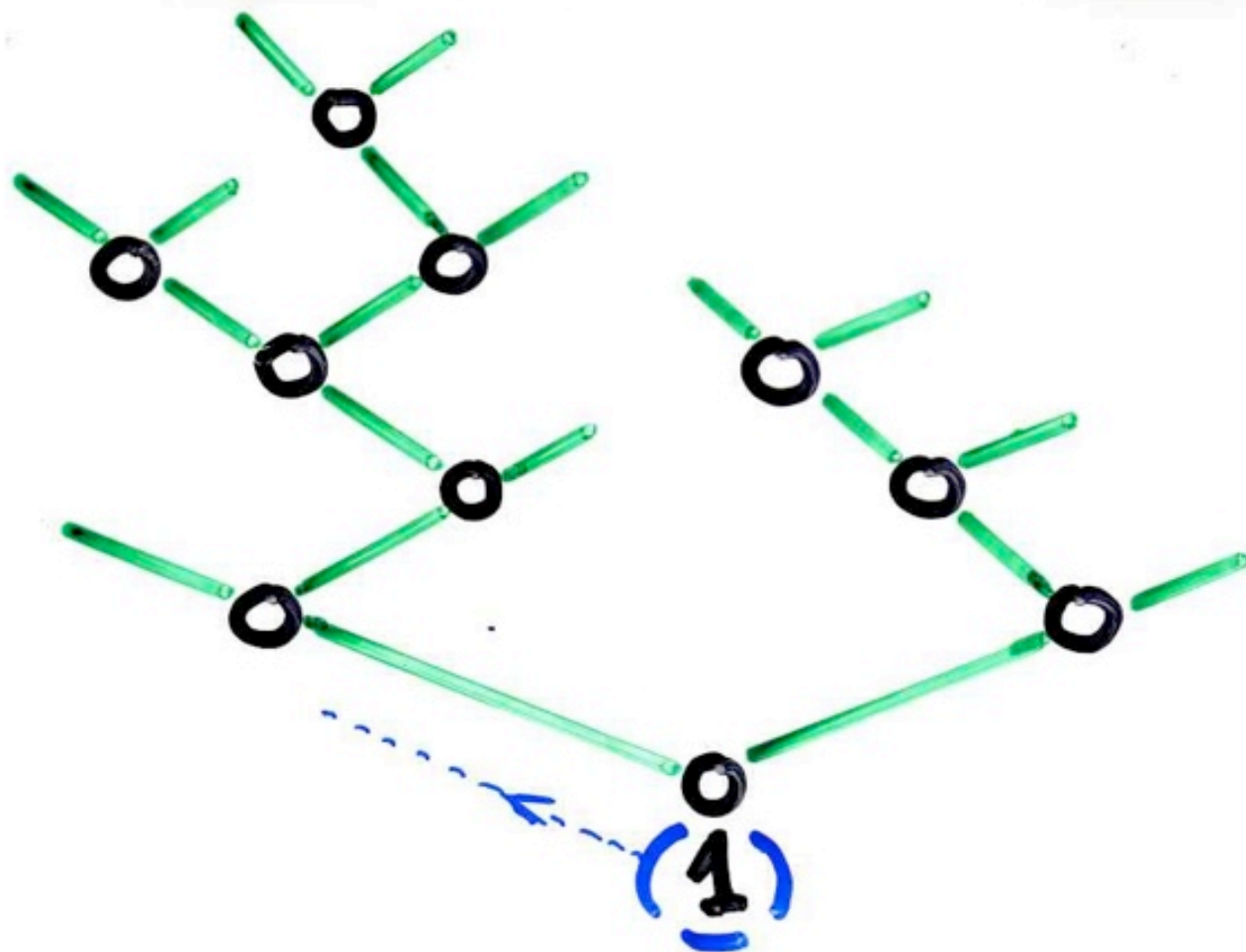


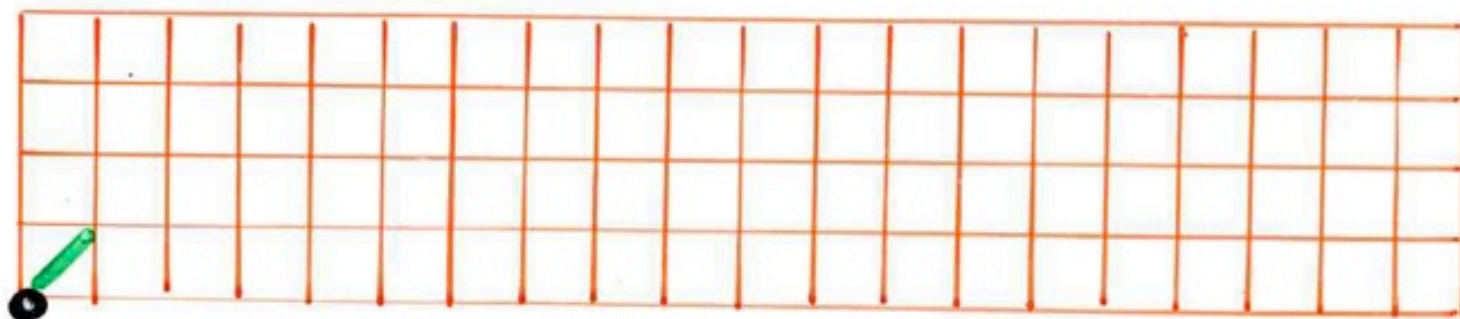
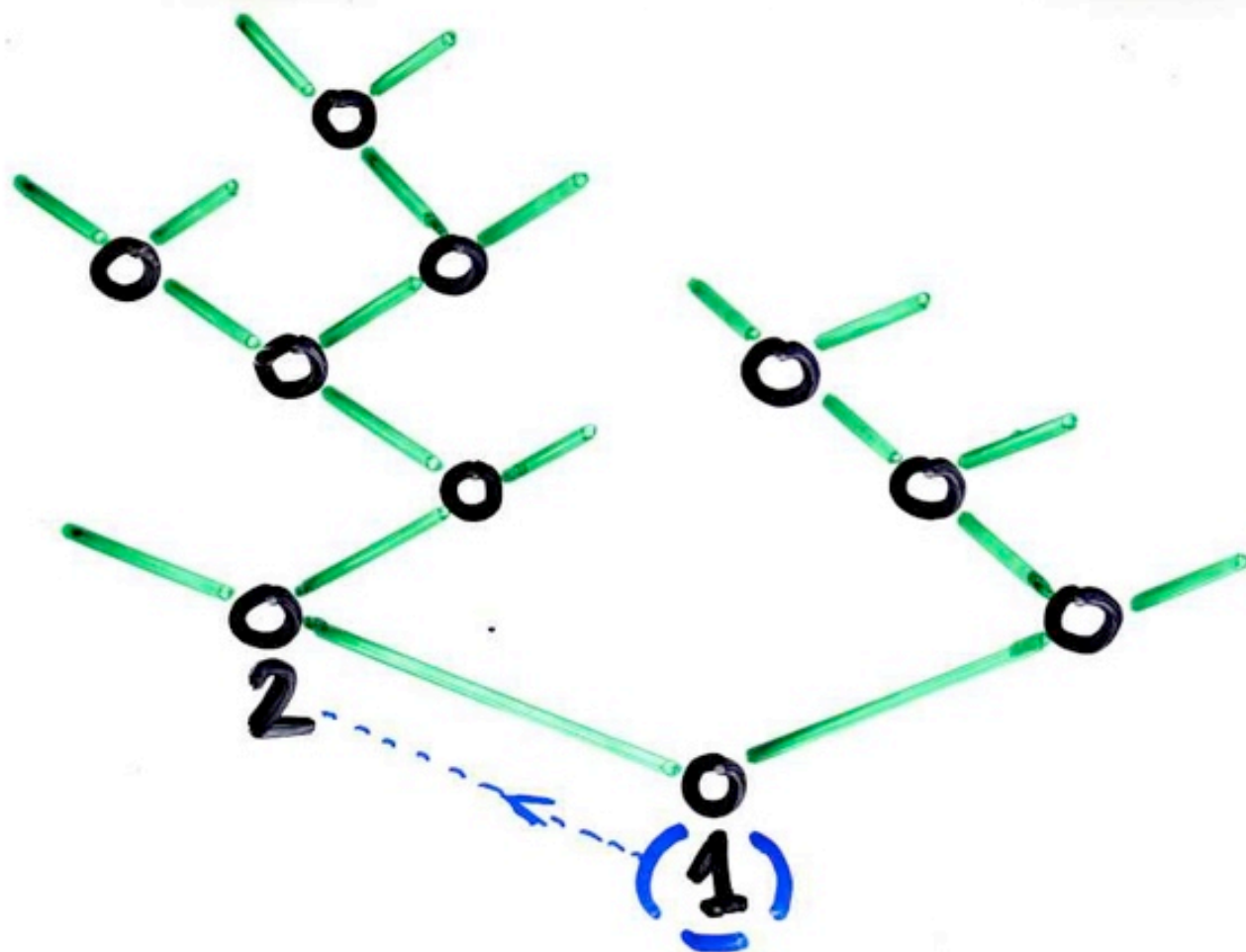


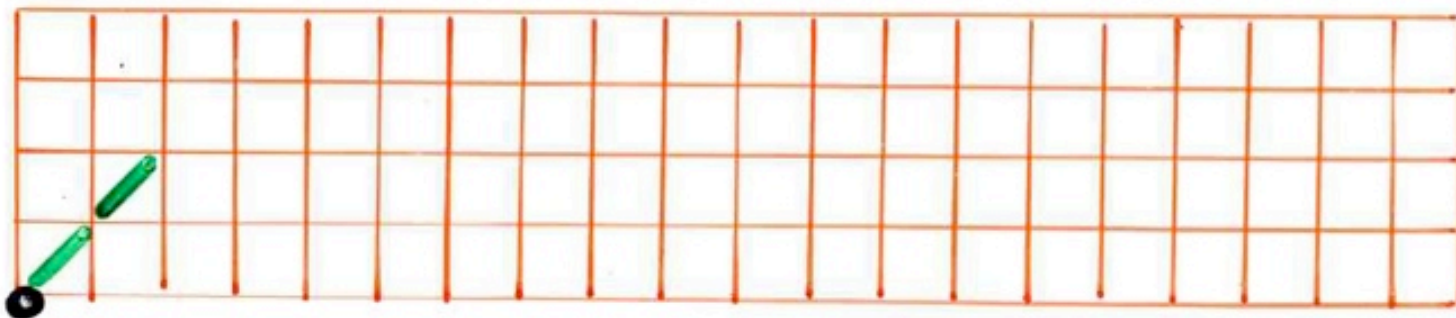
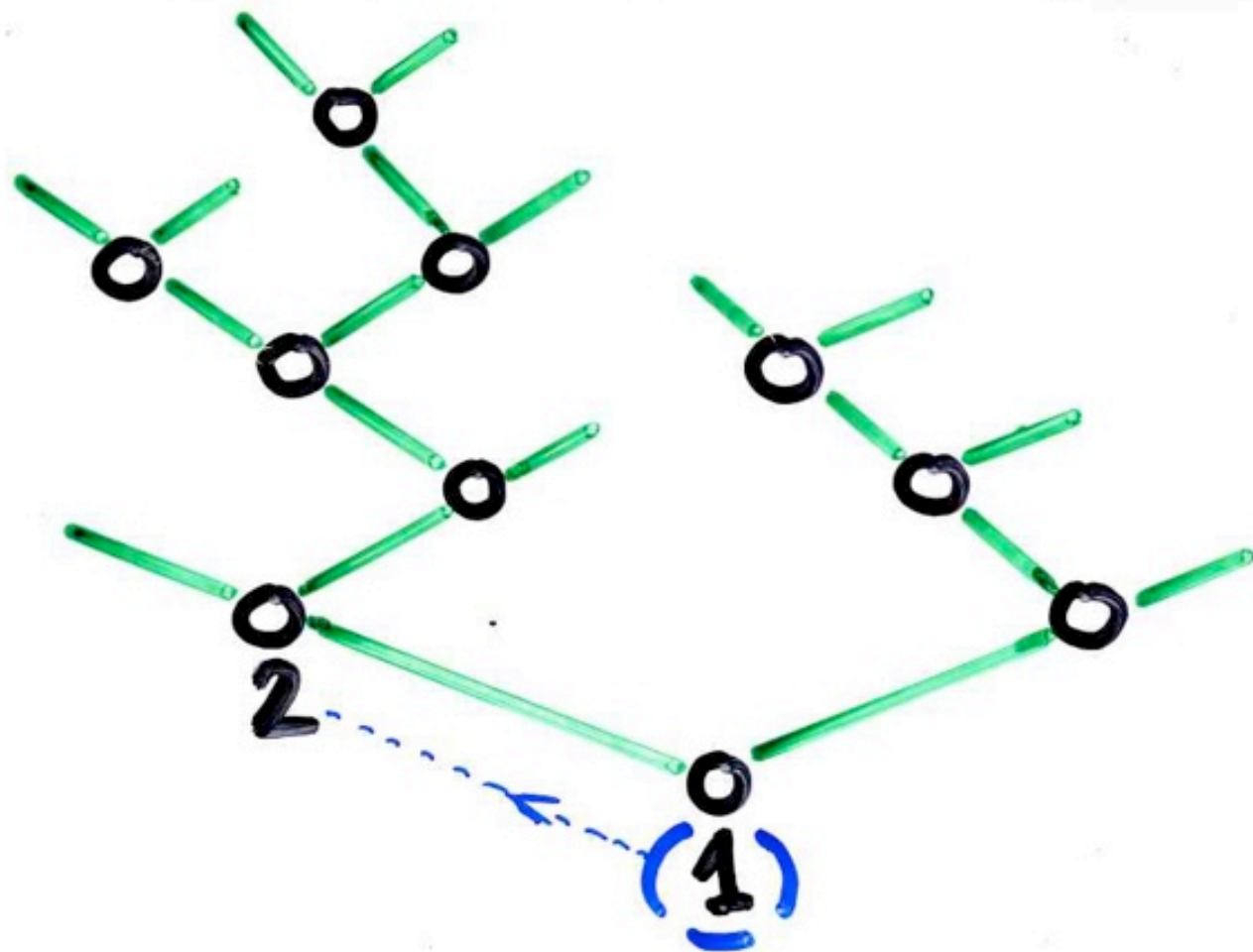


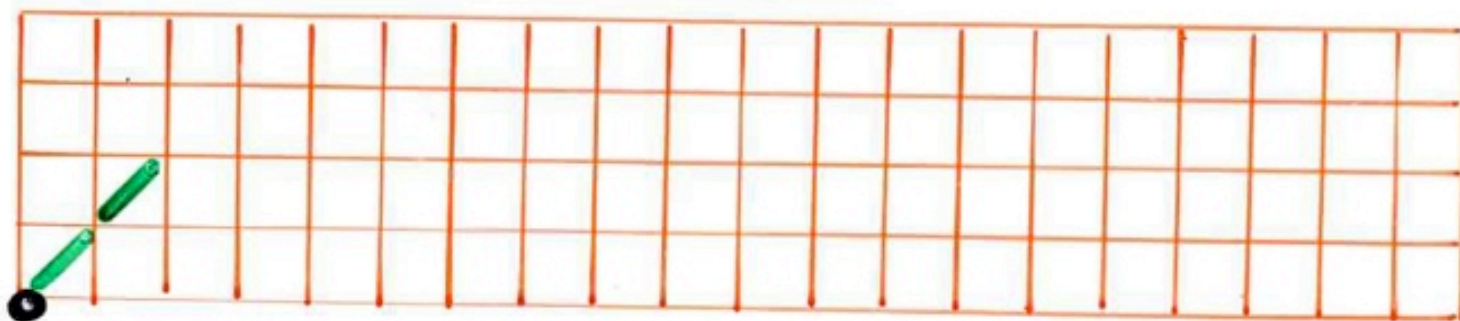
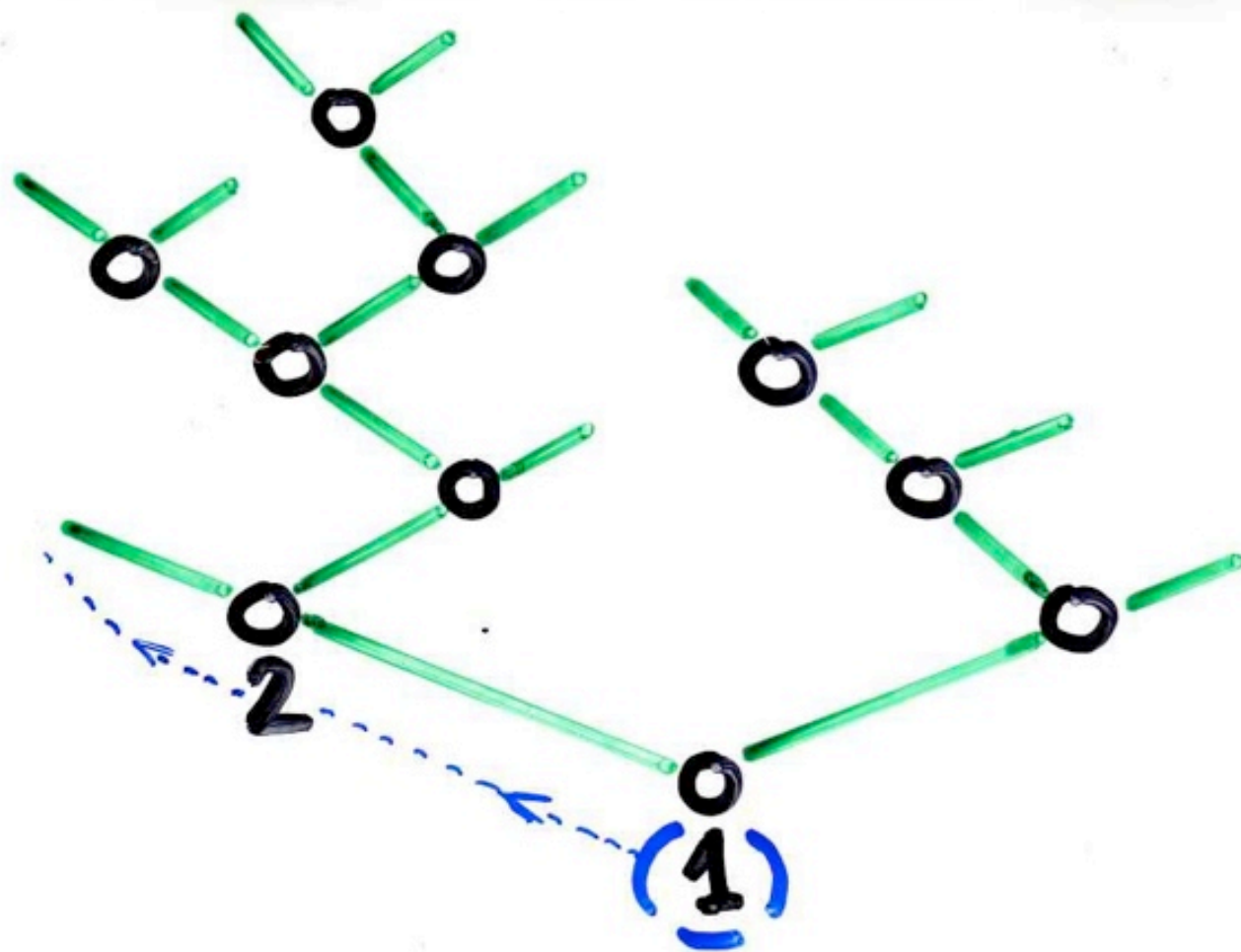


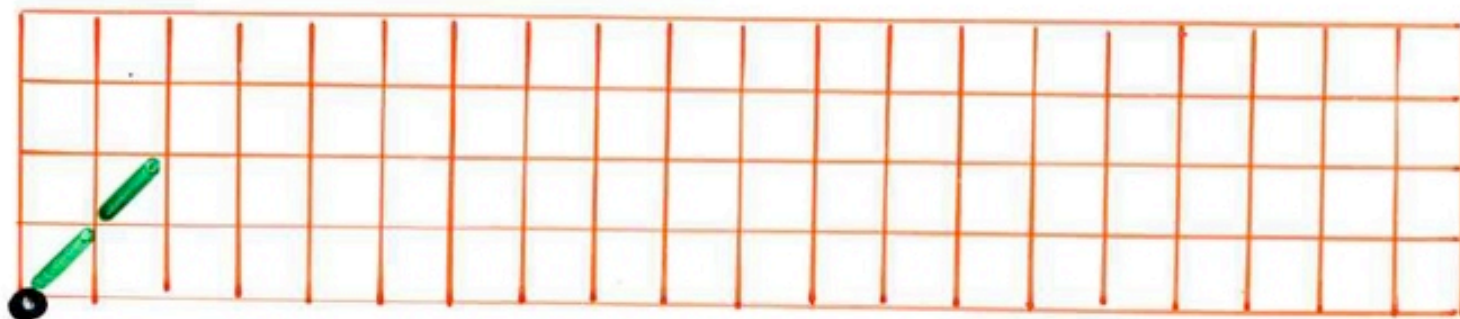
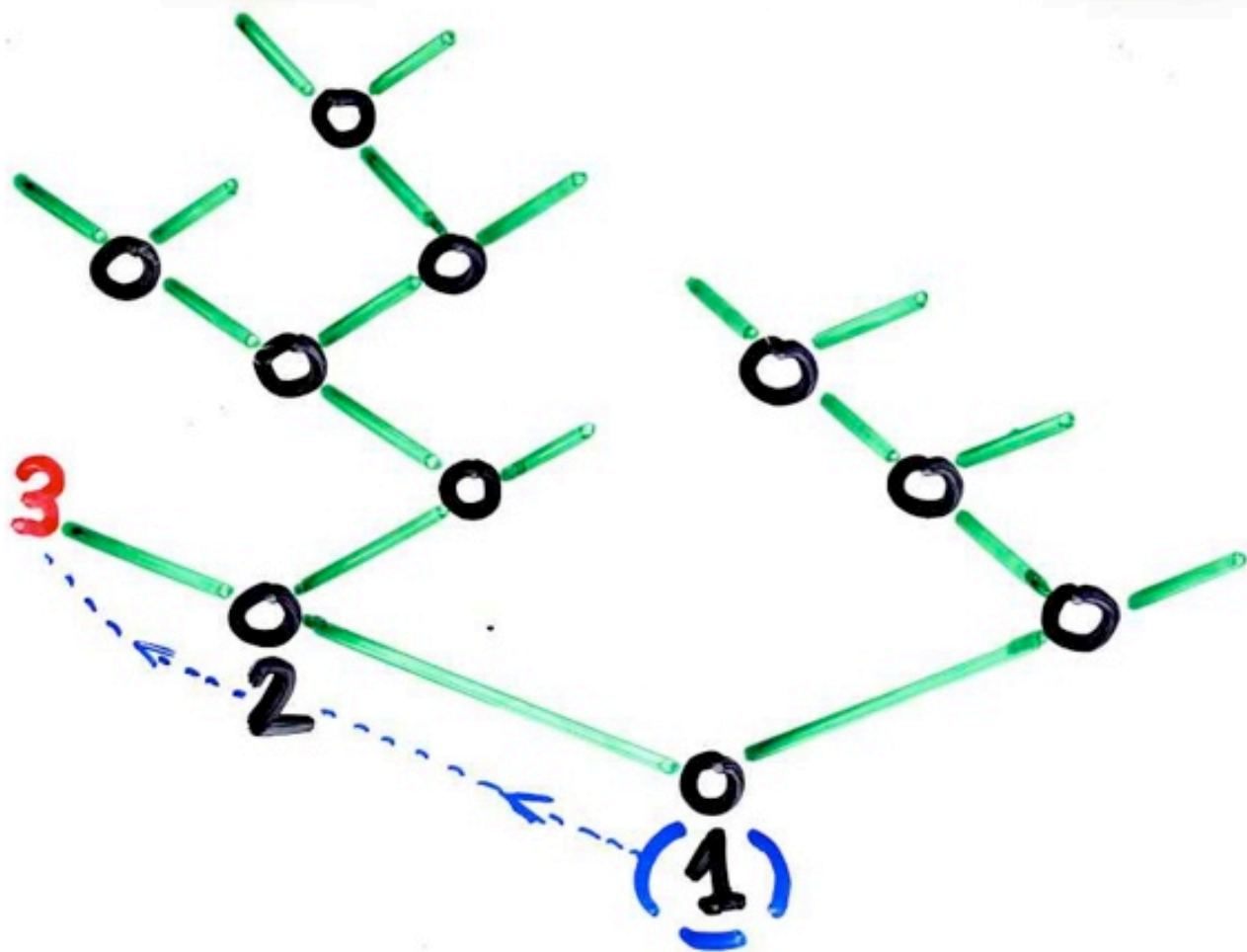
A grid consisting of 20 columns and 5 rows, outlined in orange. A small green line segment is attached to the bottom-left corner of the grid, extending downwards and to the left.

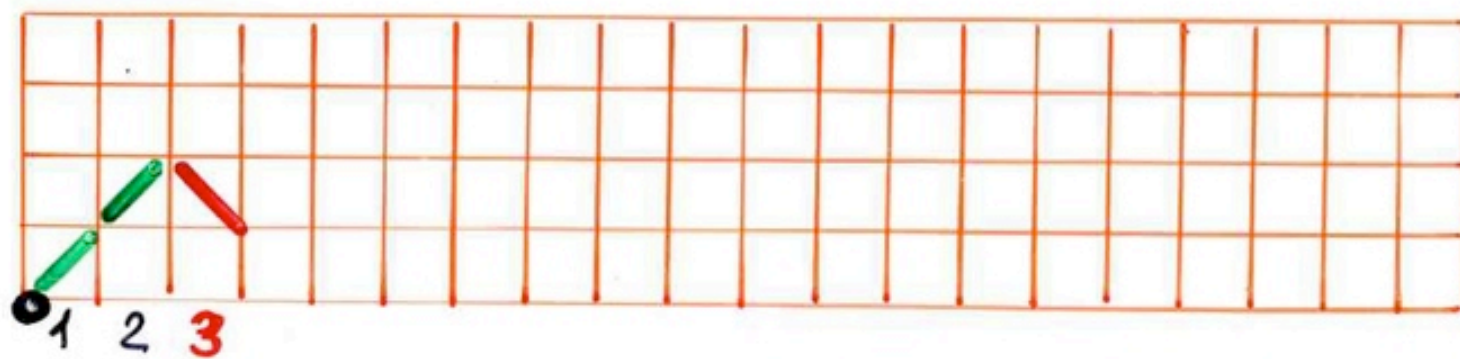
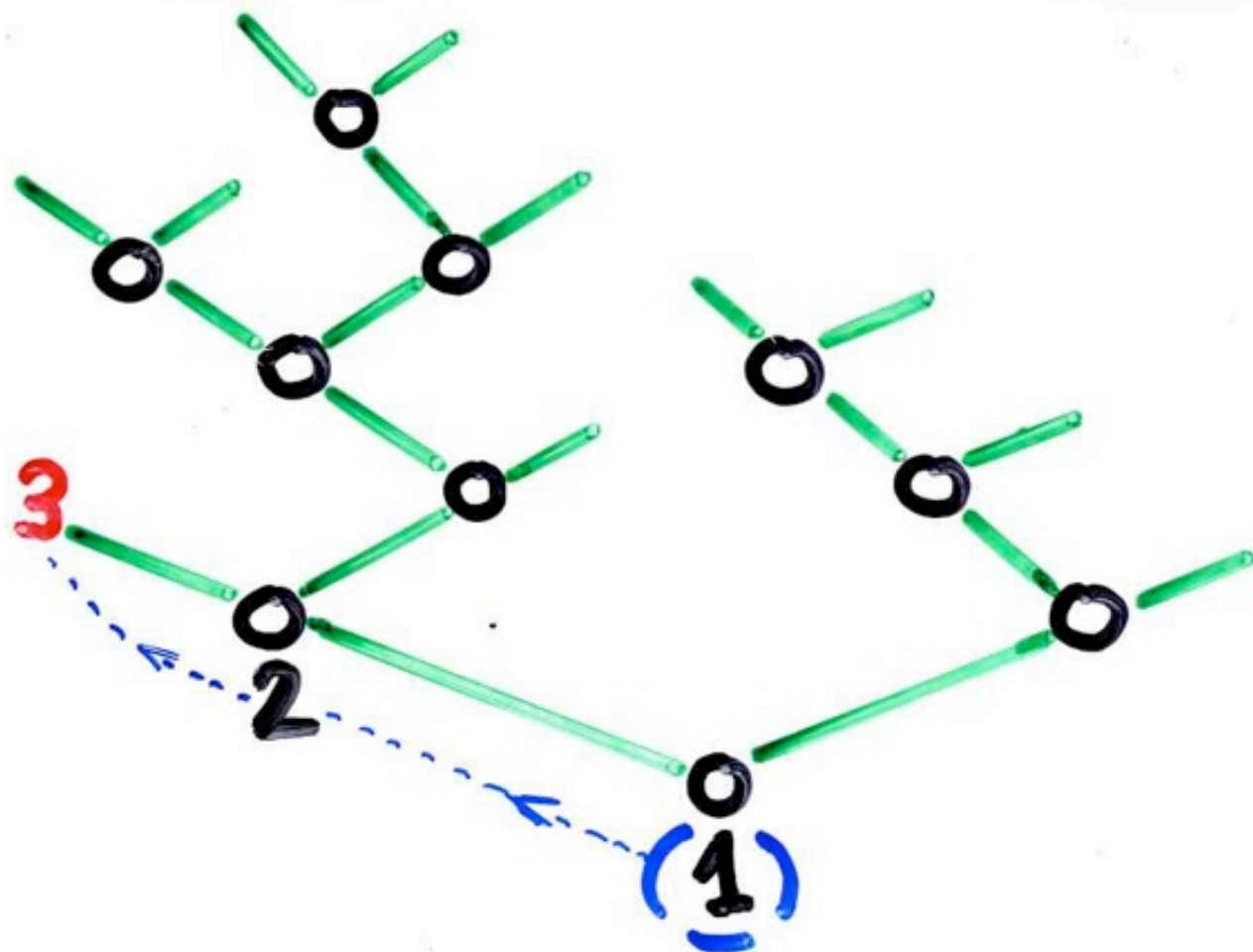


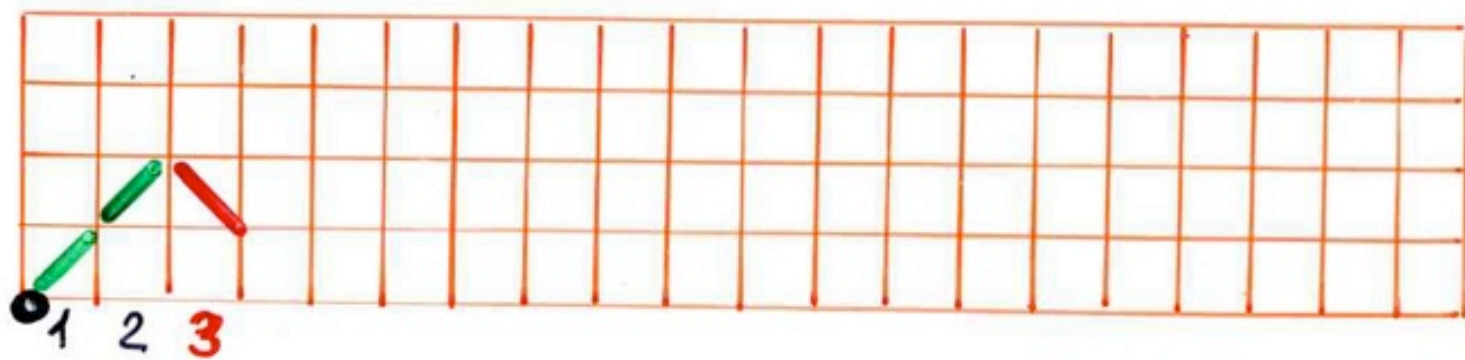
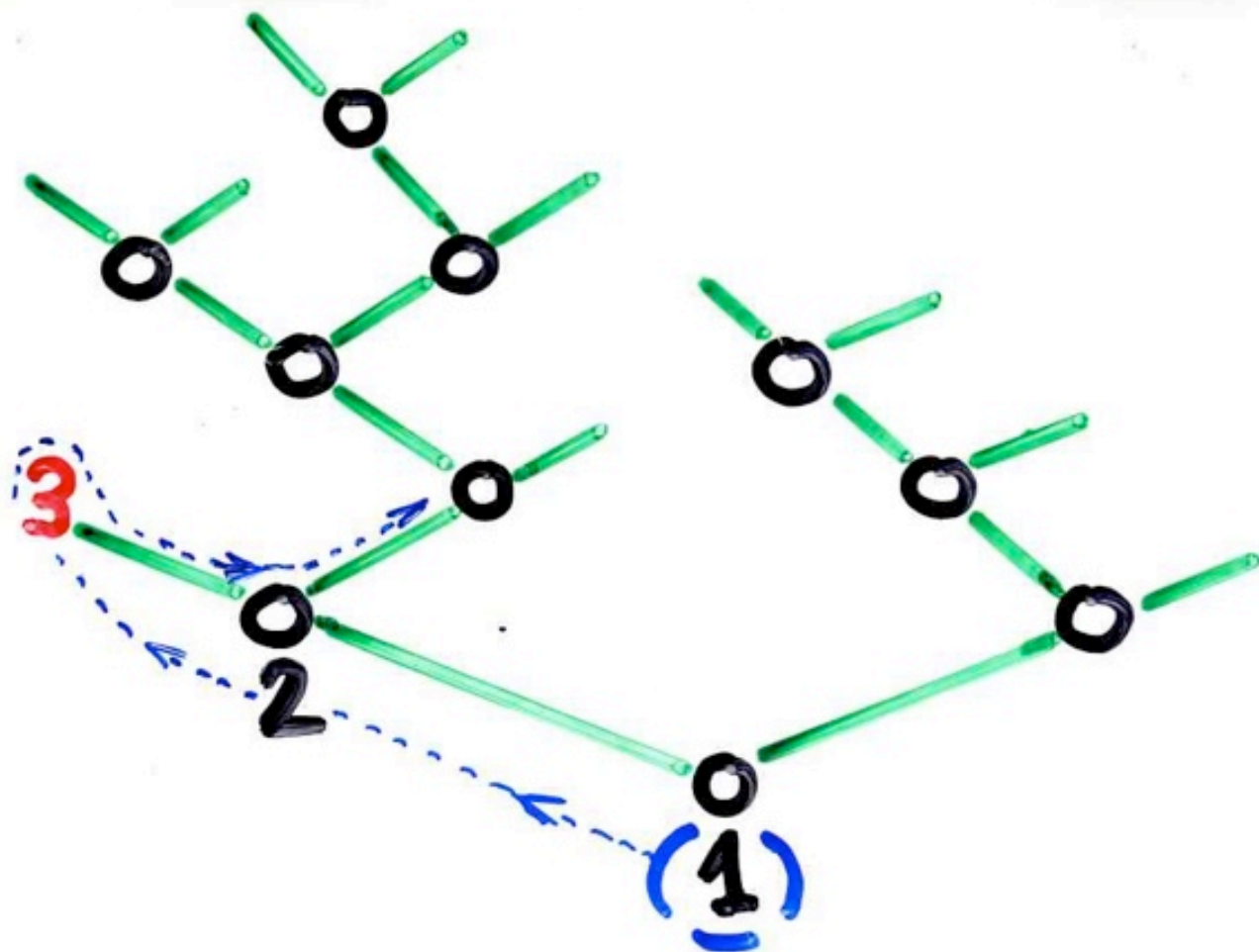


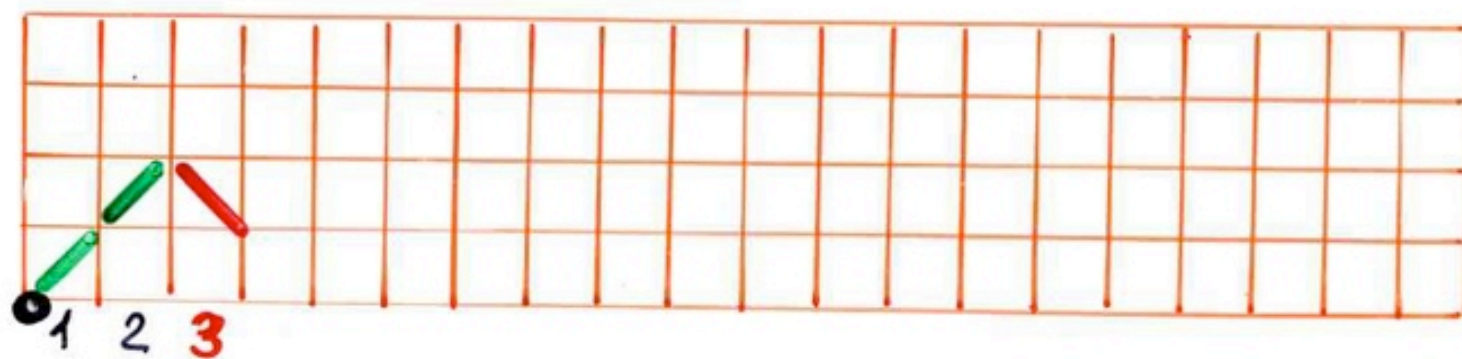
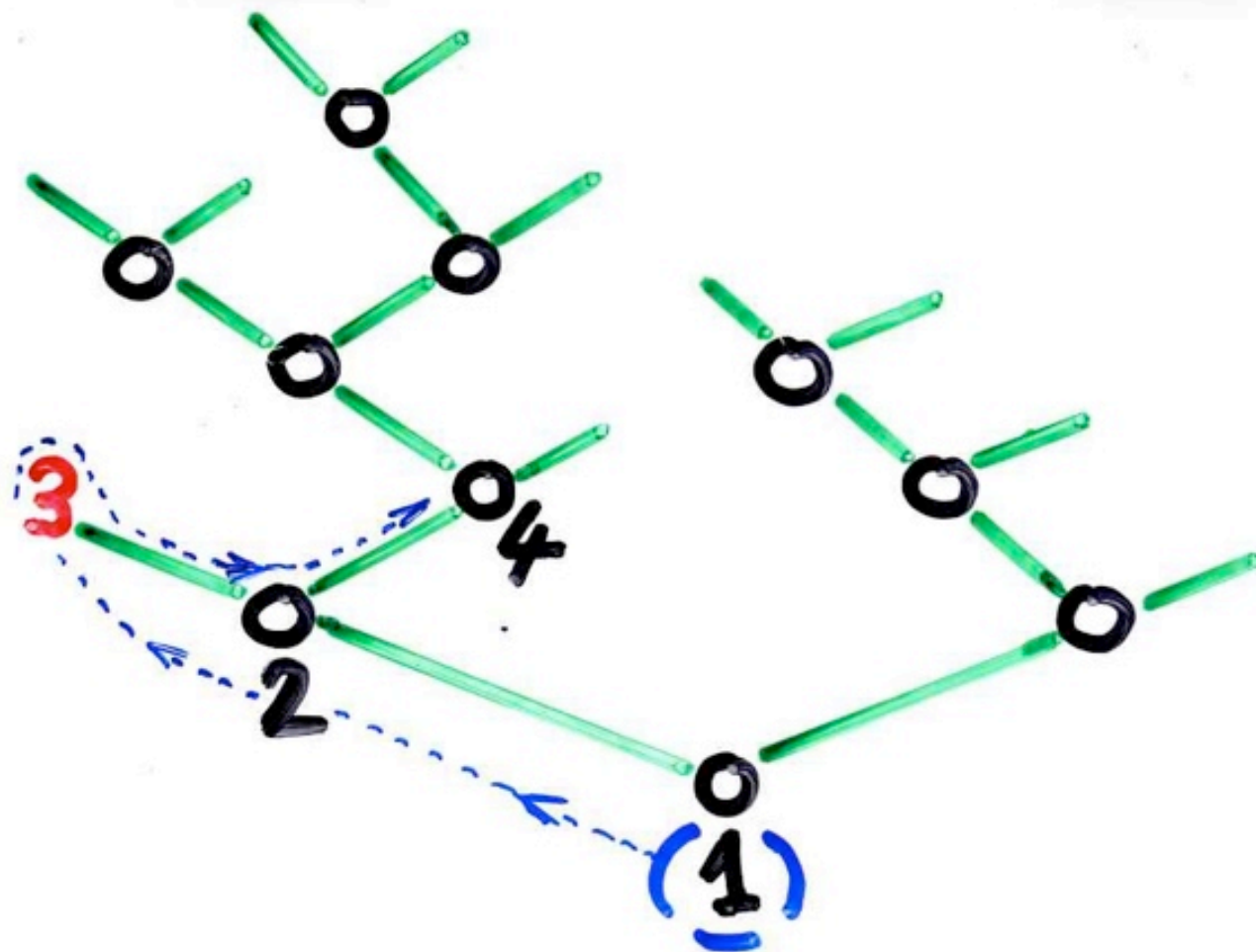


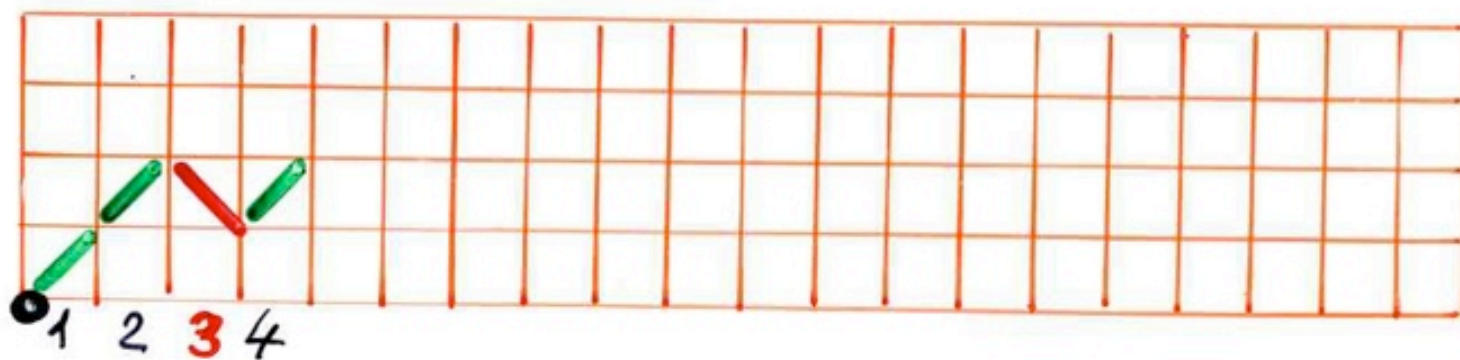
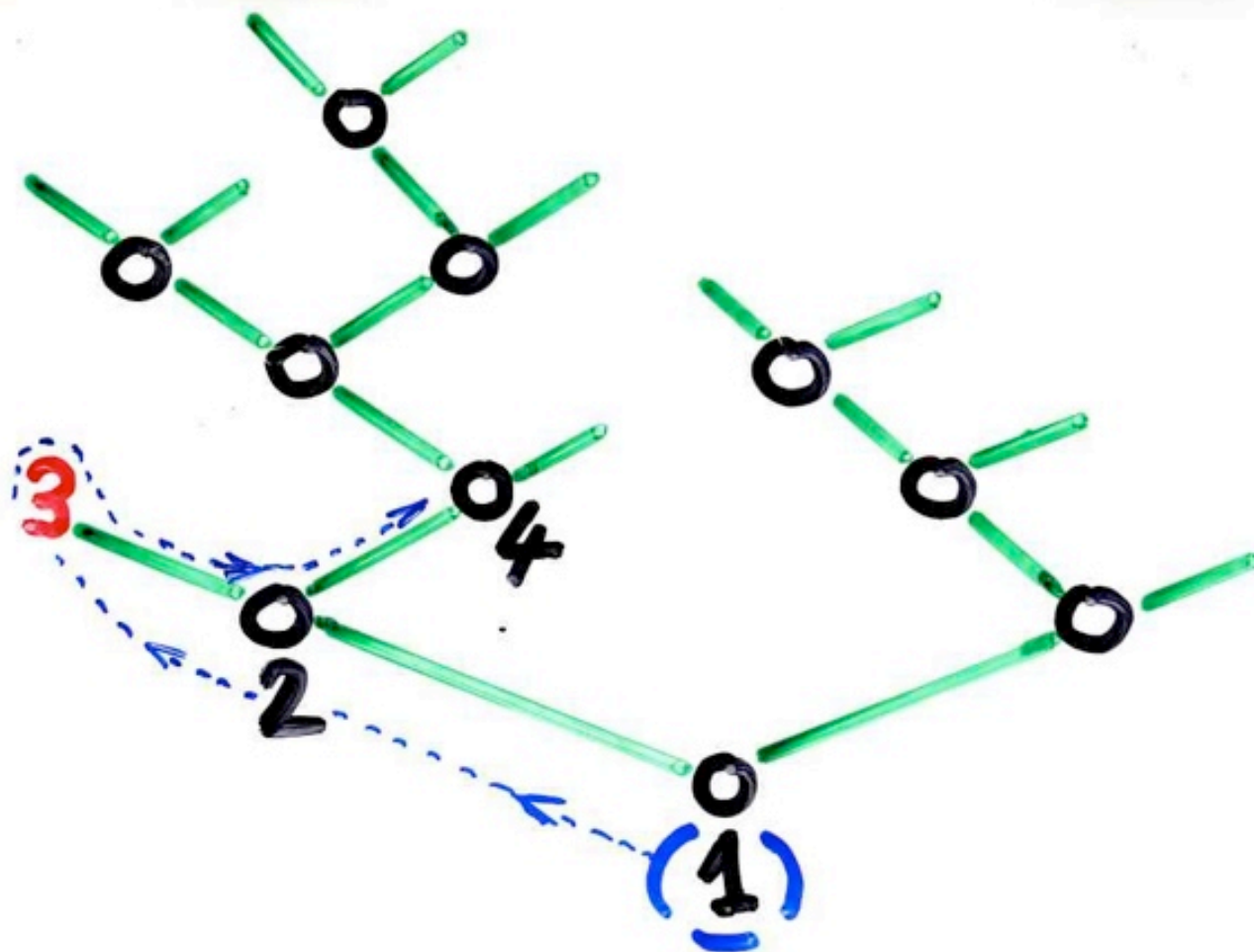


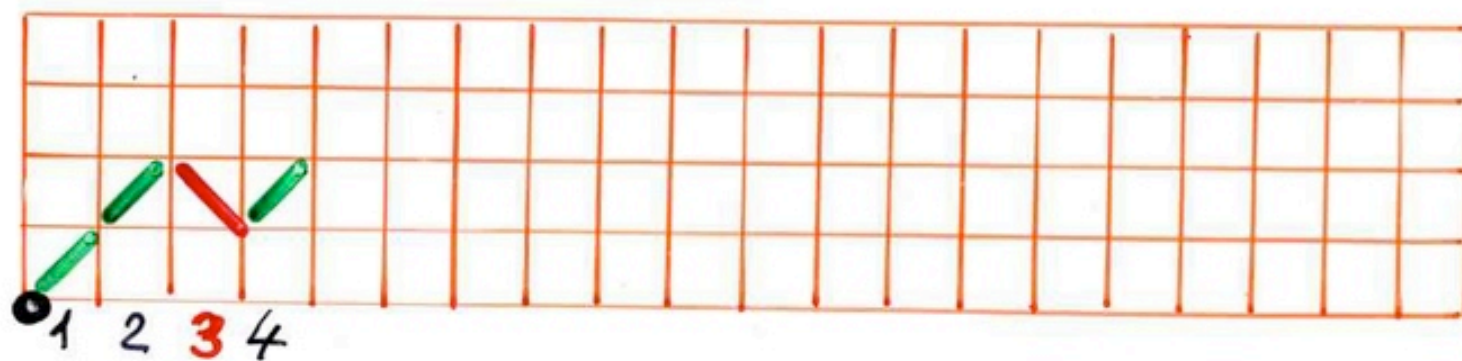
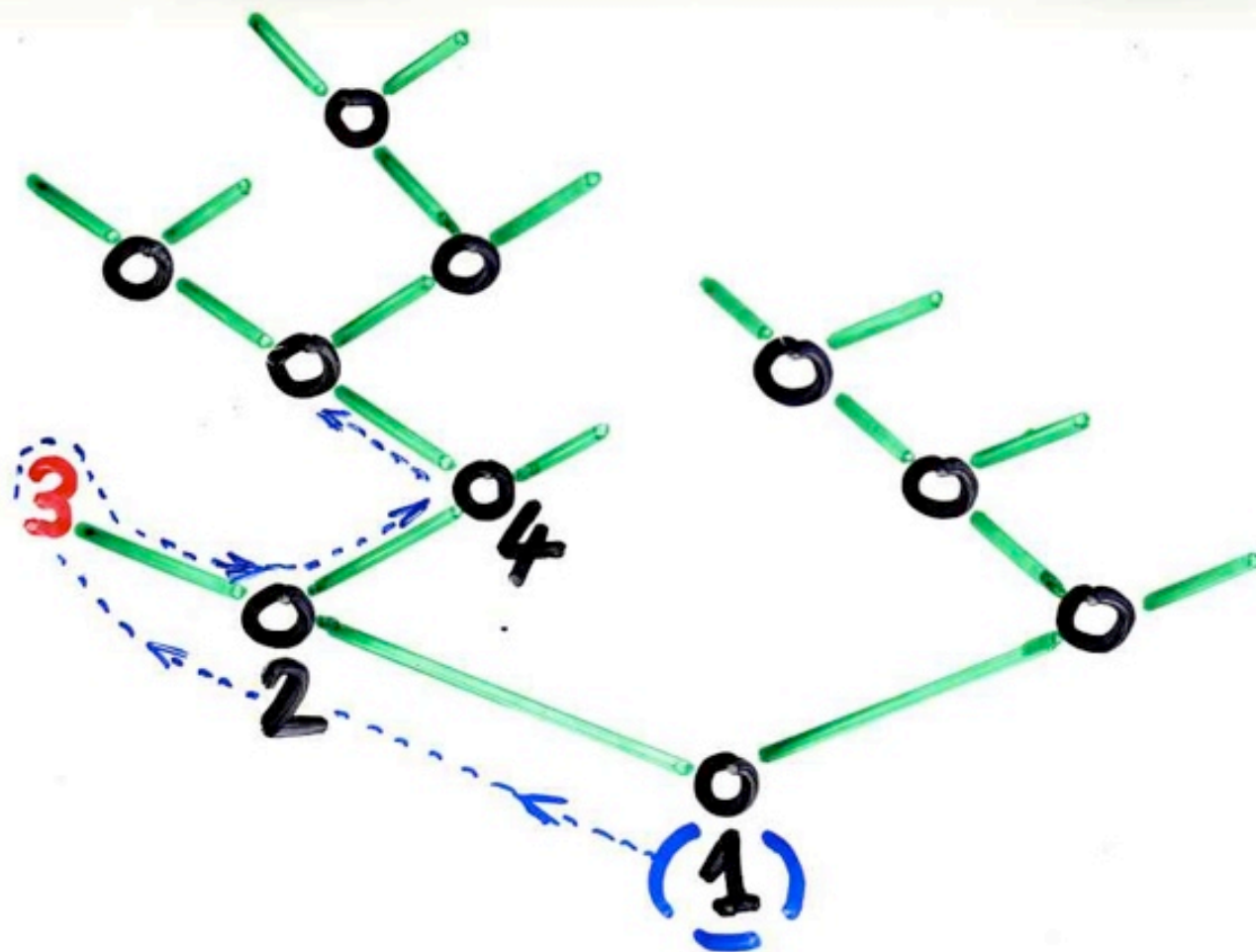


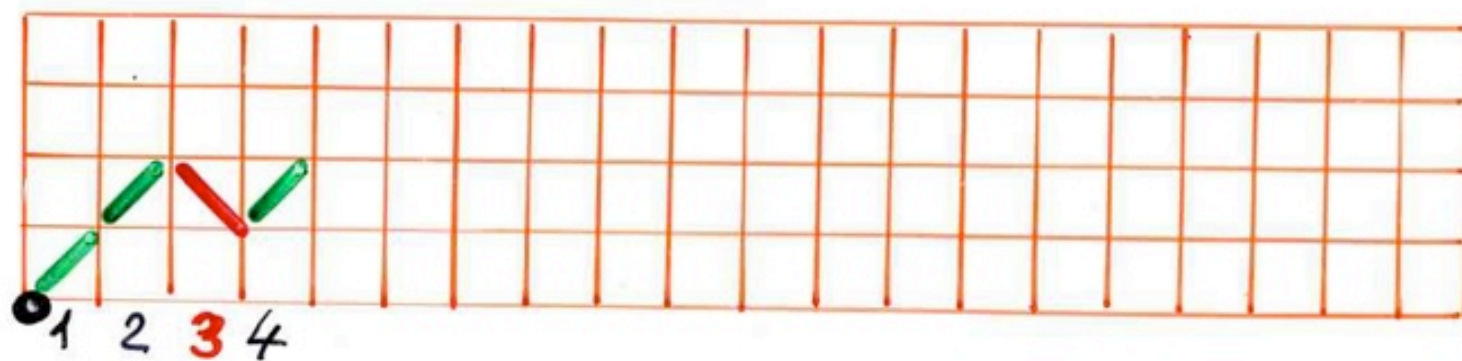
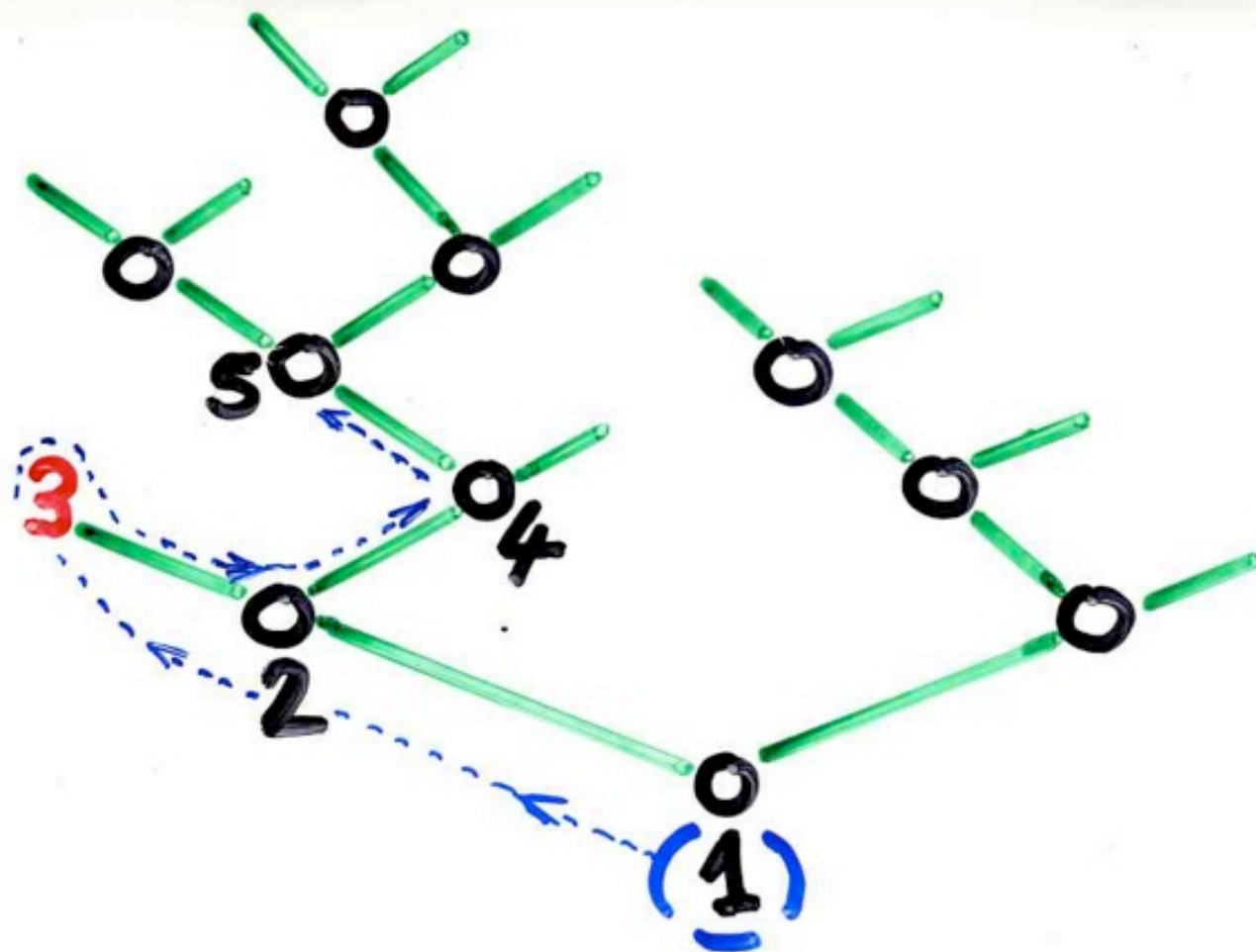


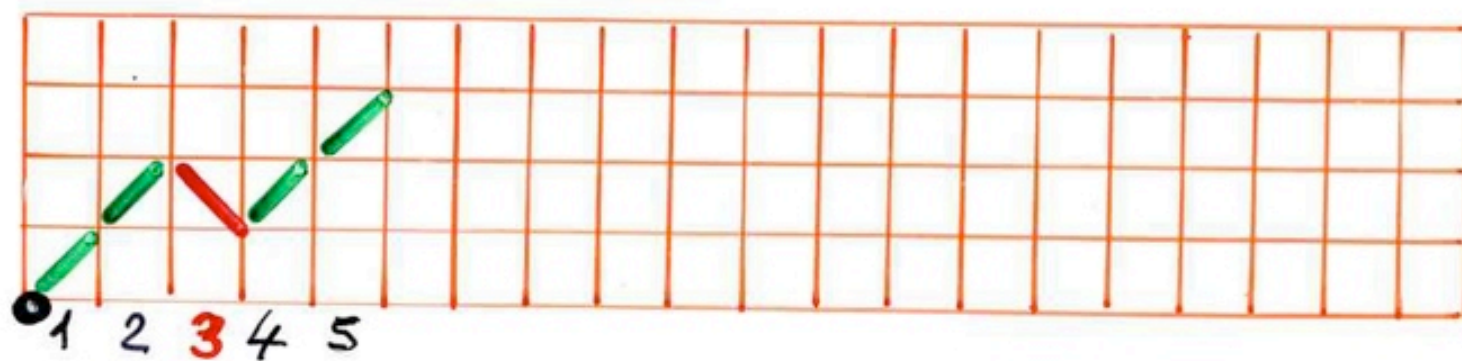
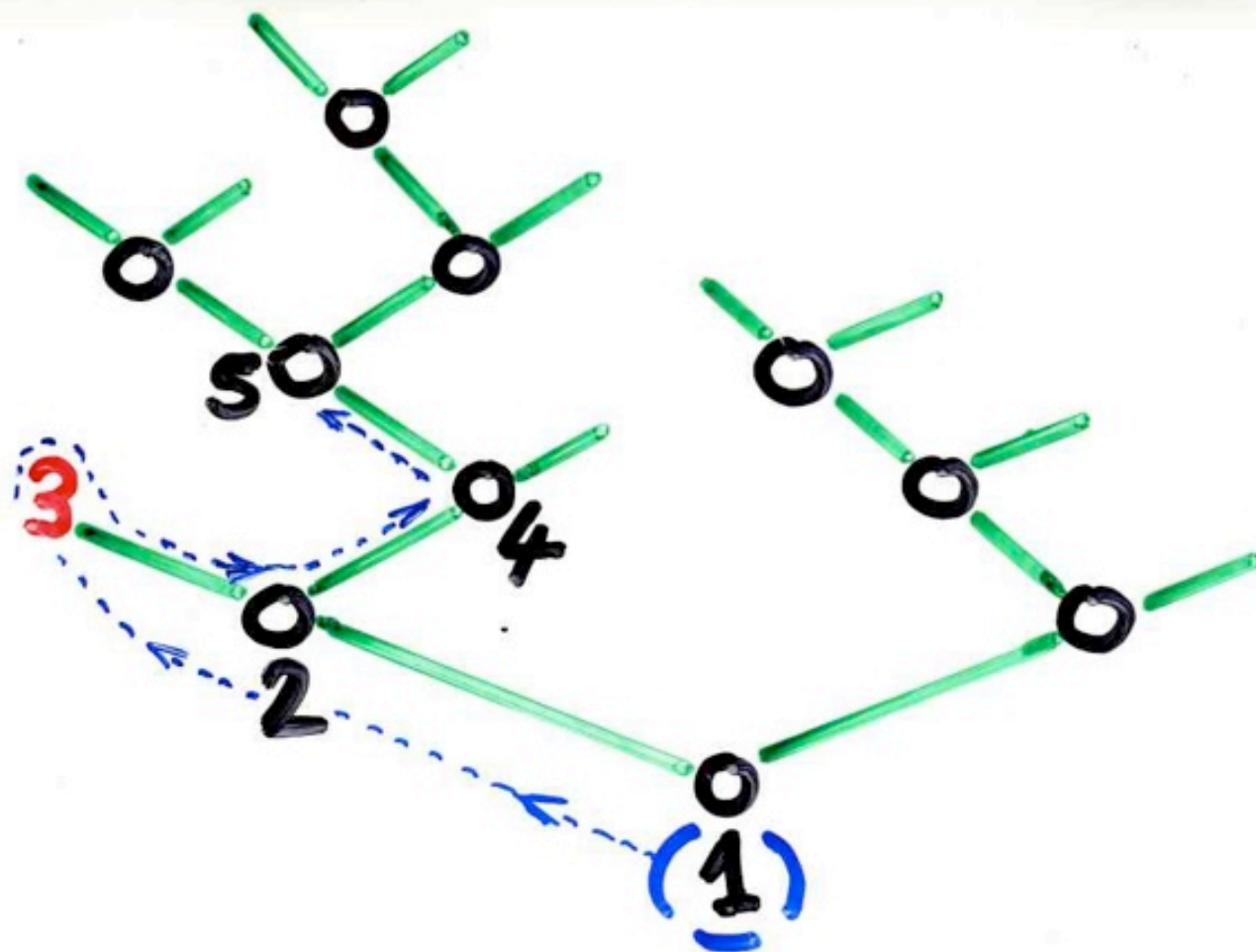


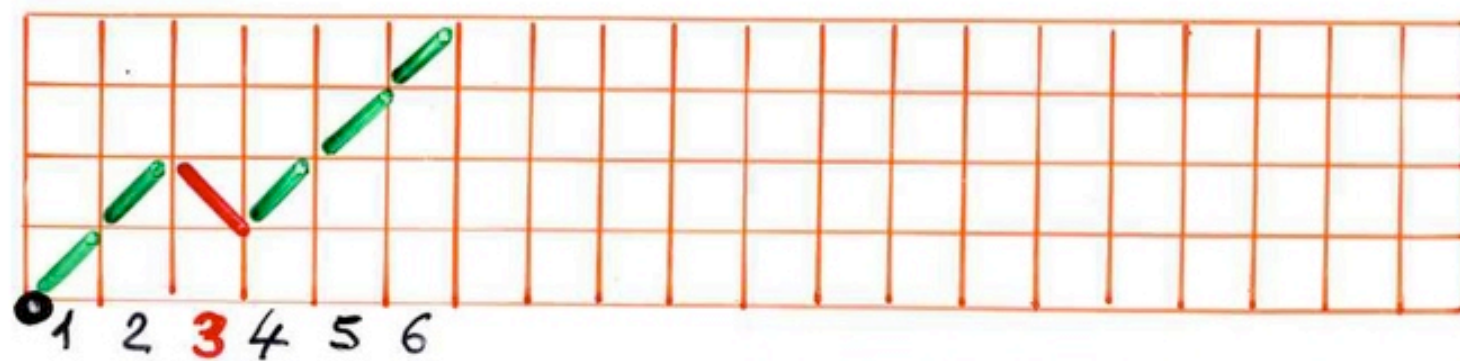
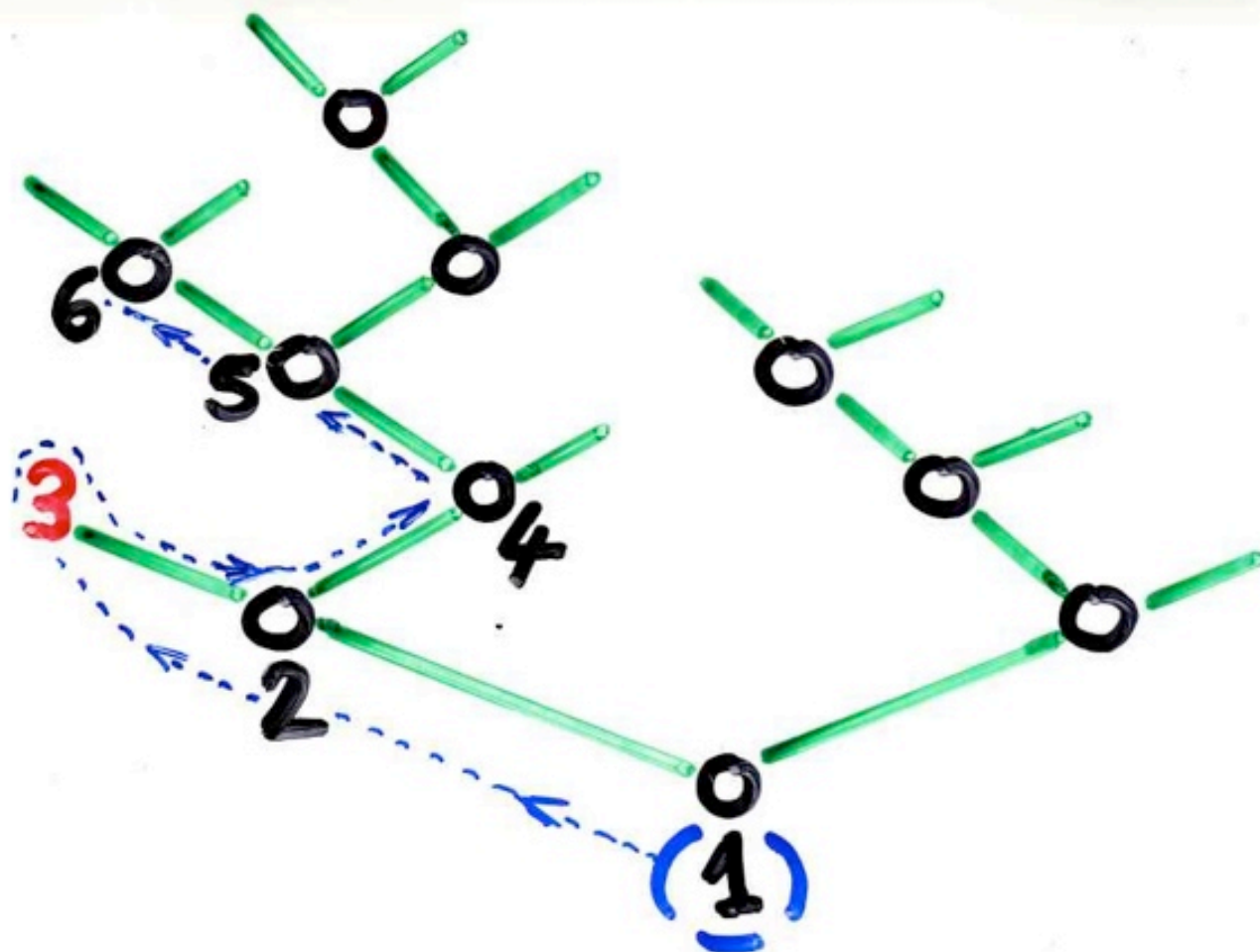


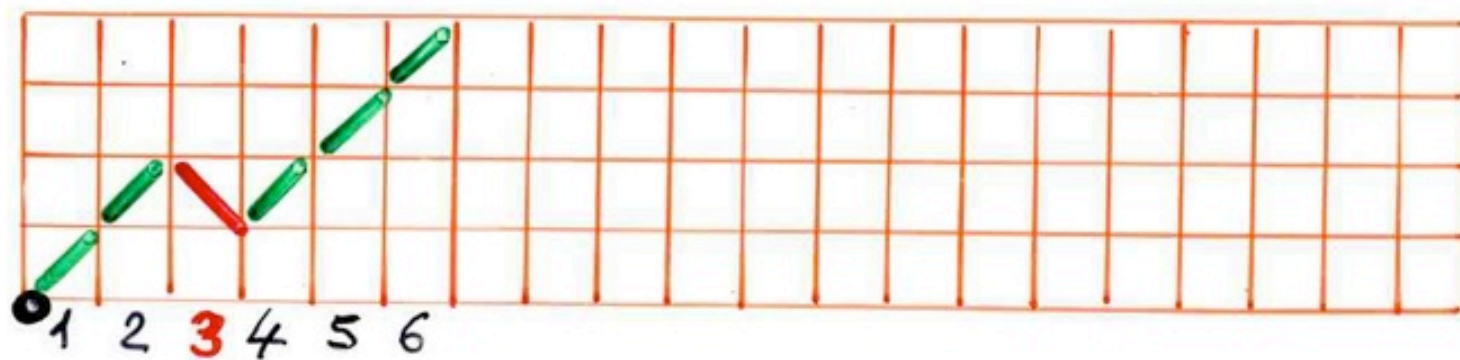
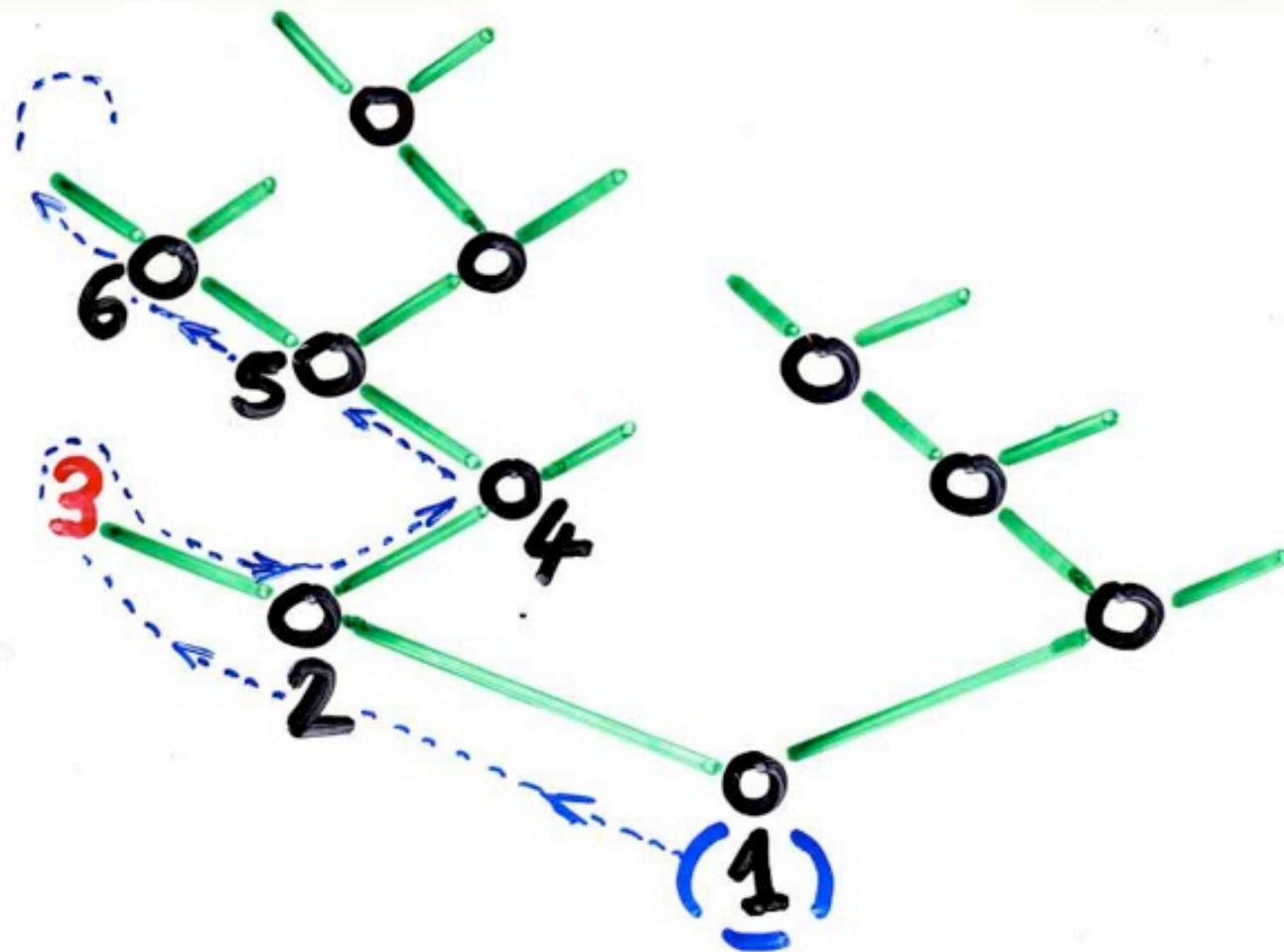


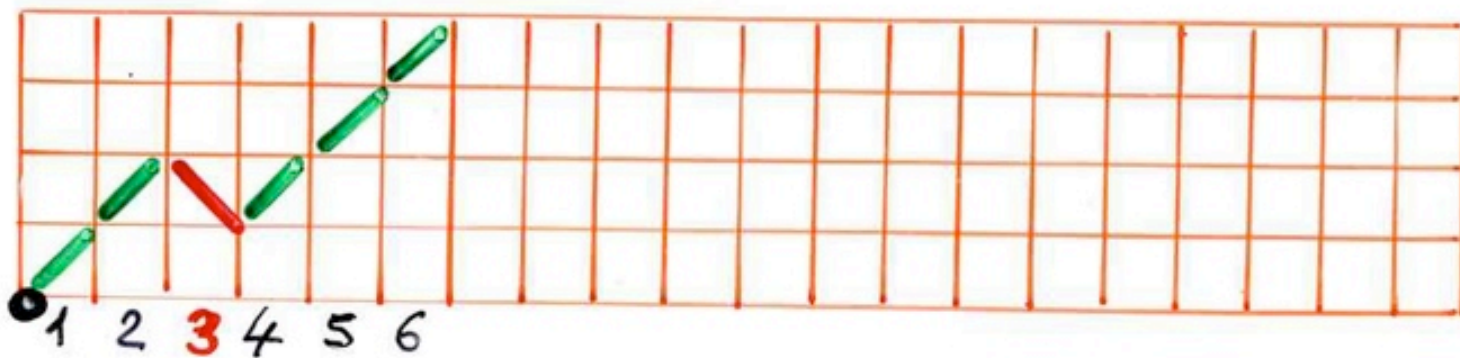


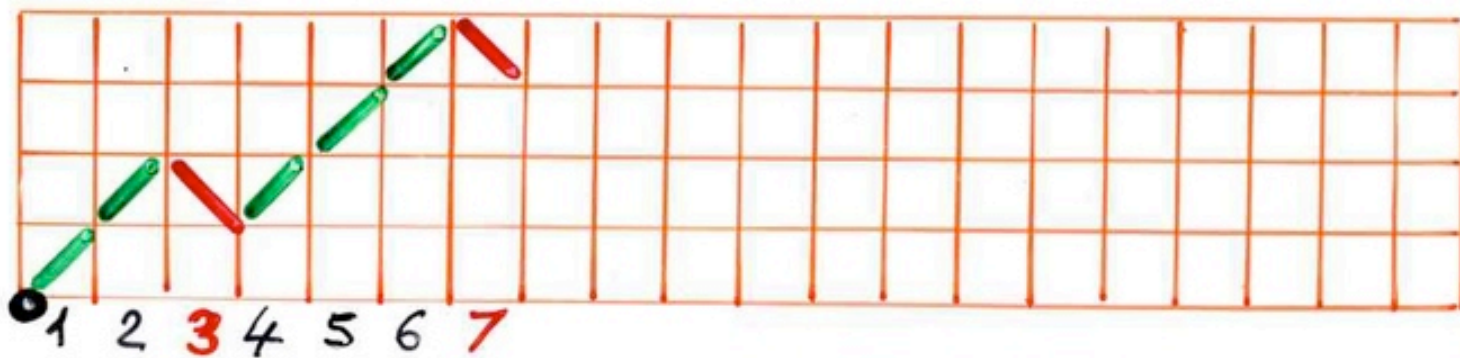
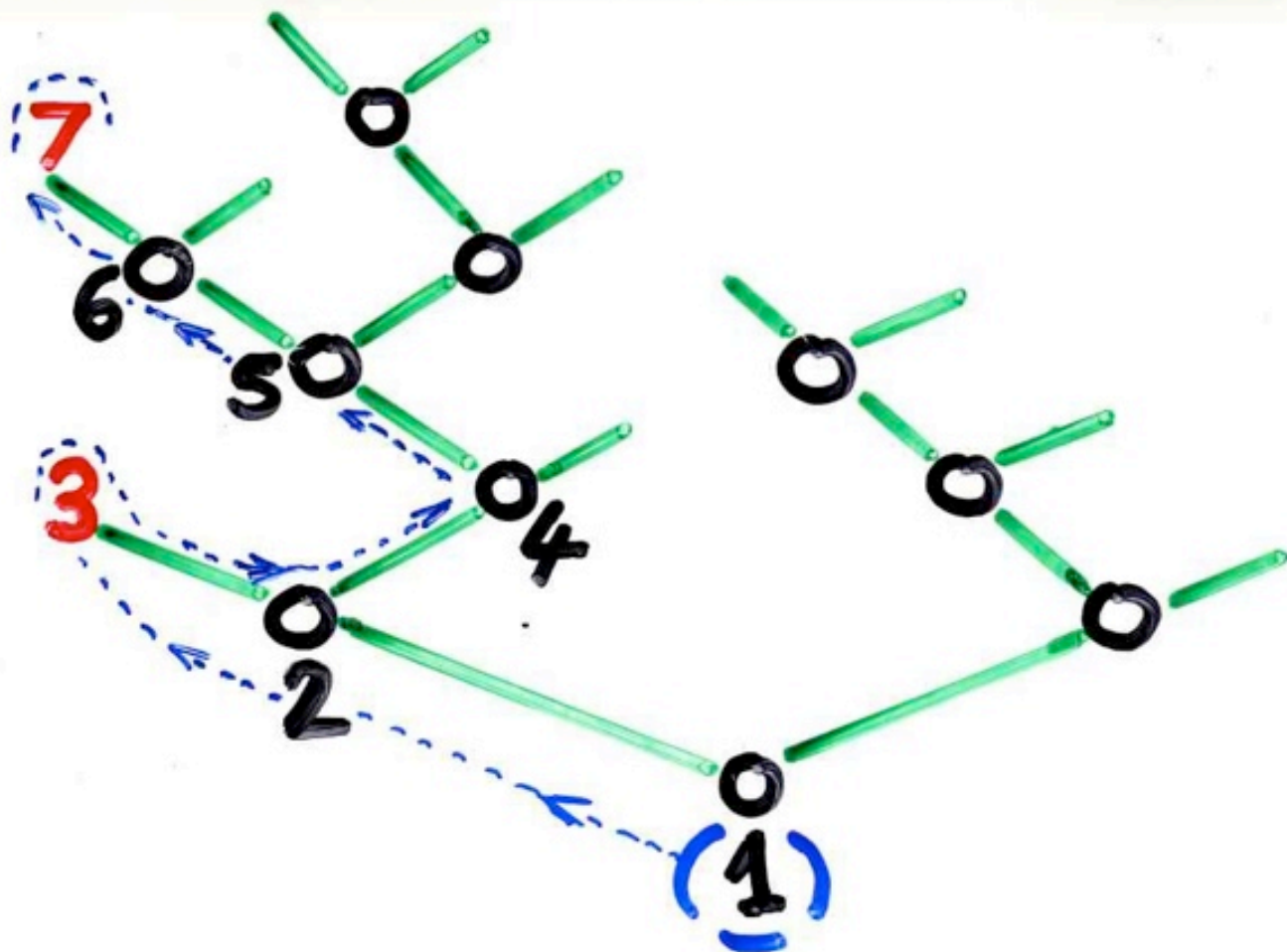


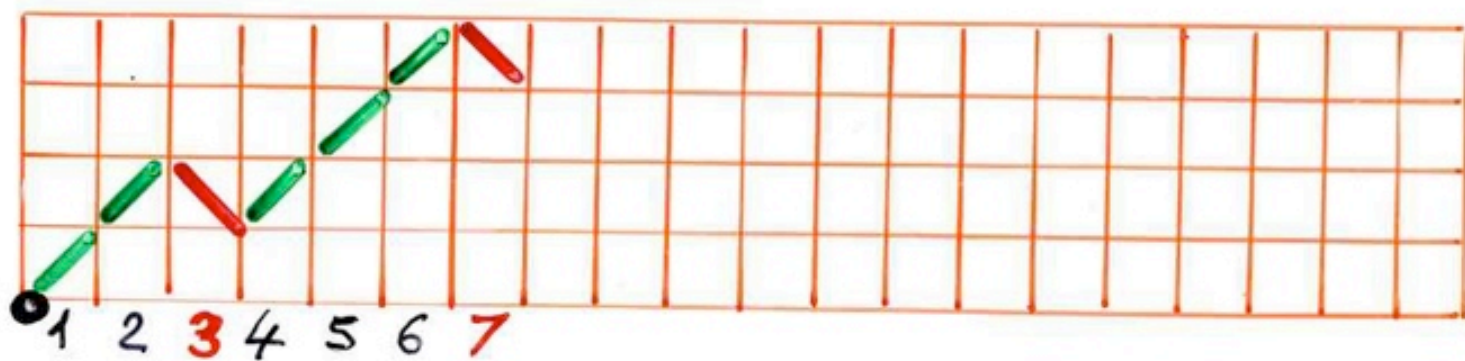
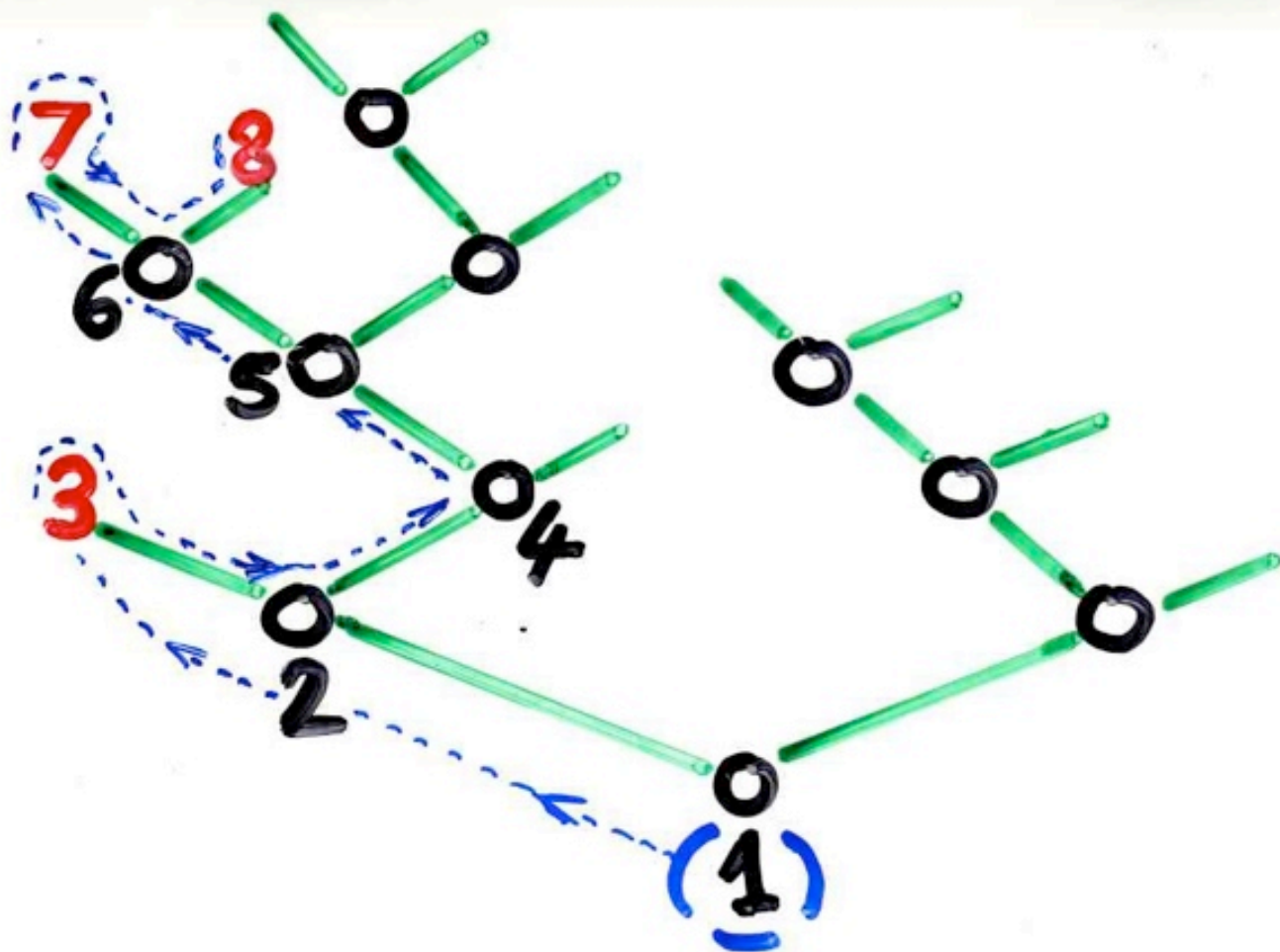


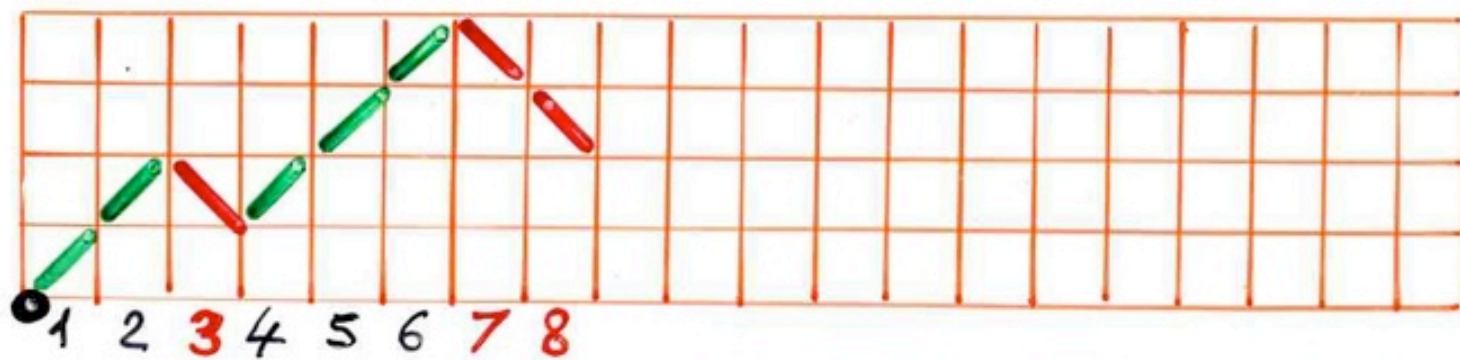
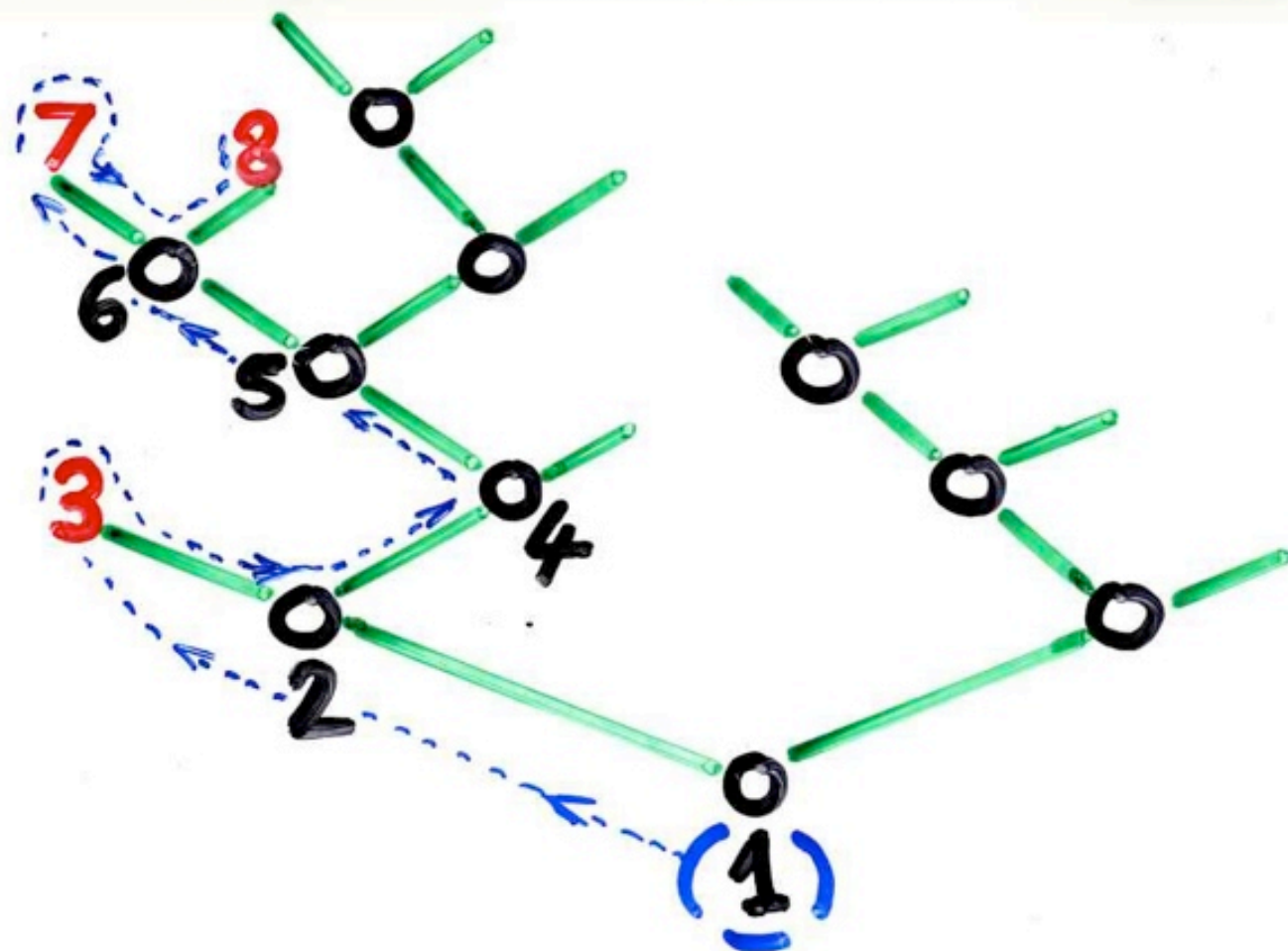


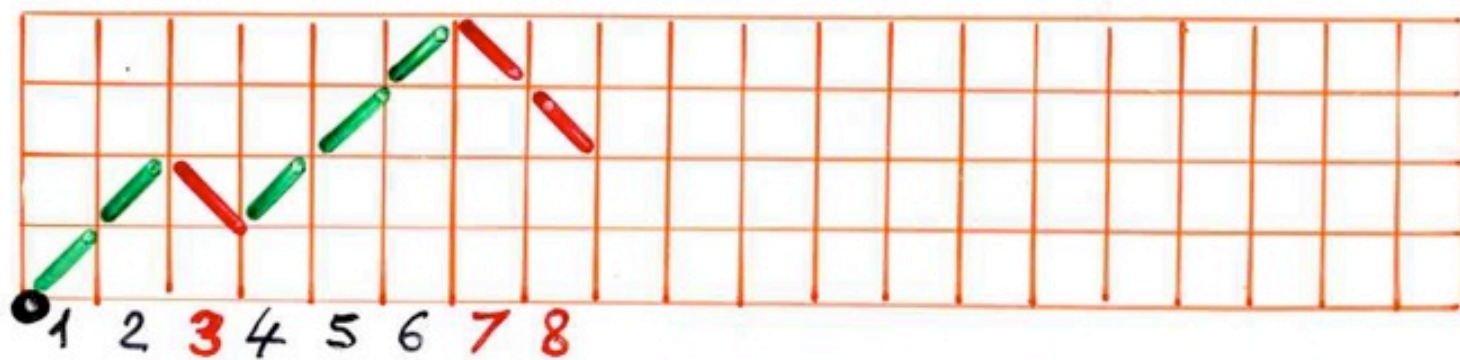


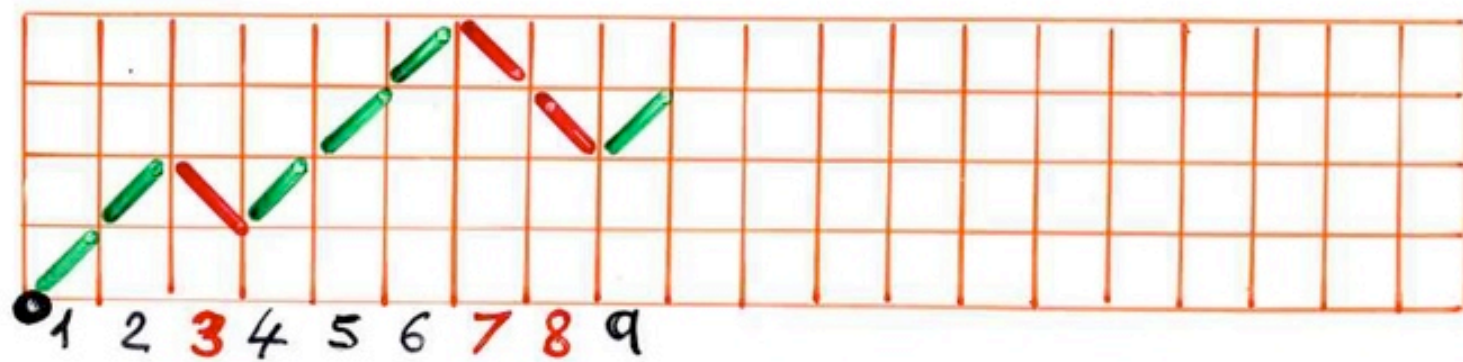
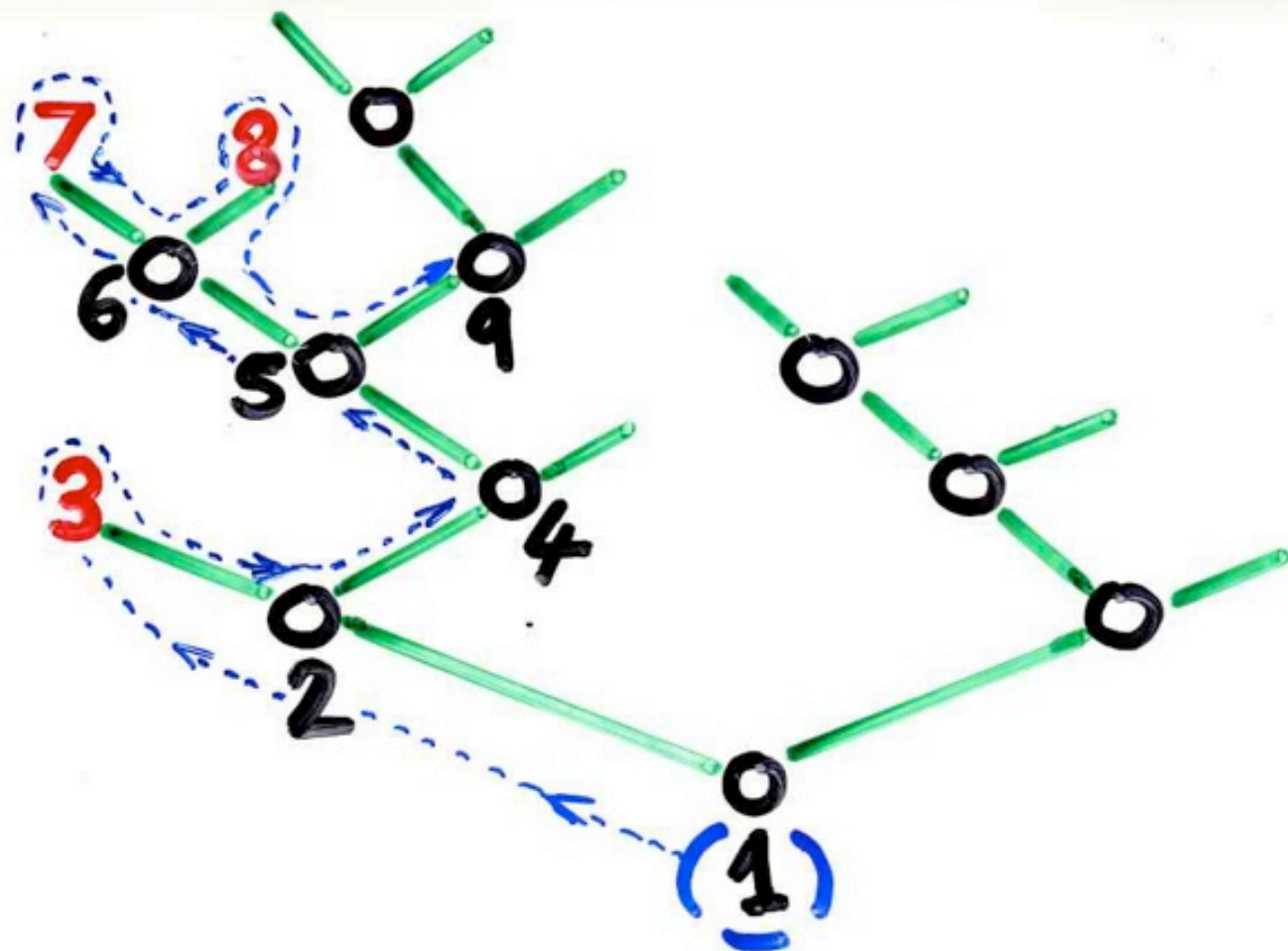


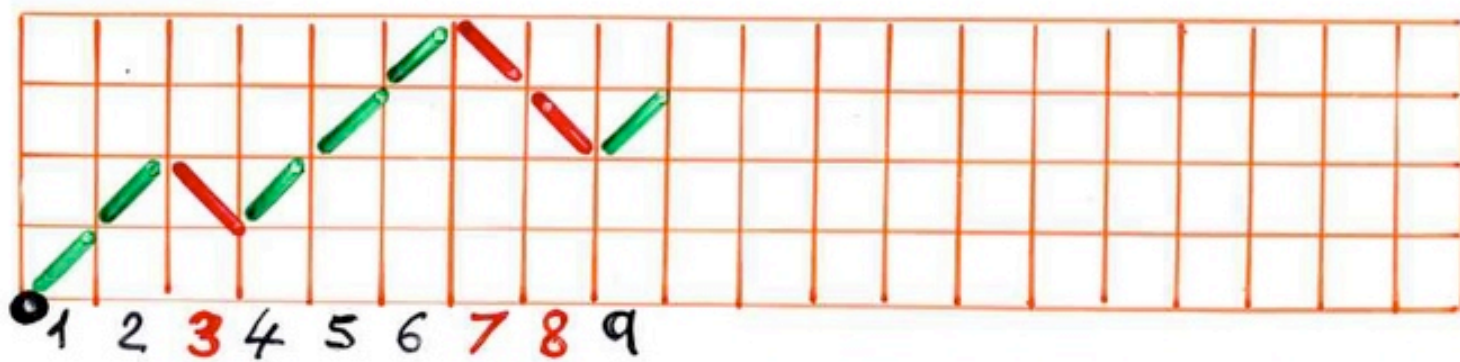
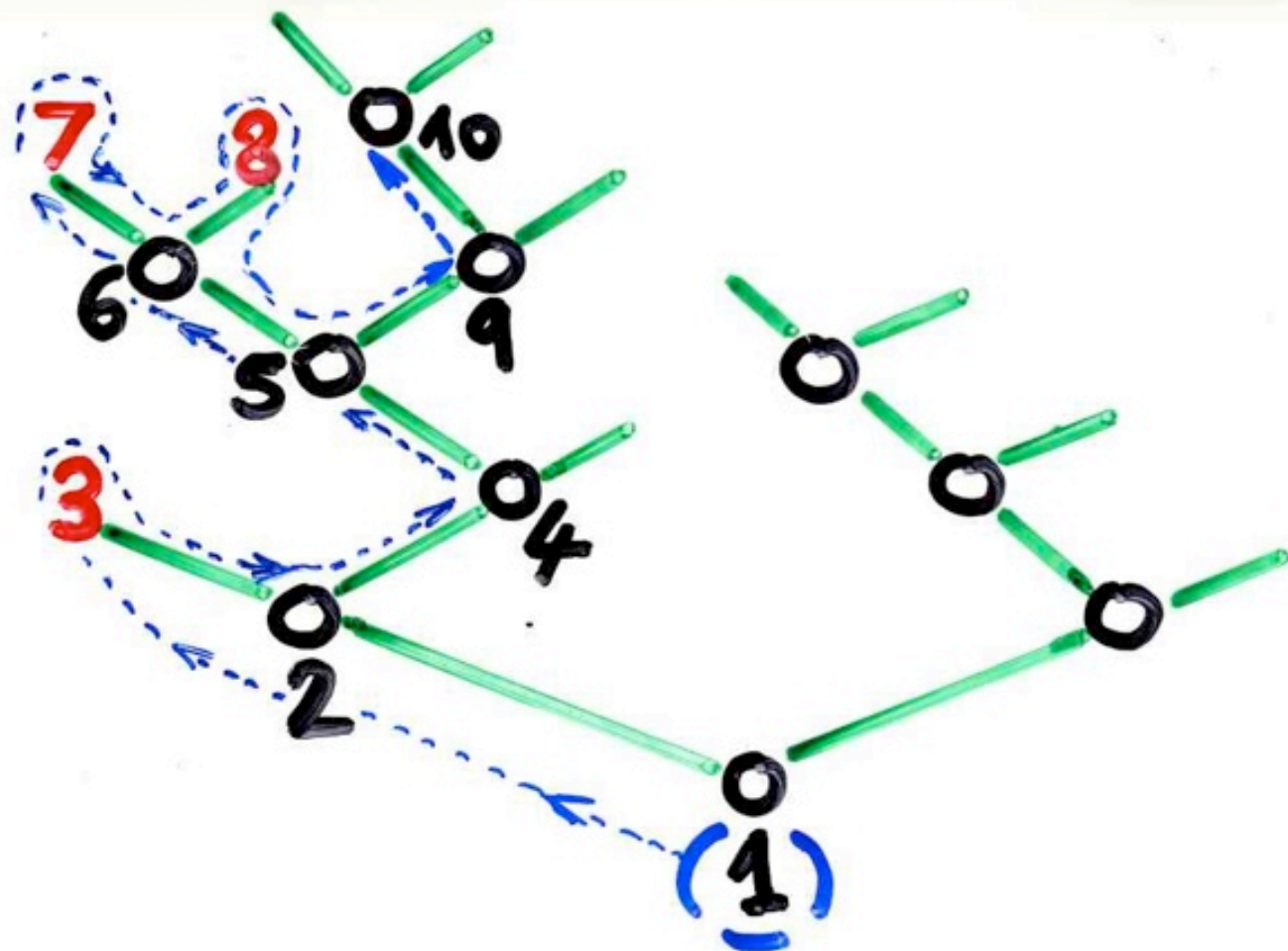


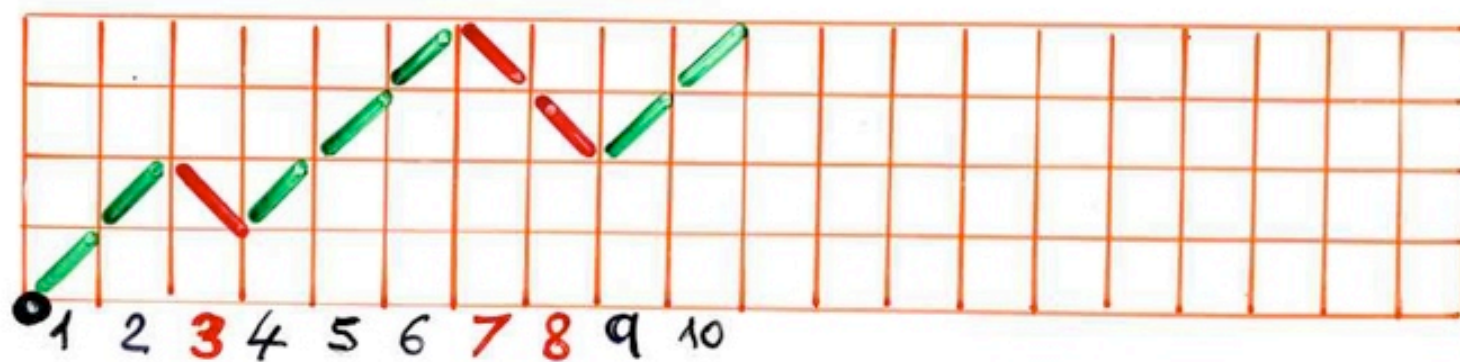
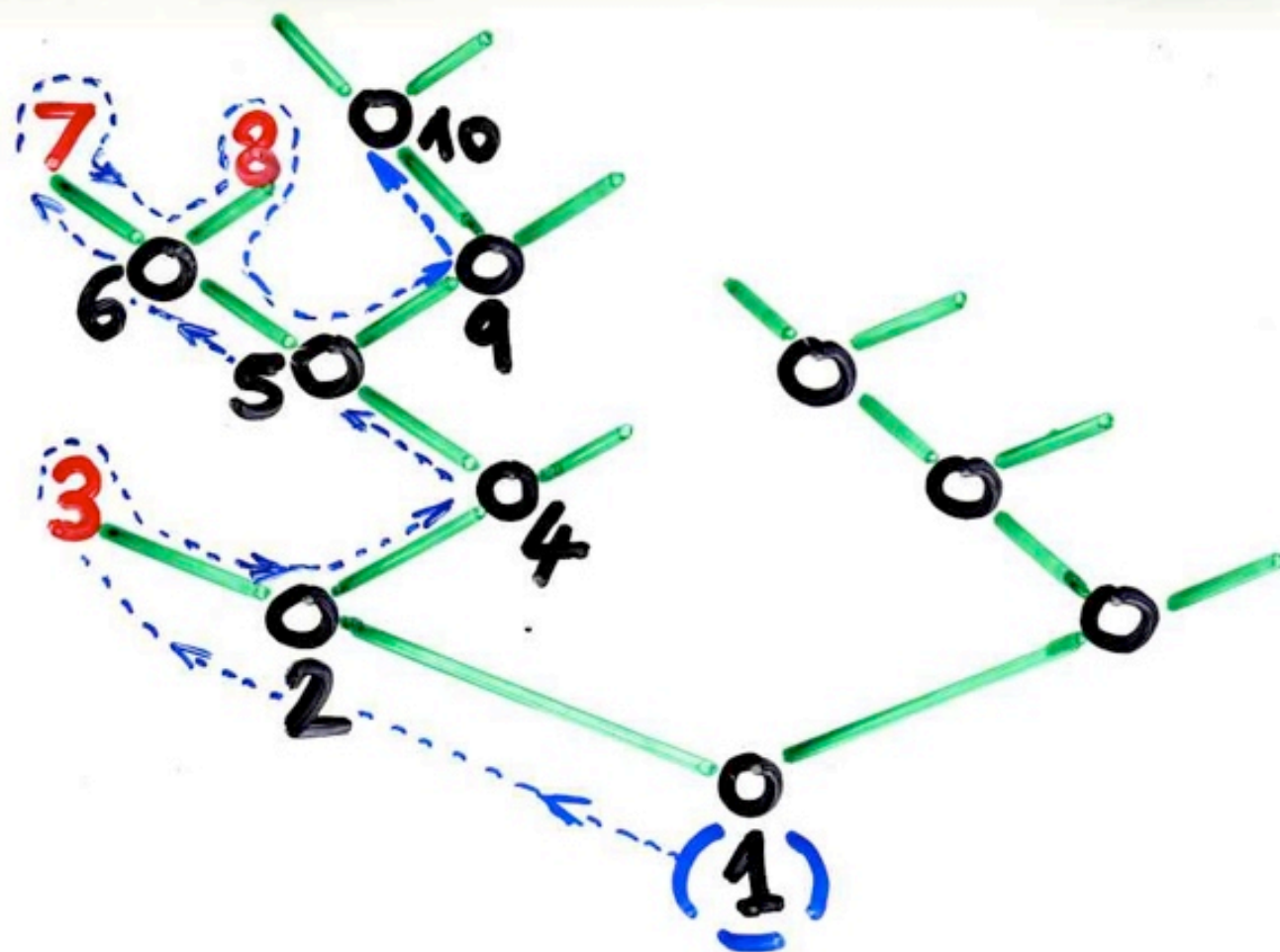


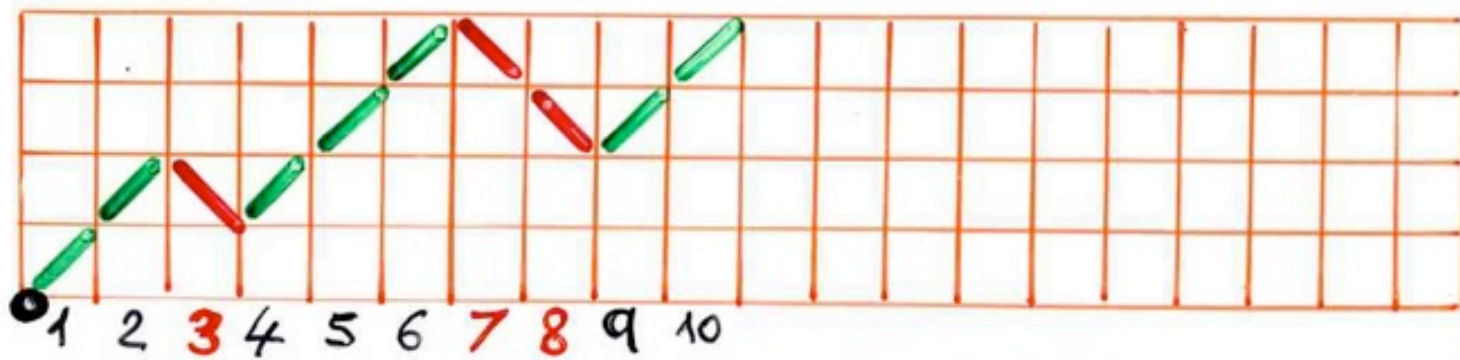
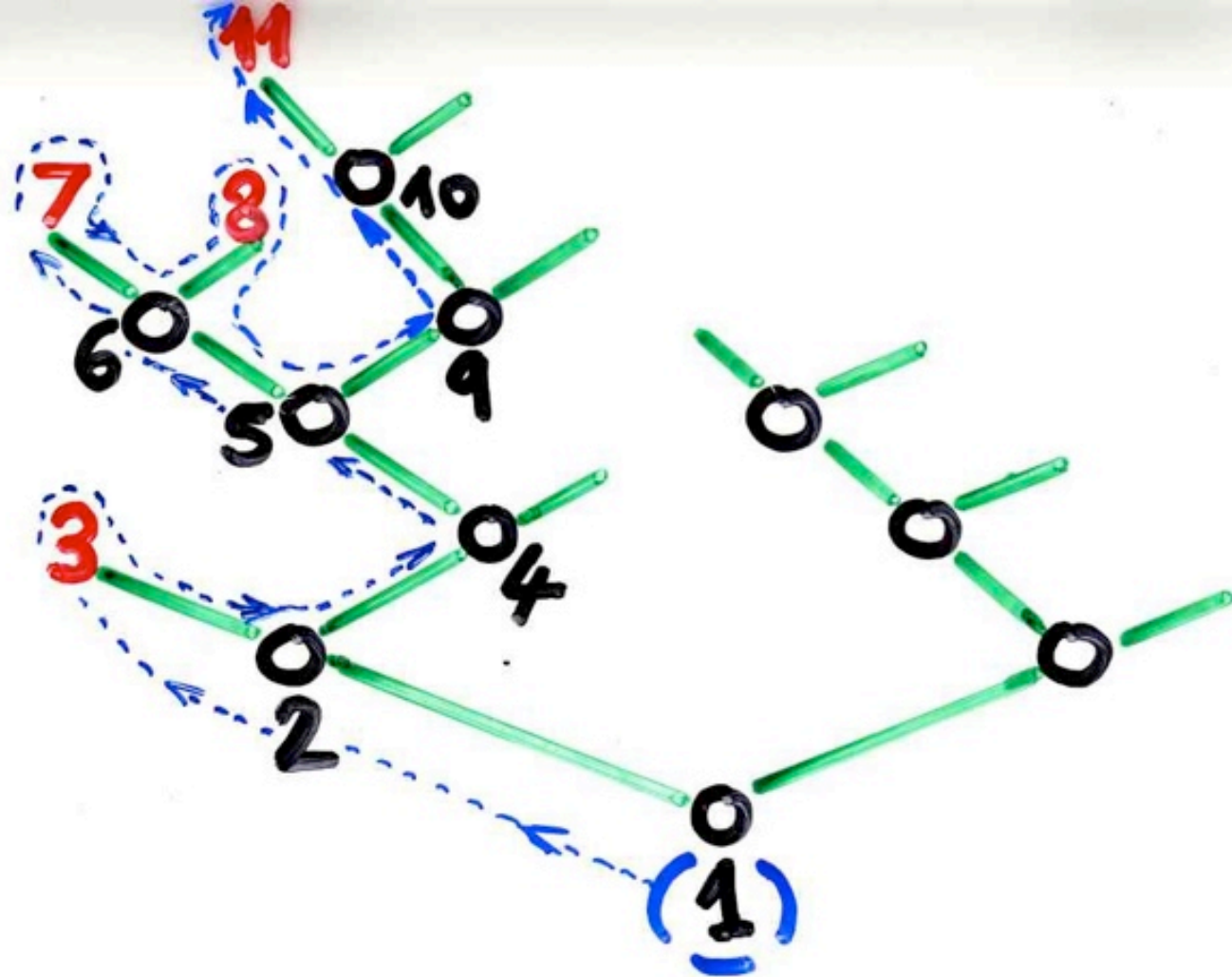


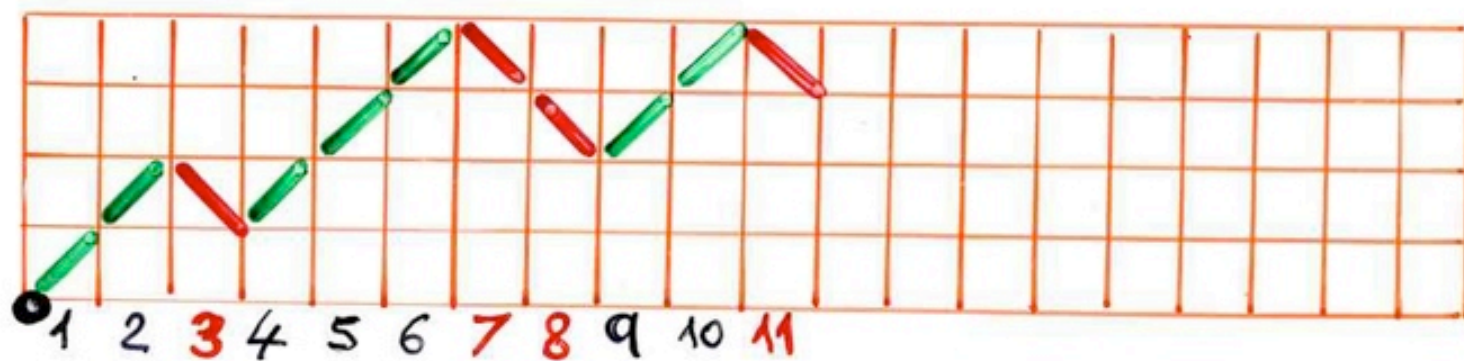
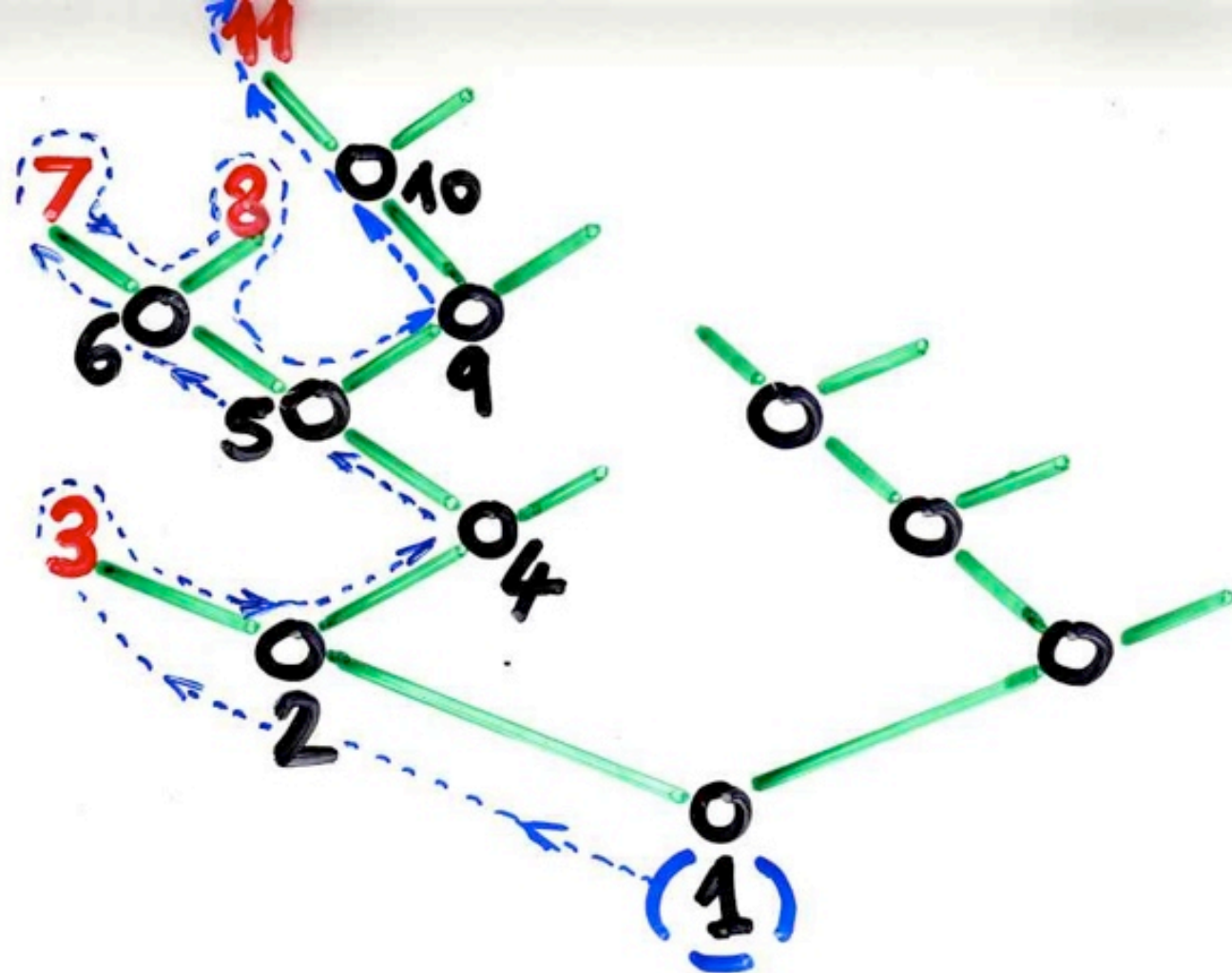


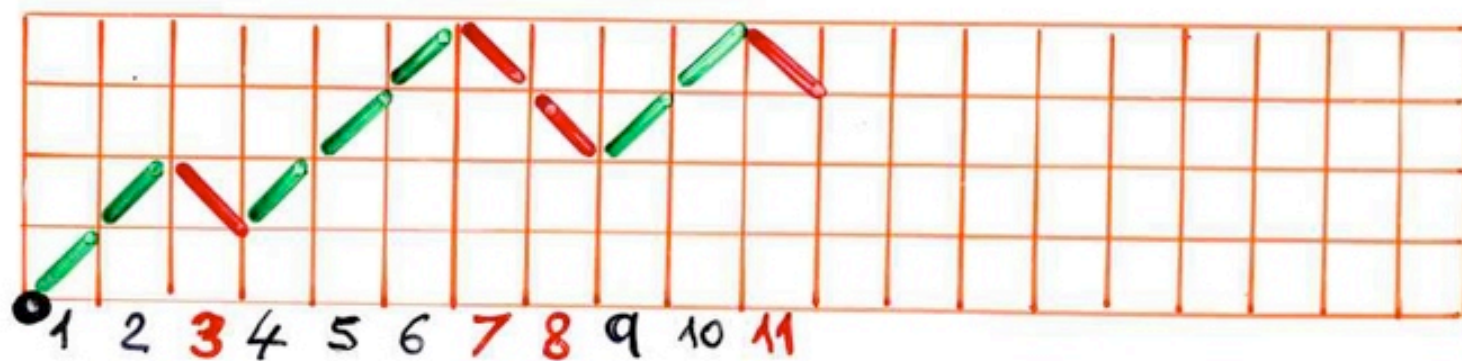
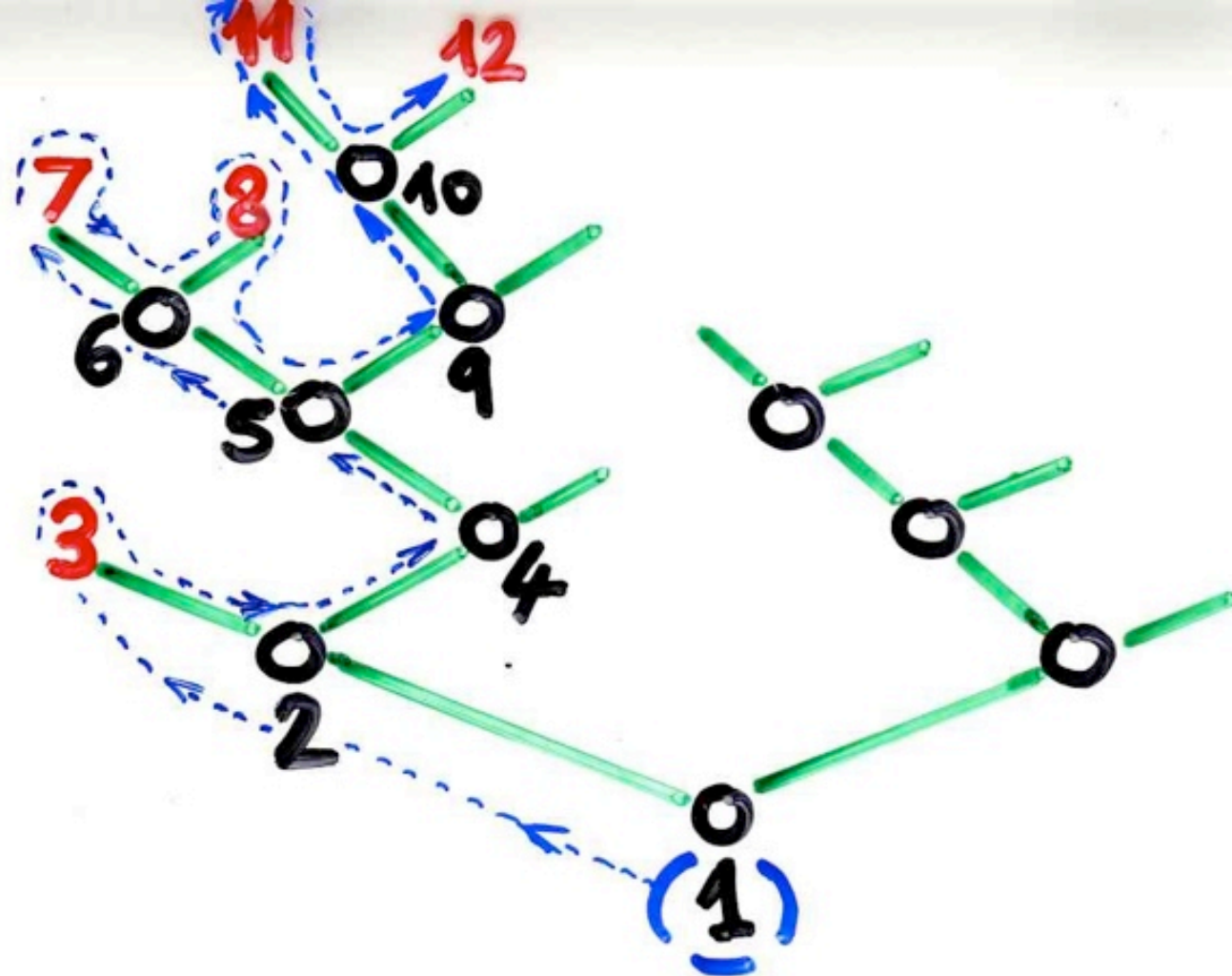


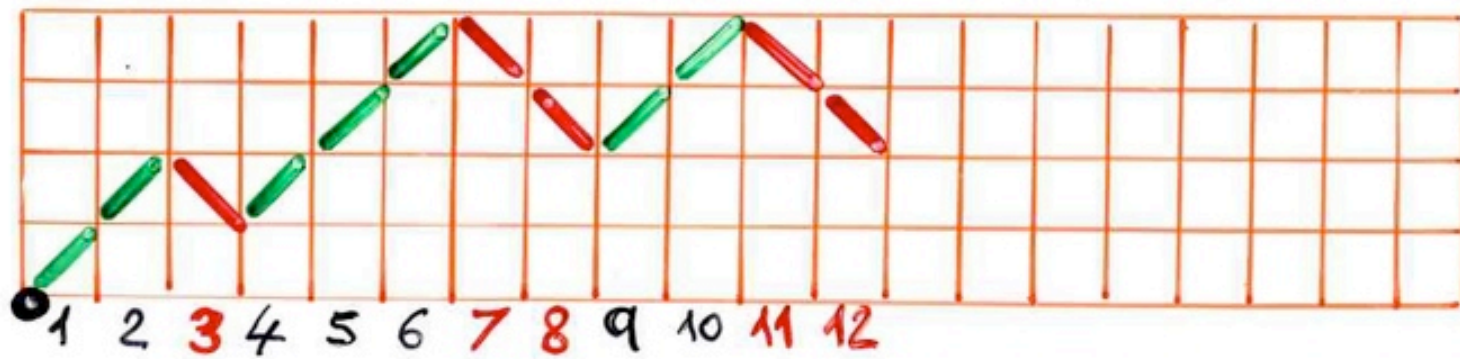
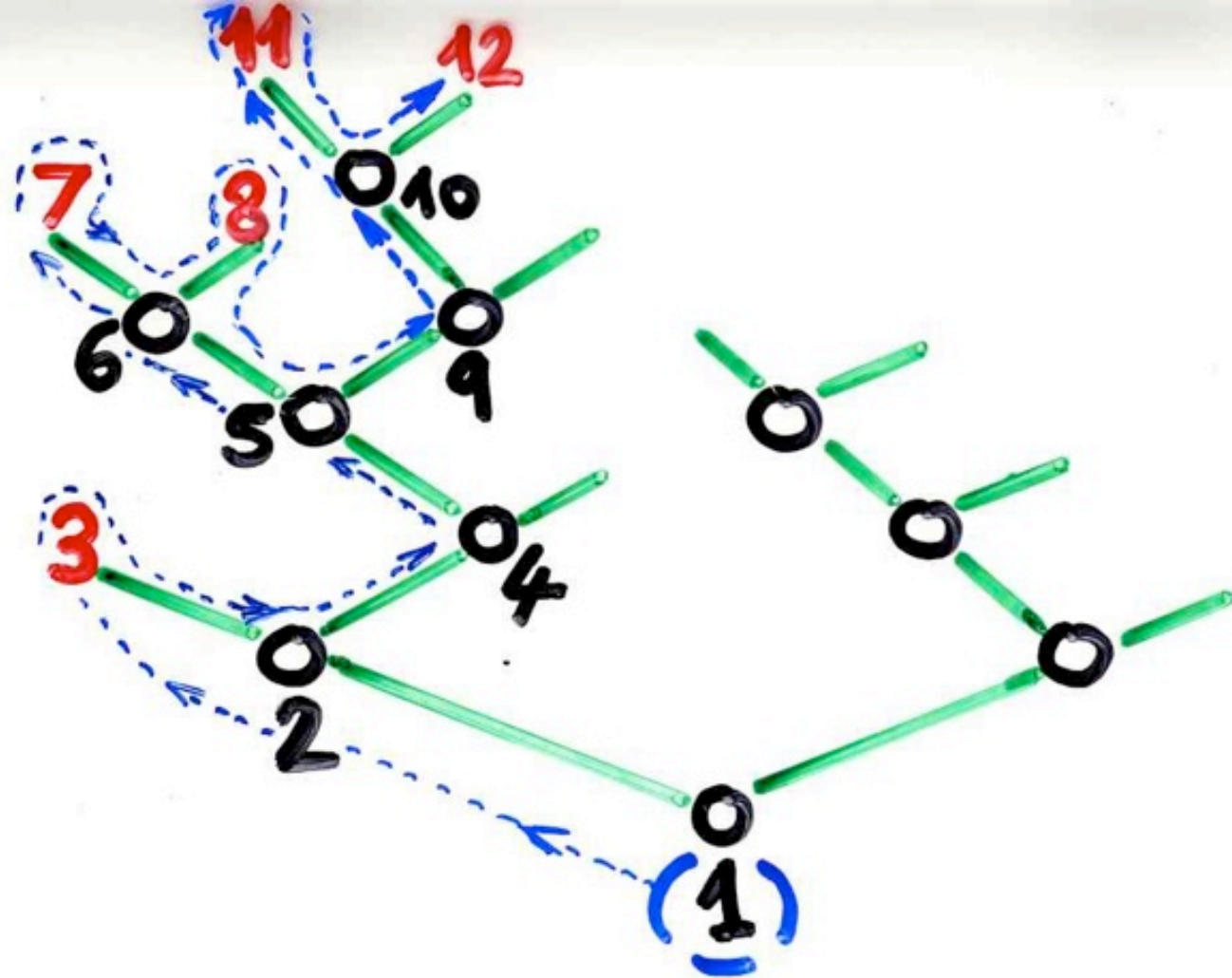


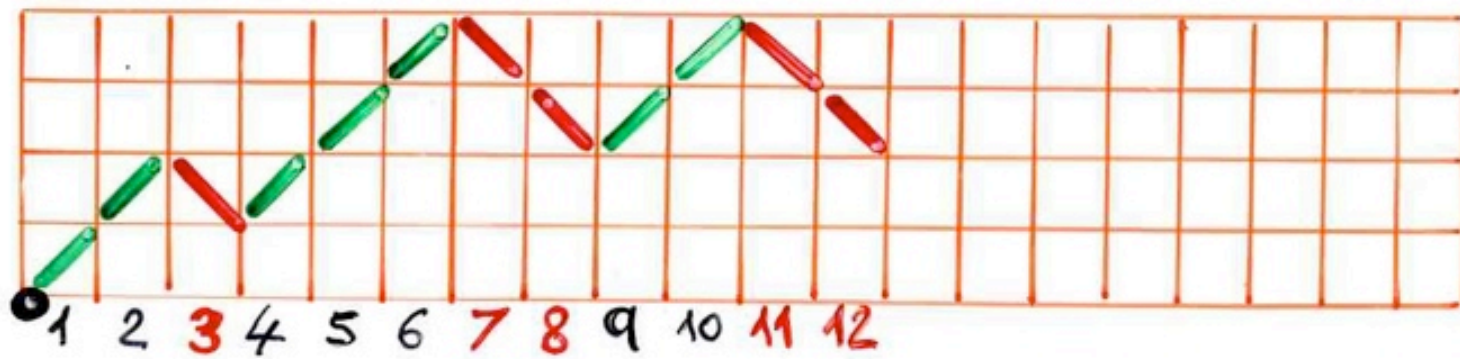
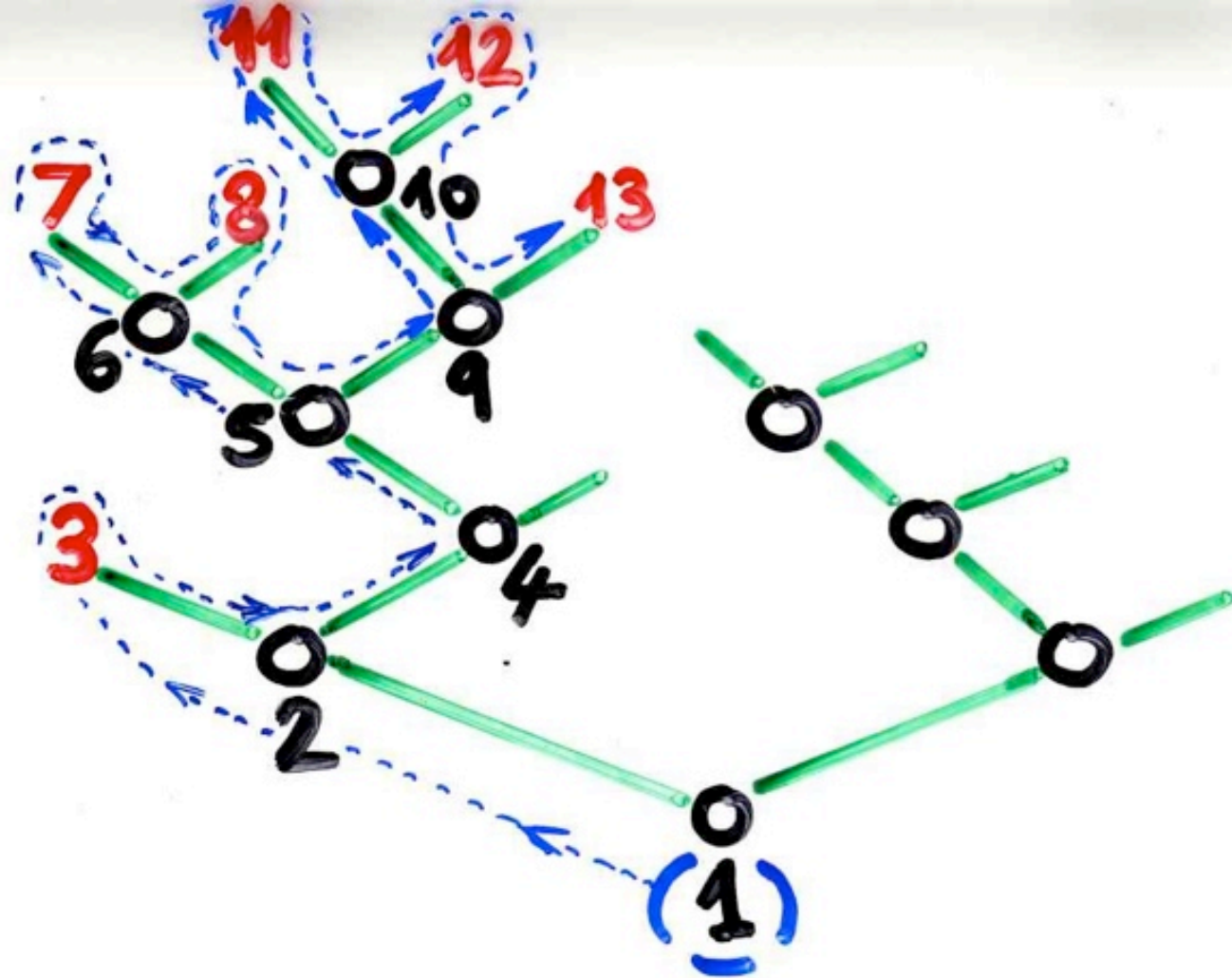


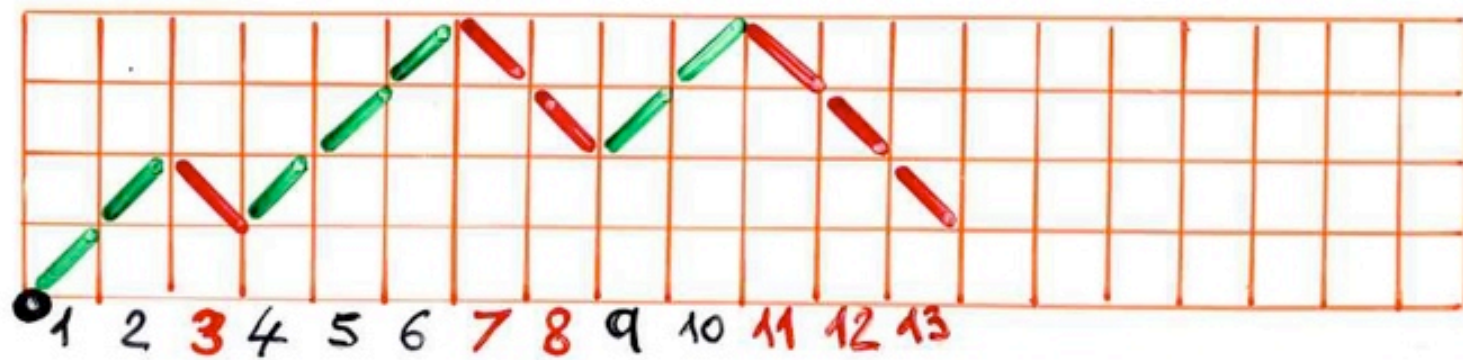
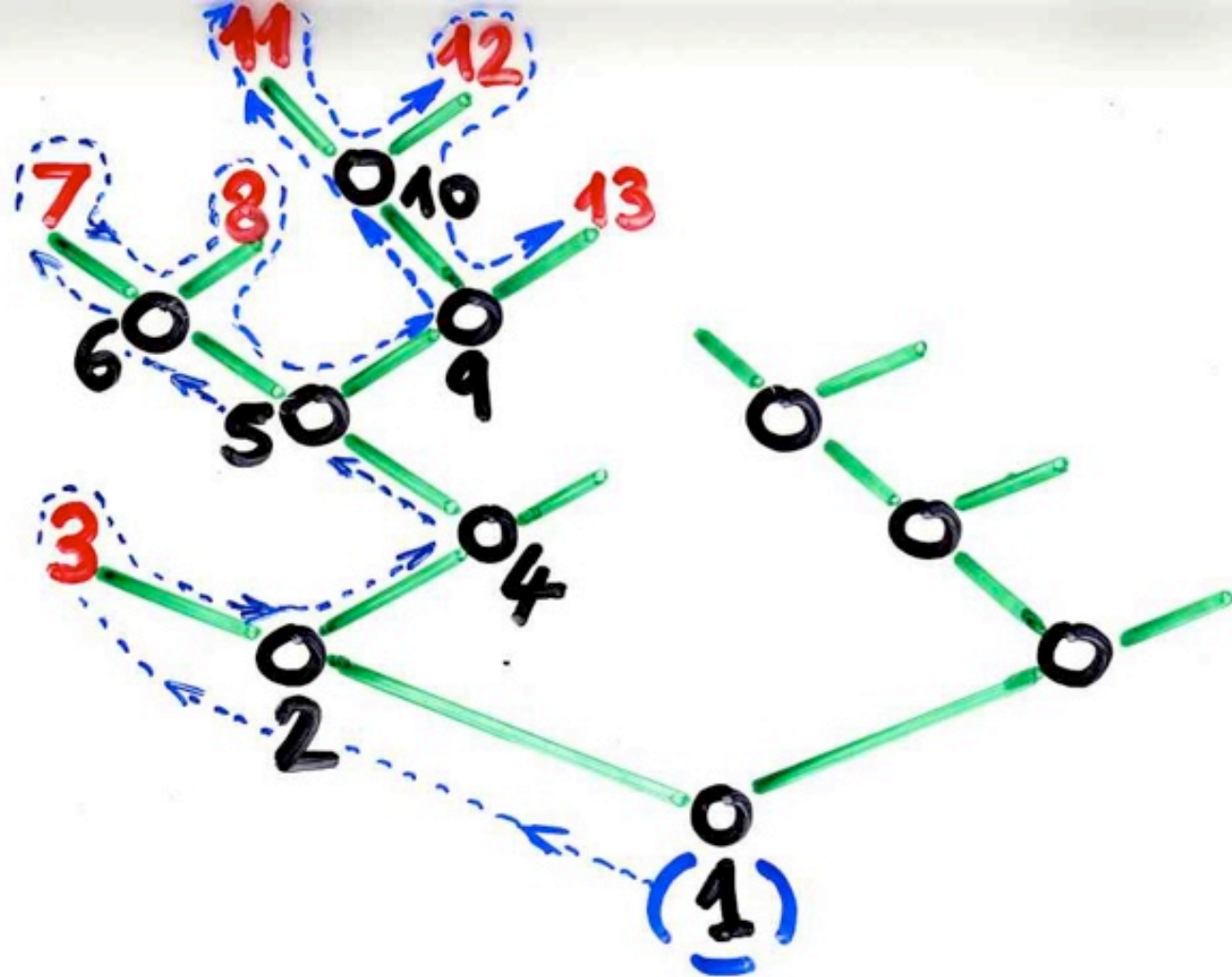


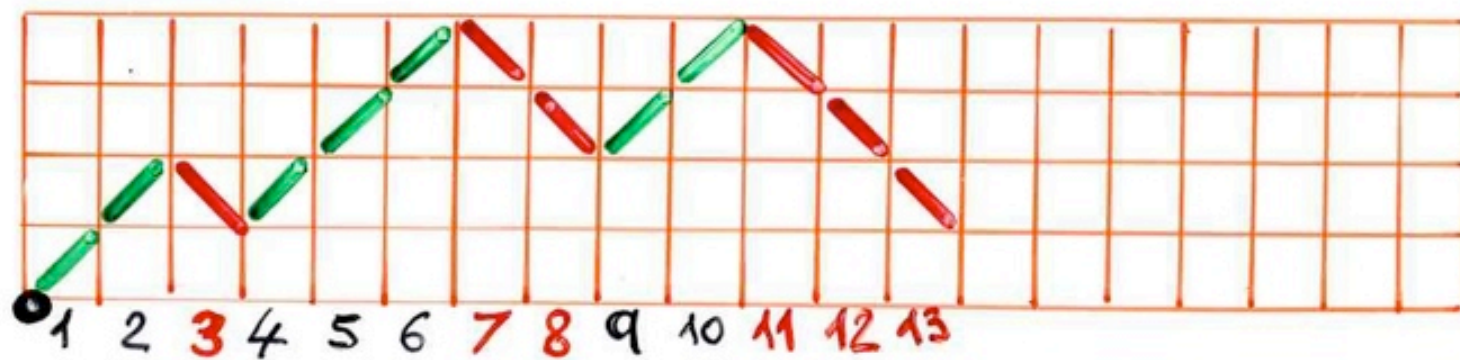
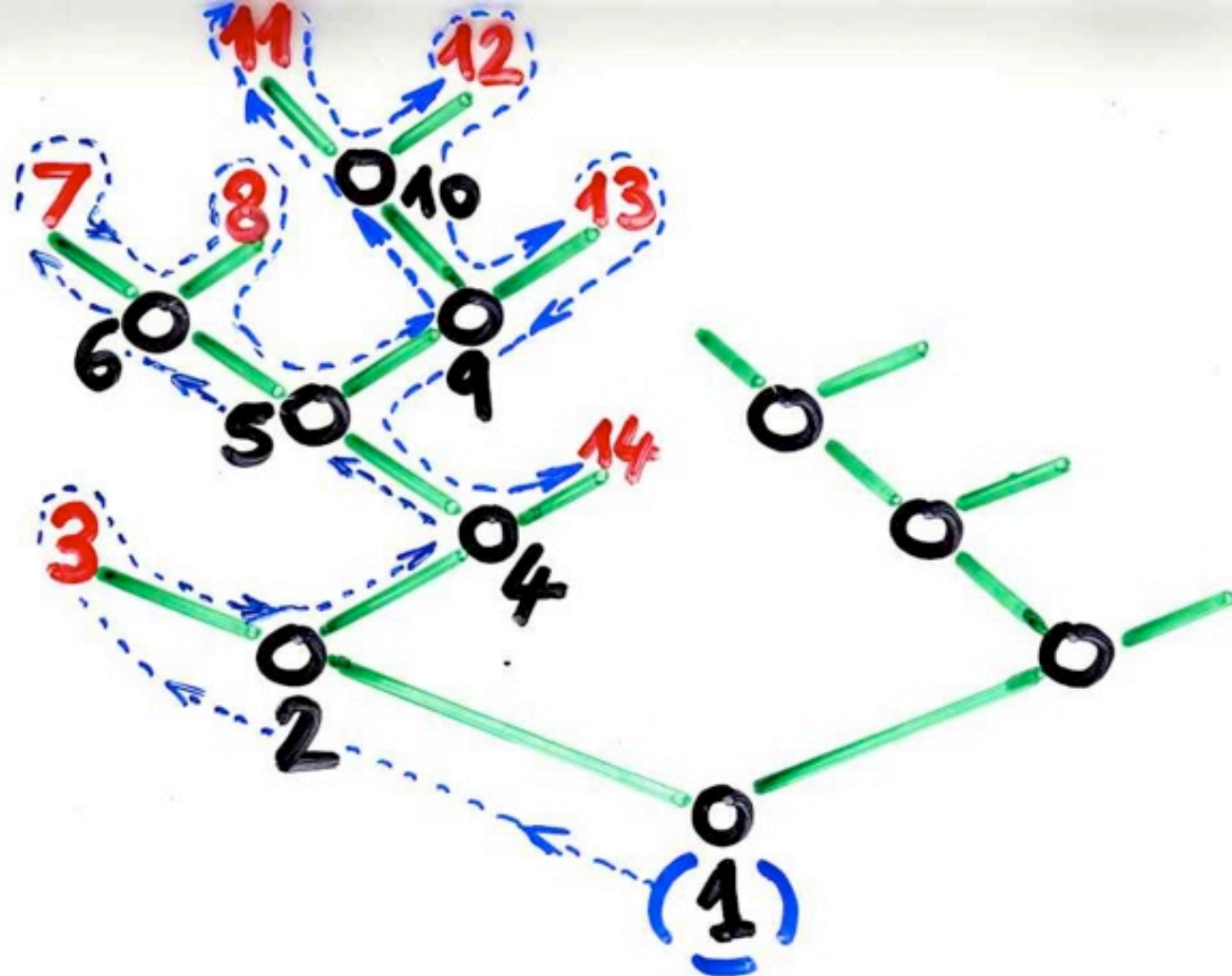


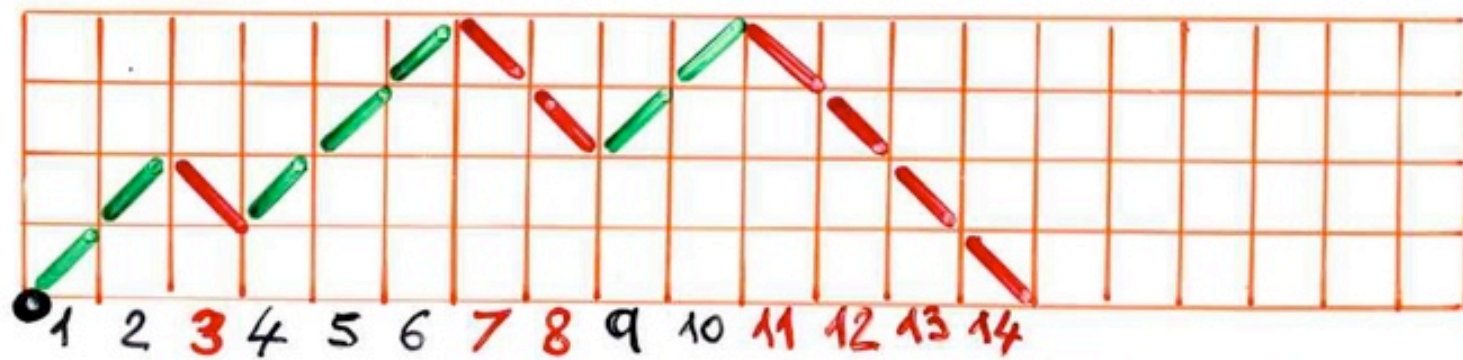
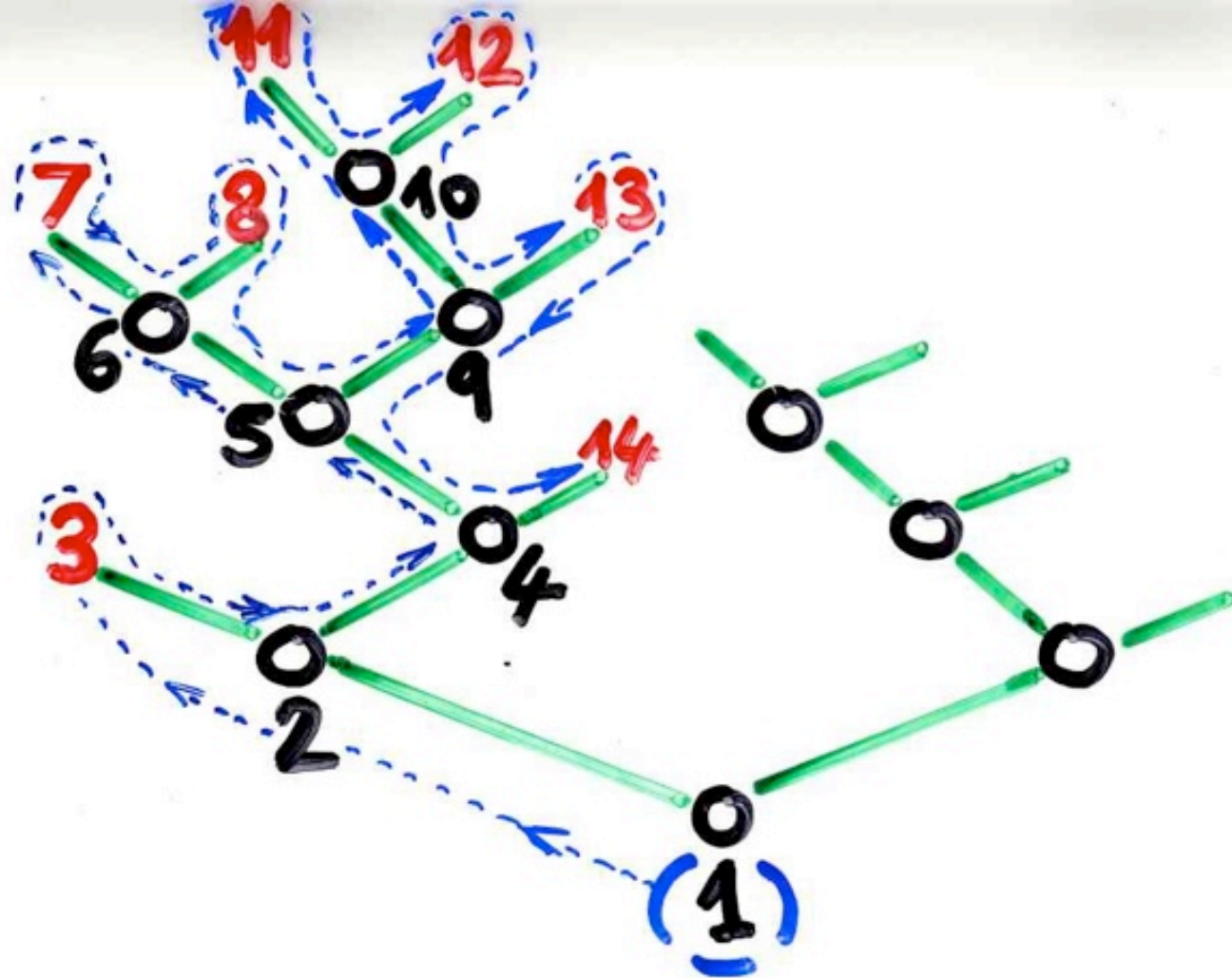


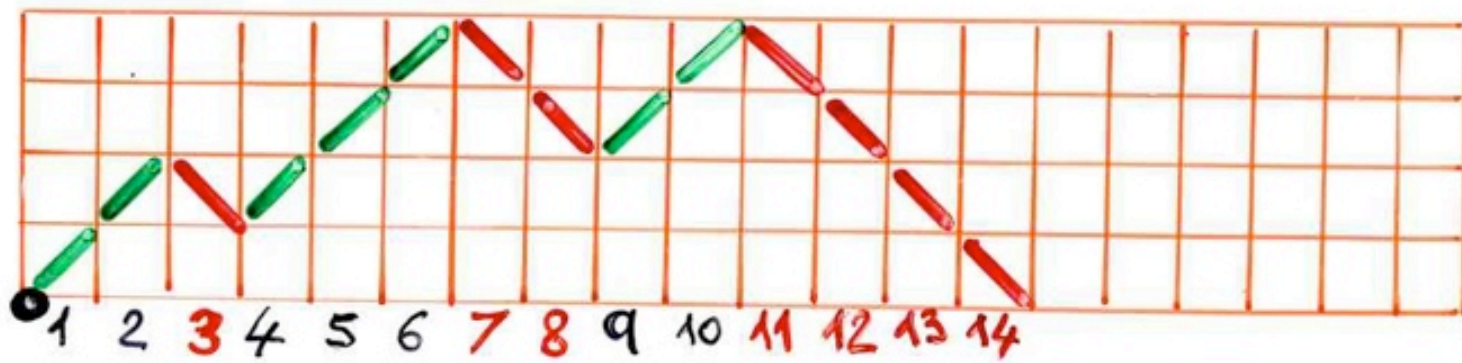
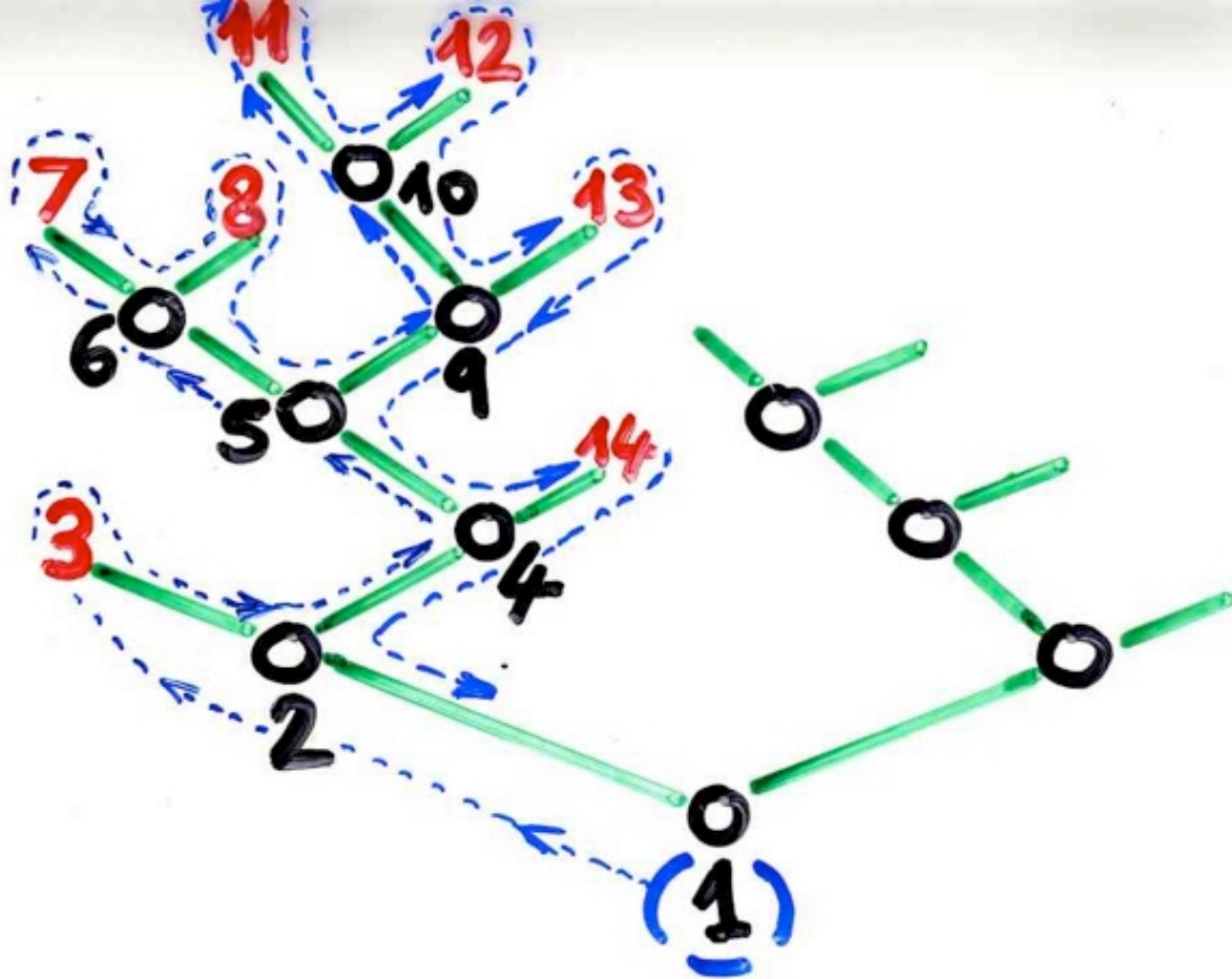


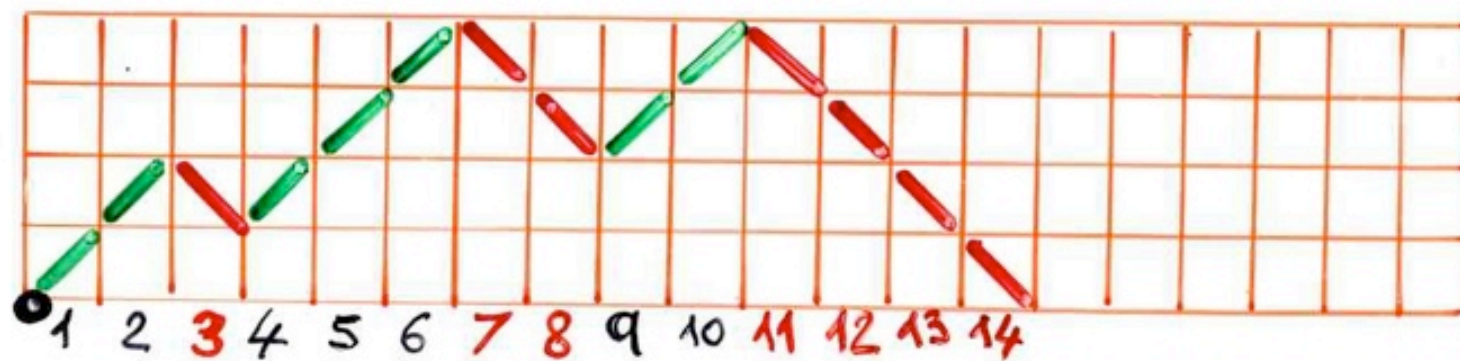
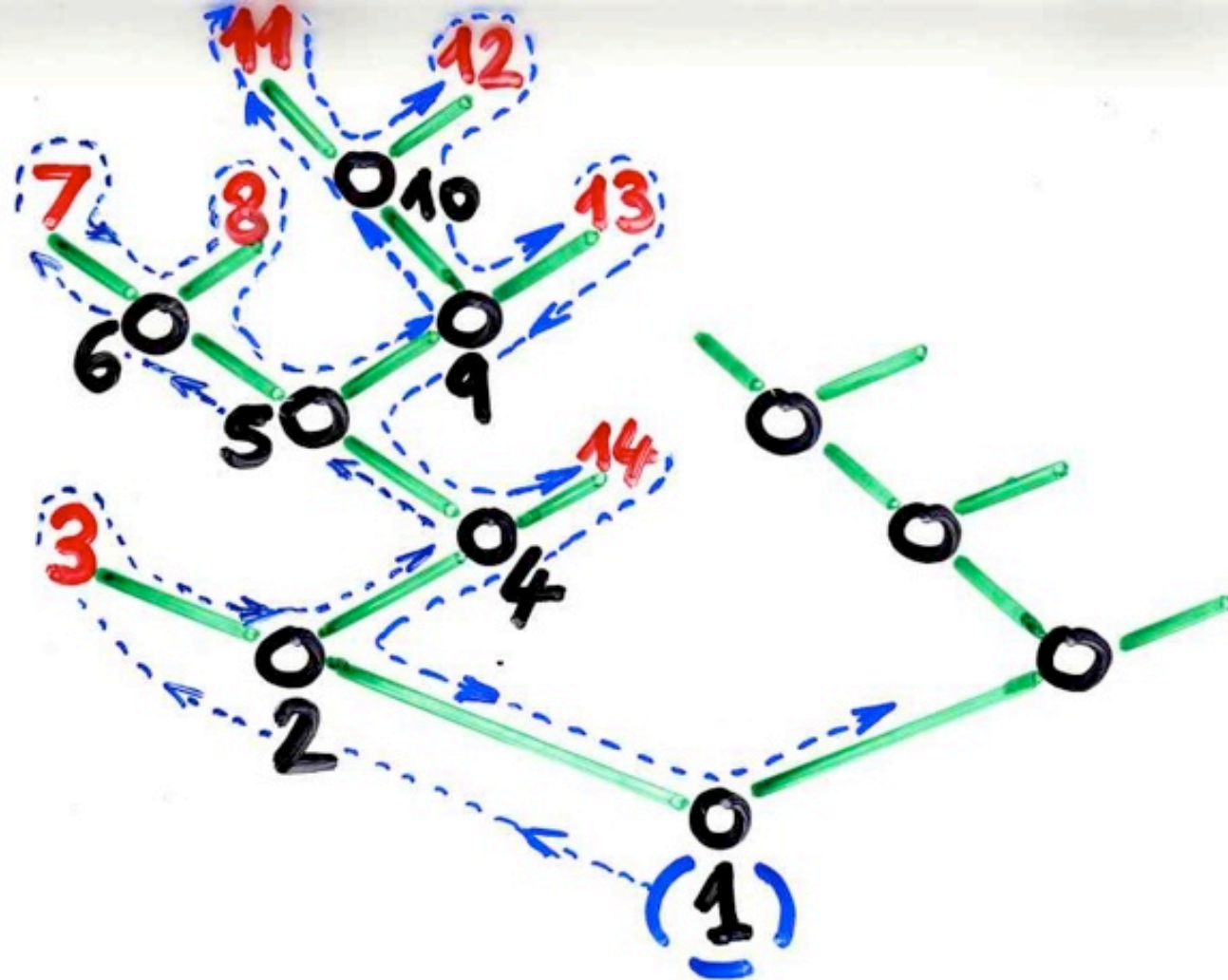


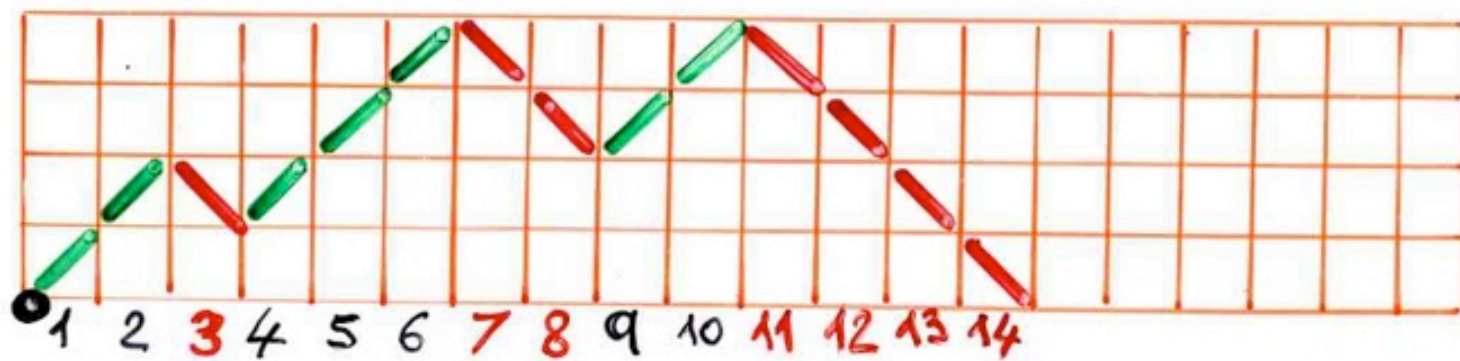
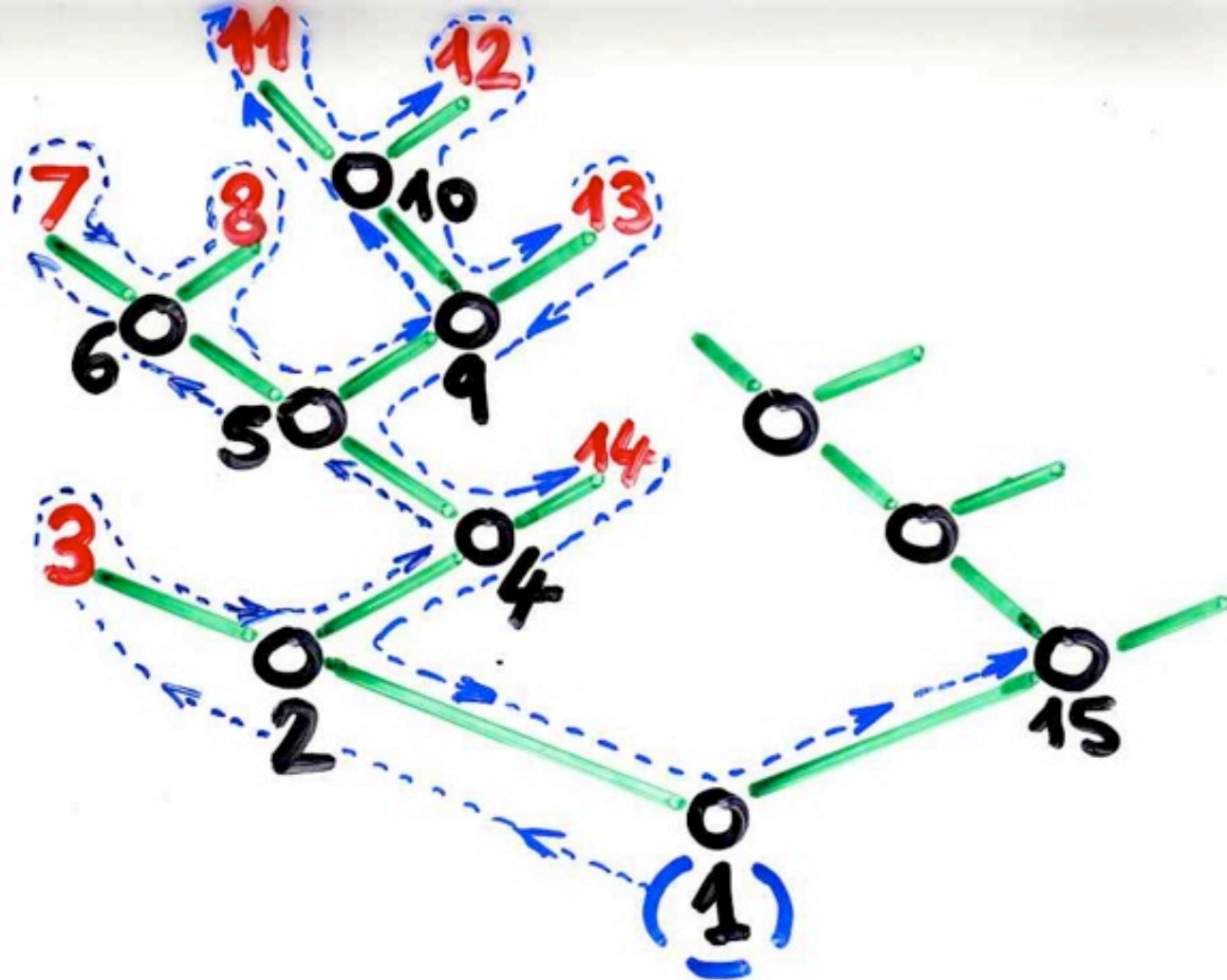


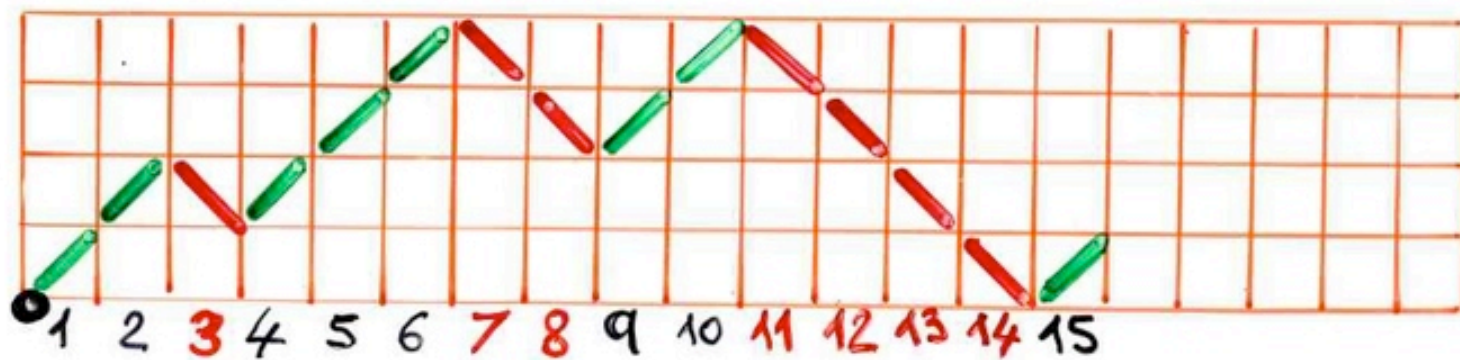


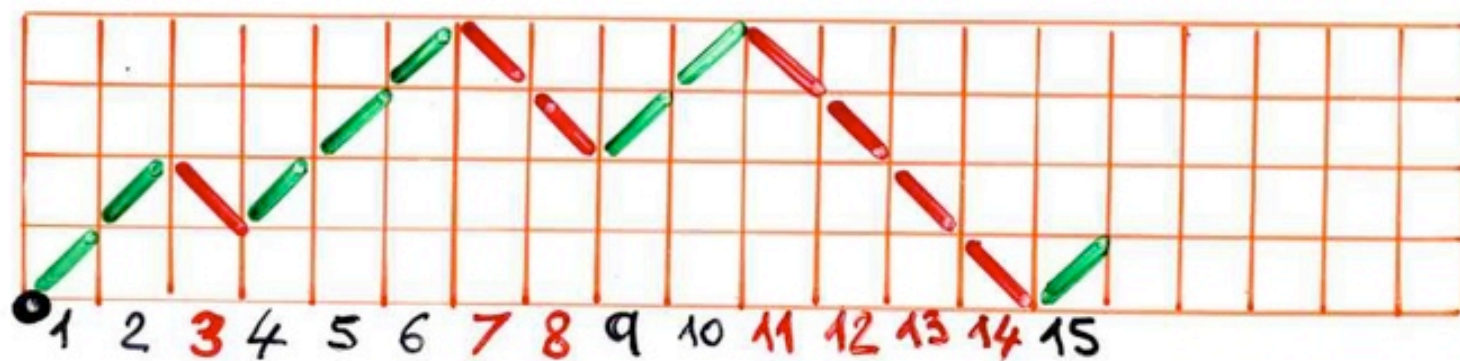
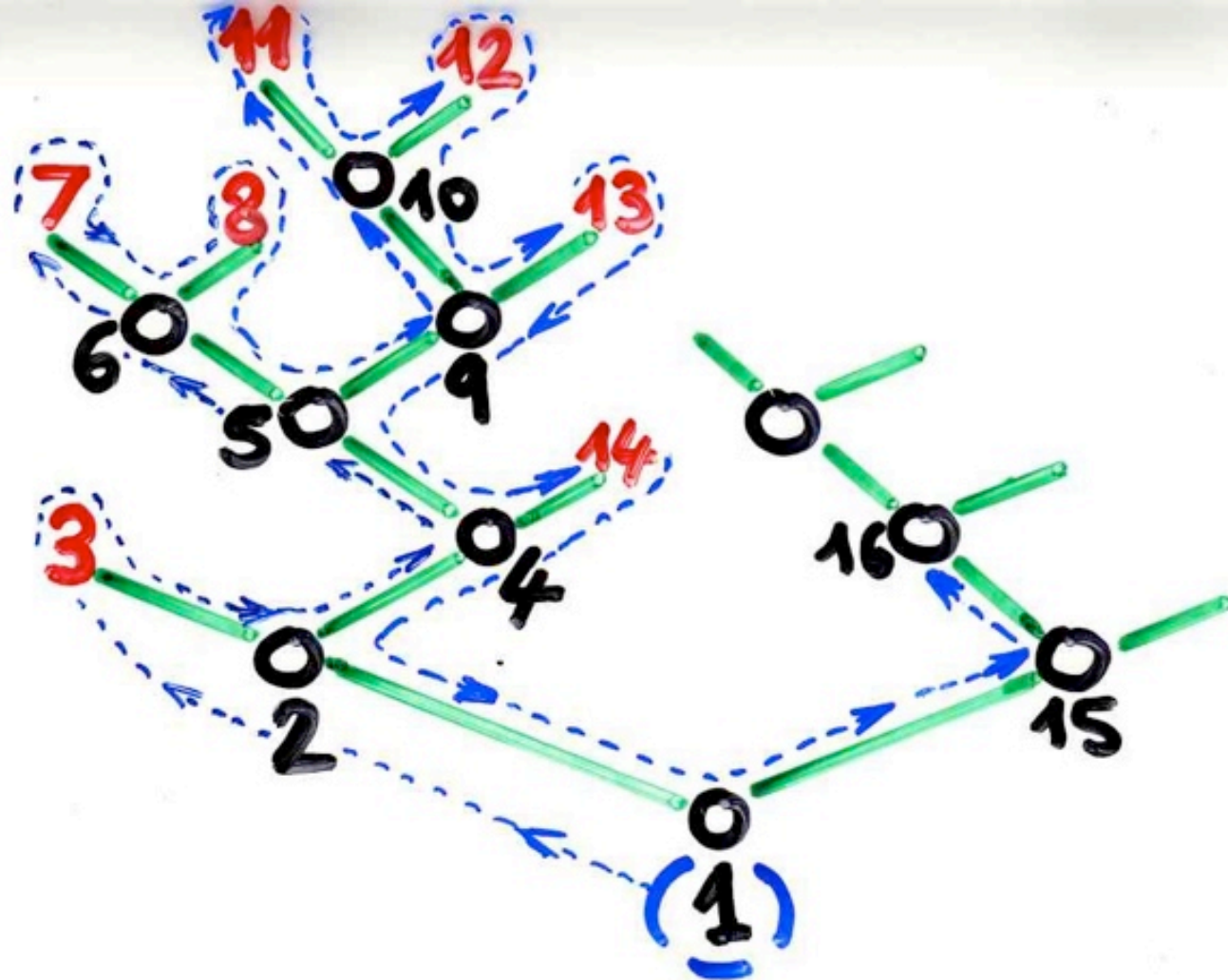


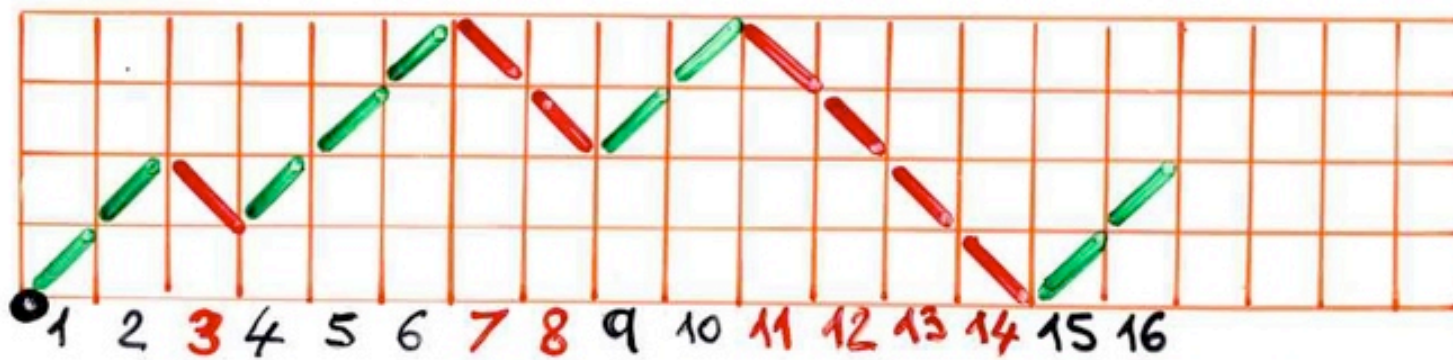
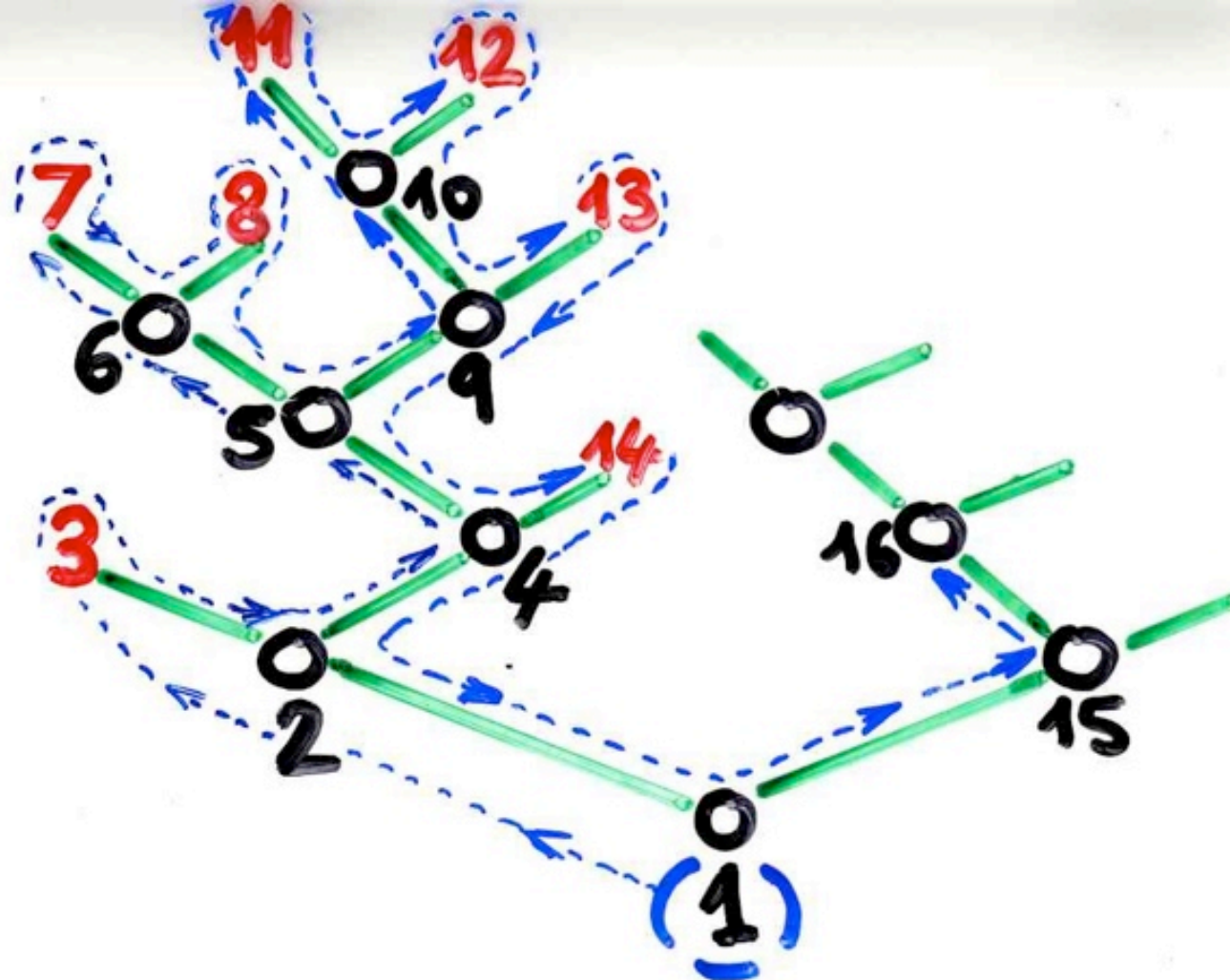


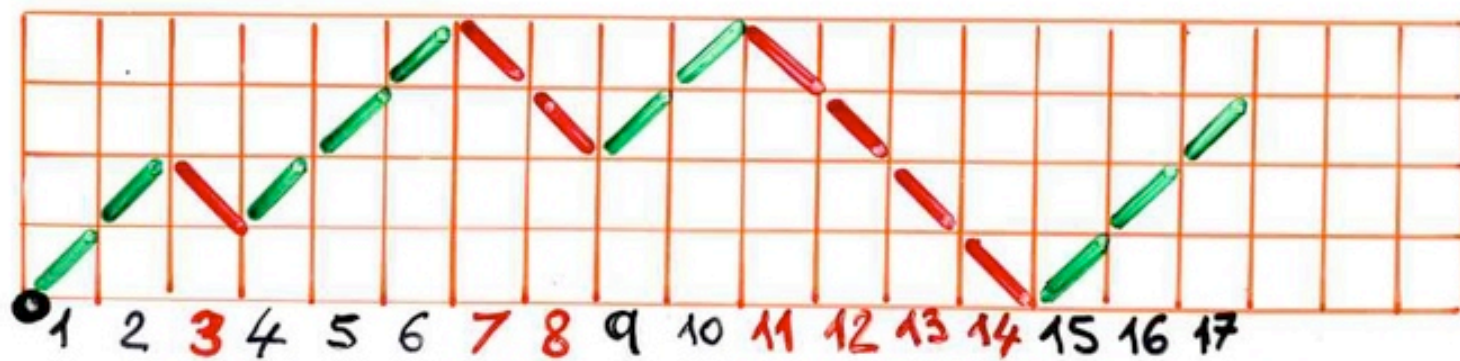


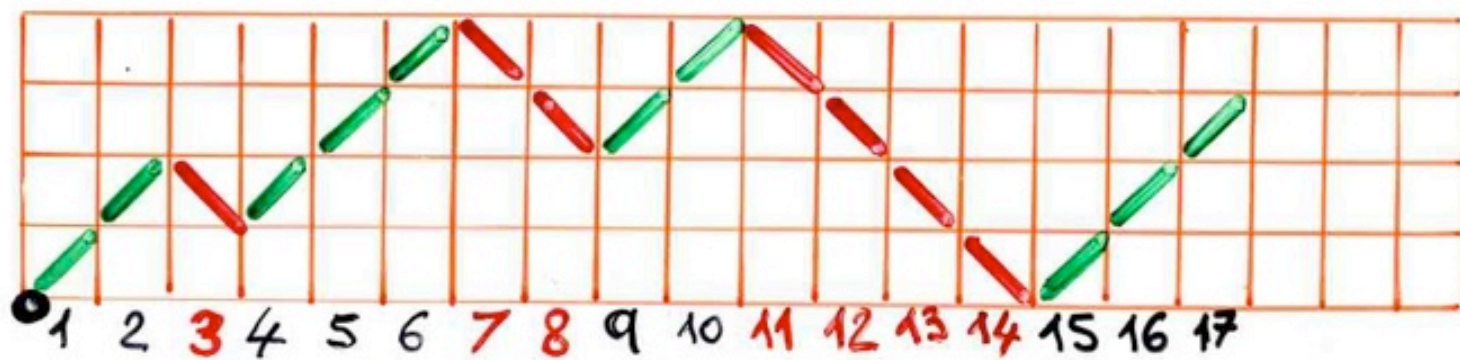
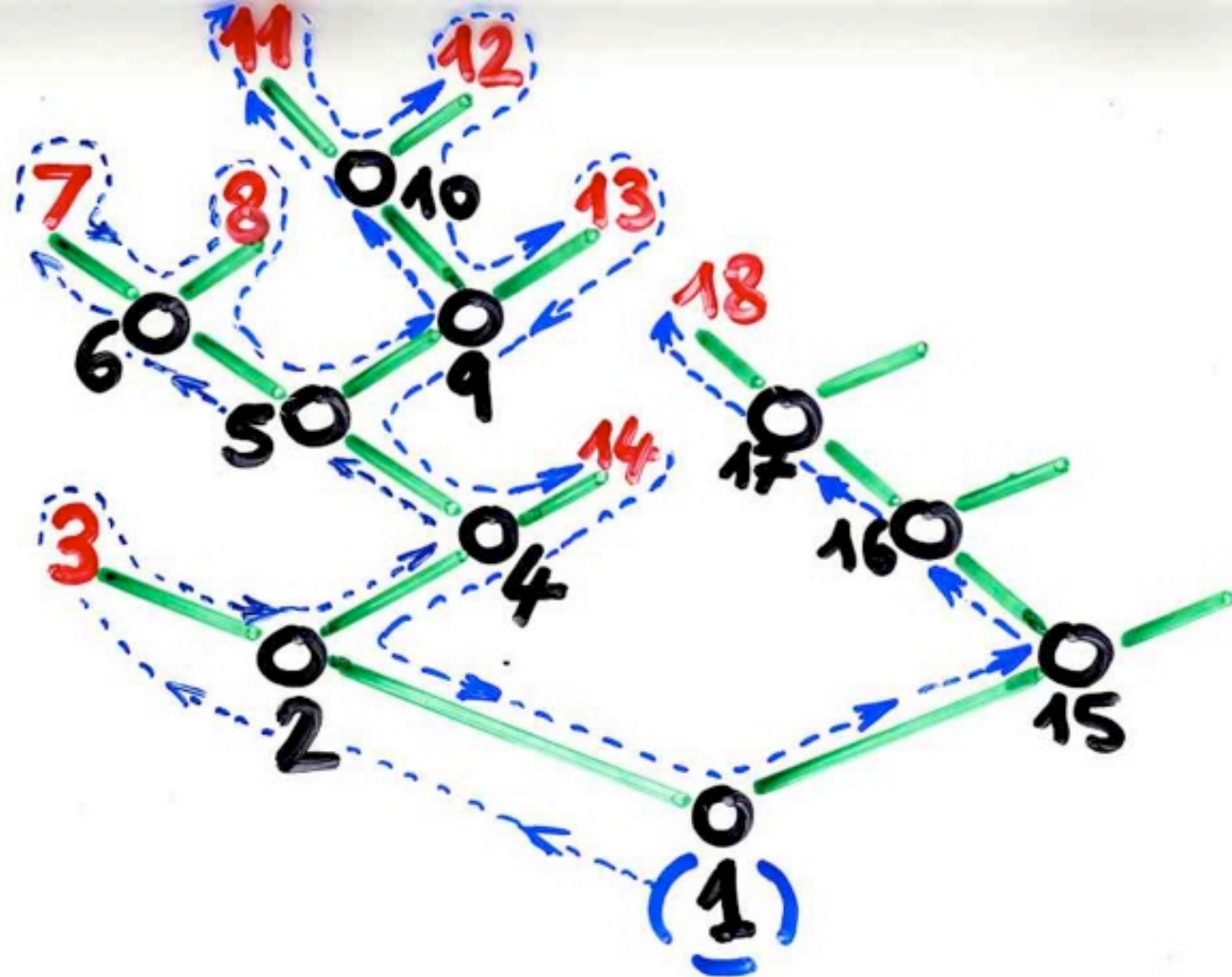


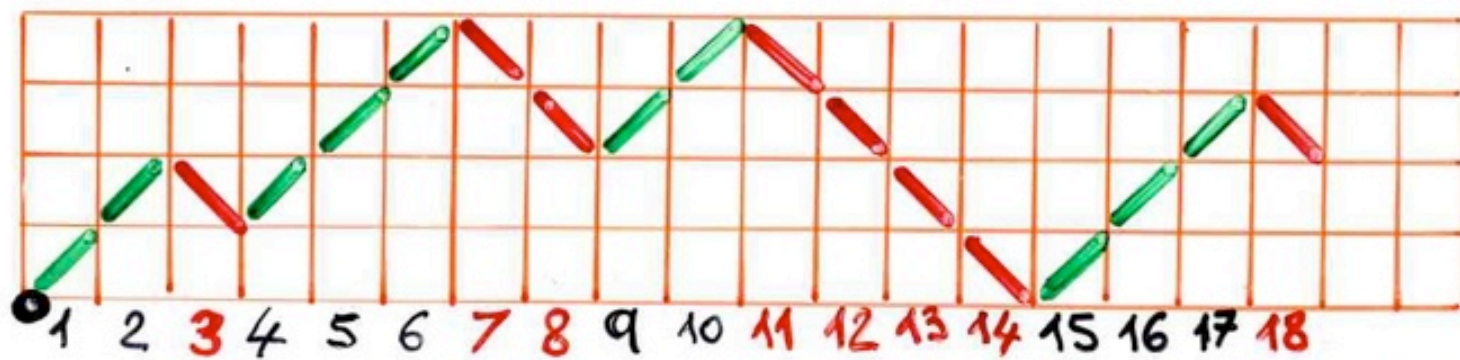


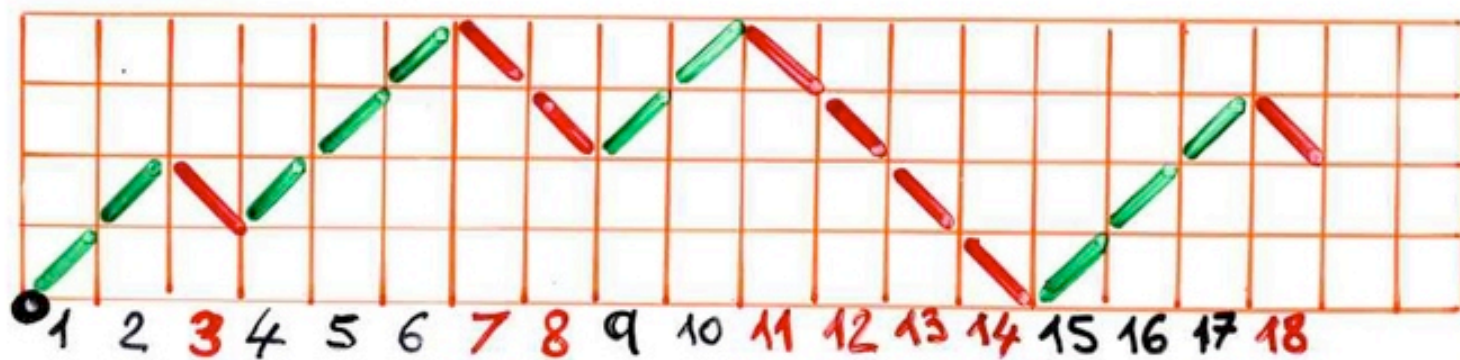
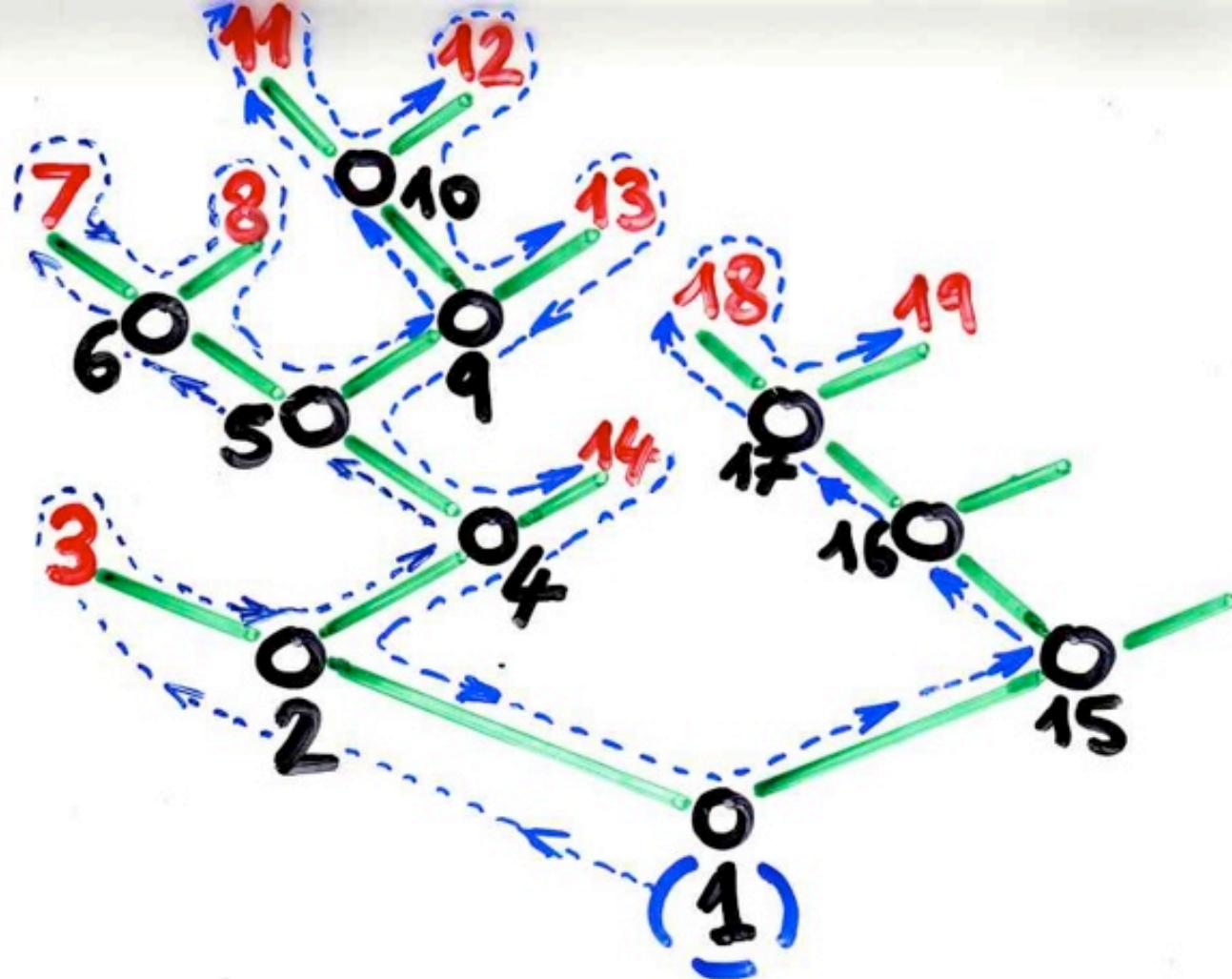


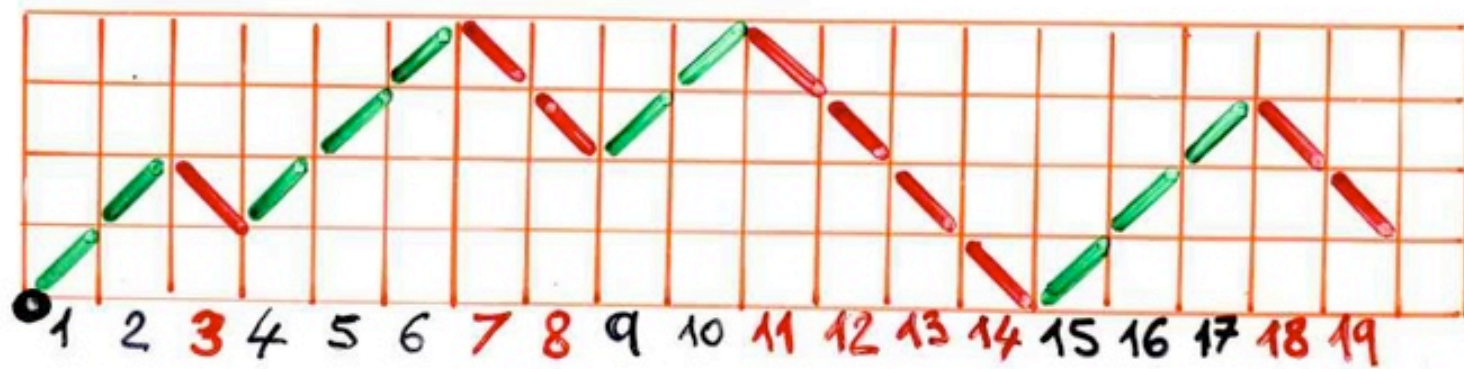
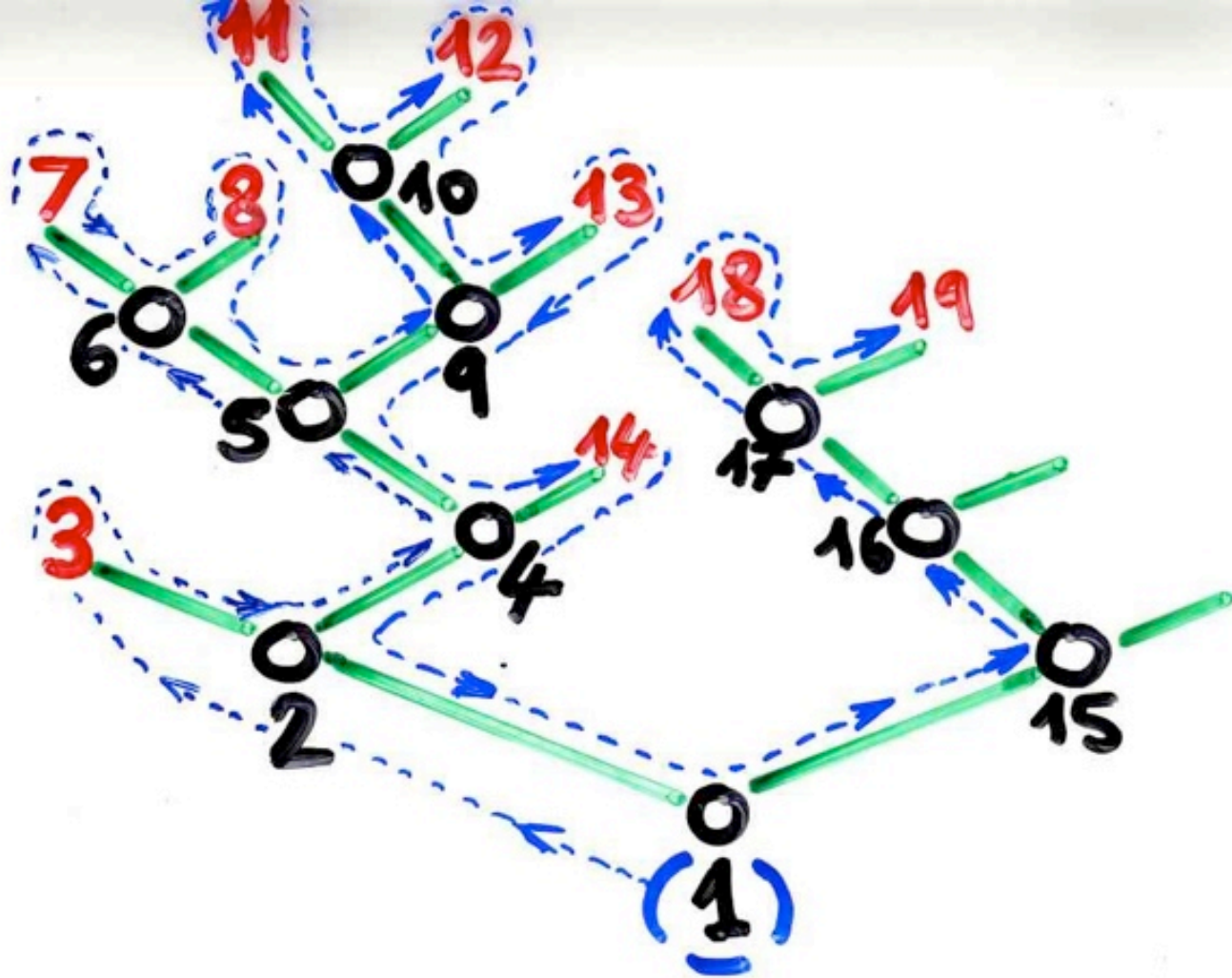


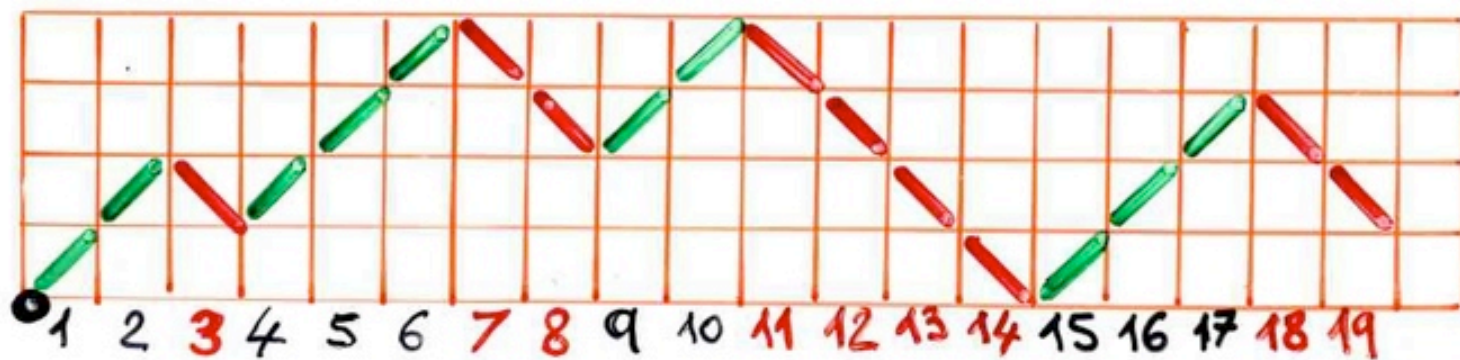


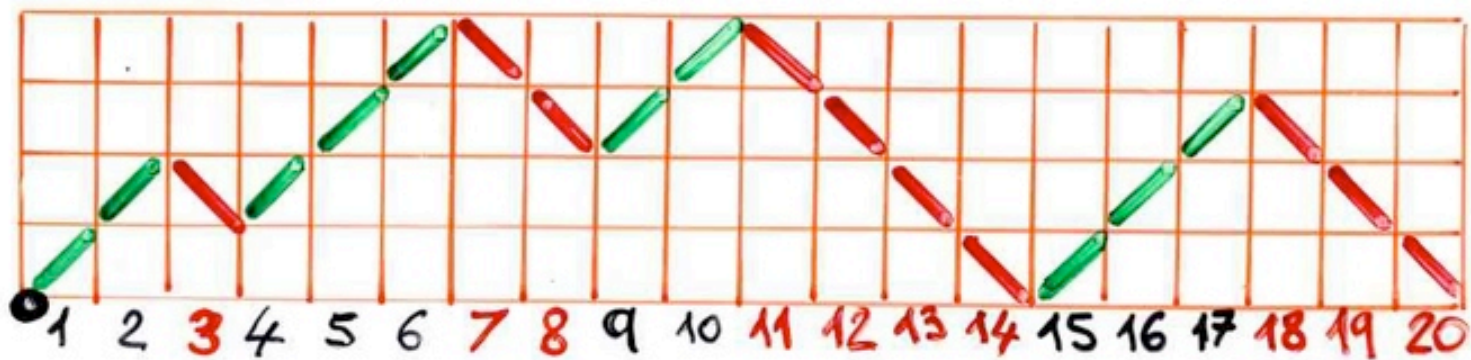
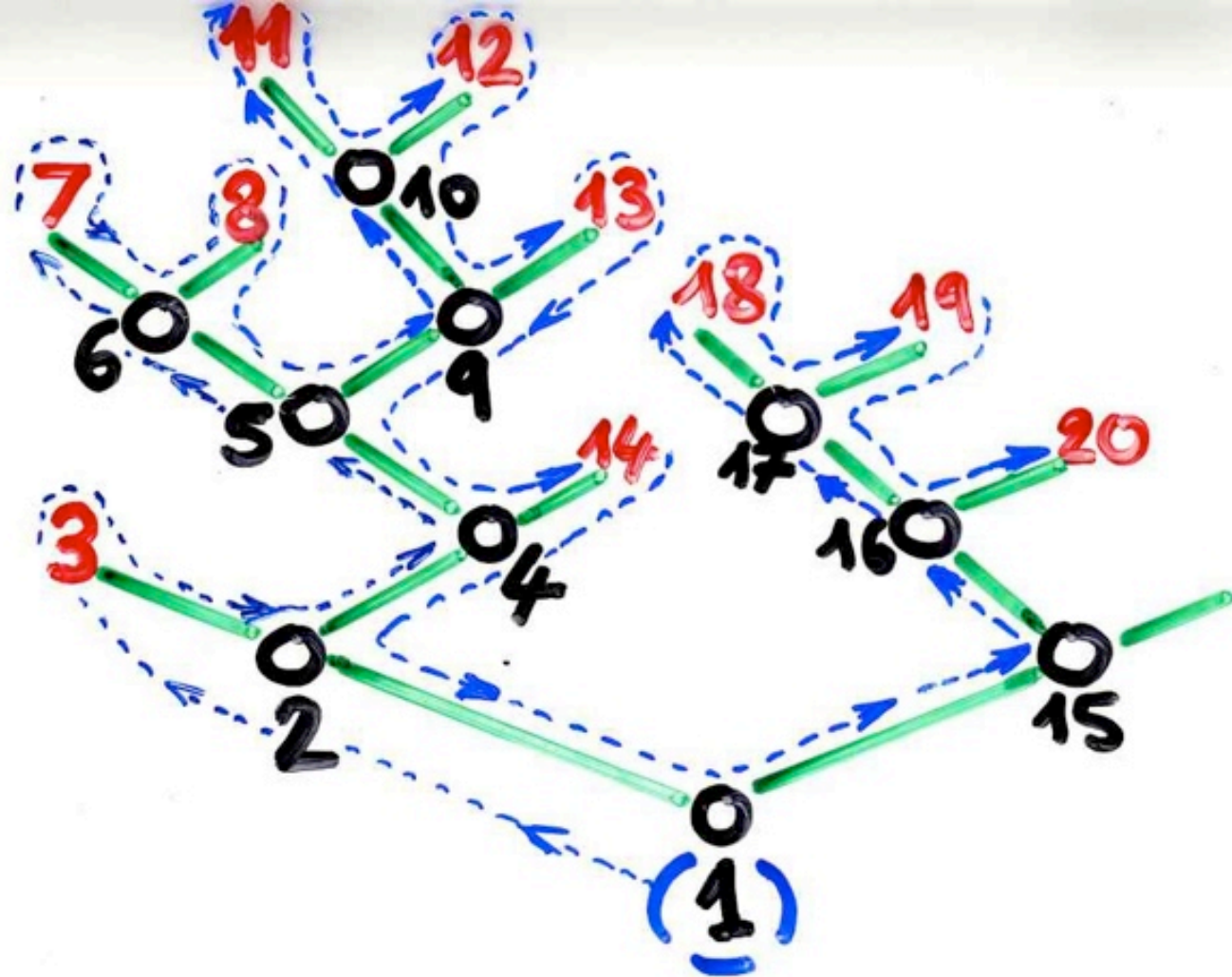


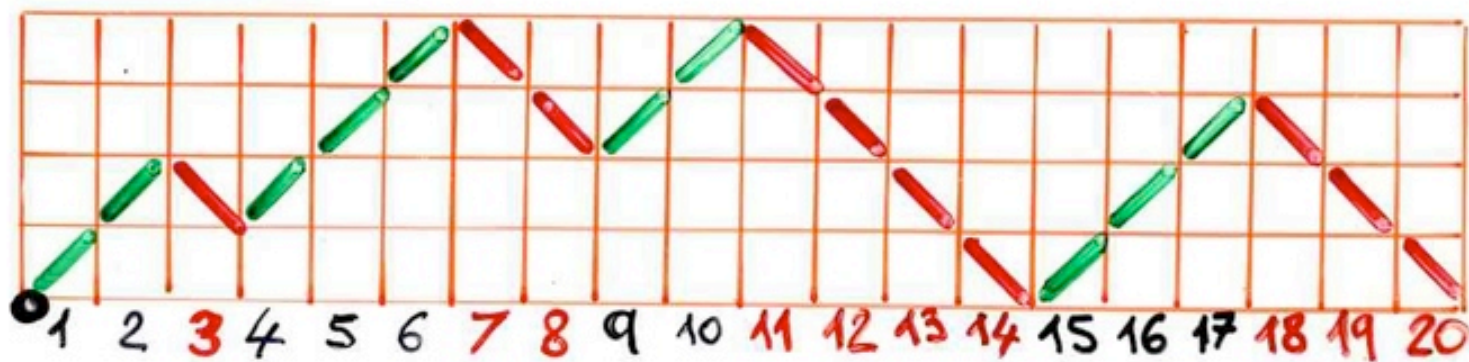
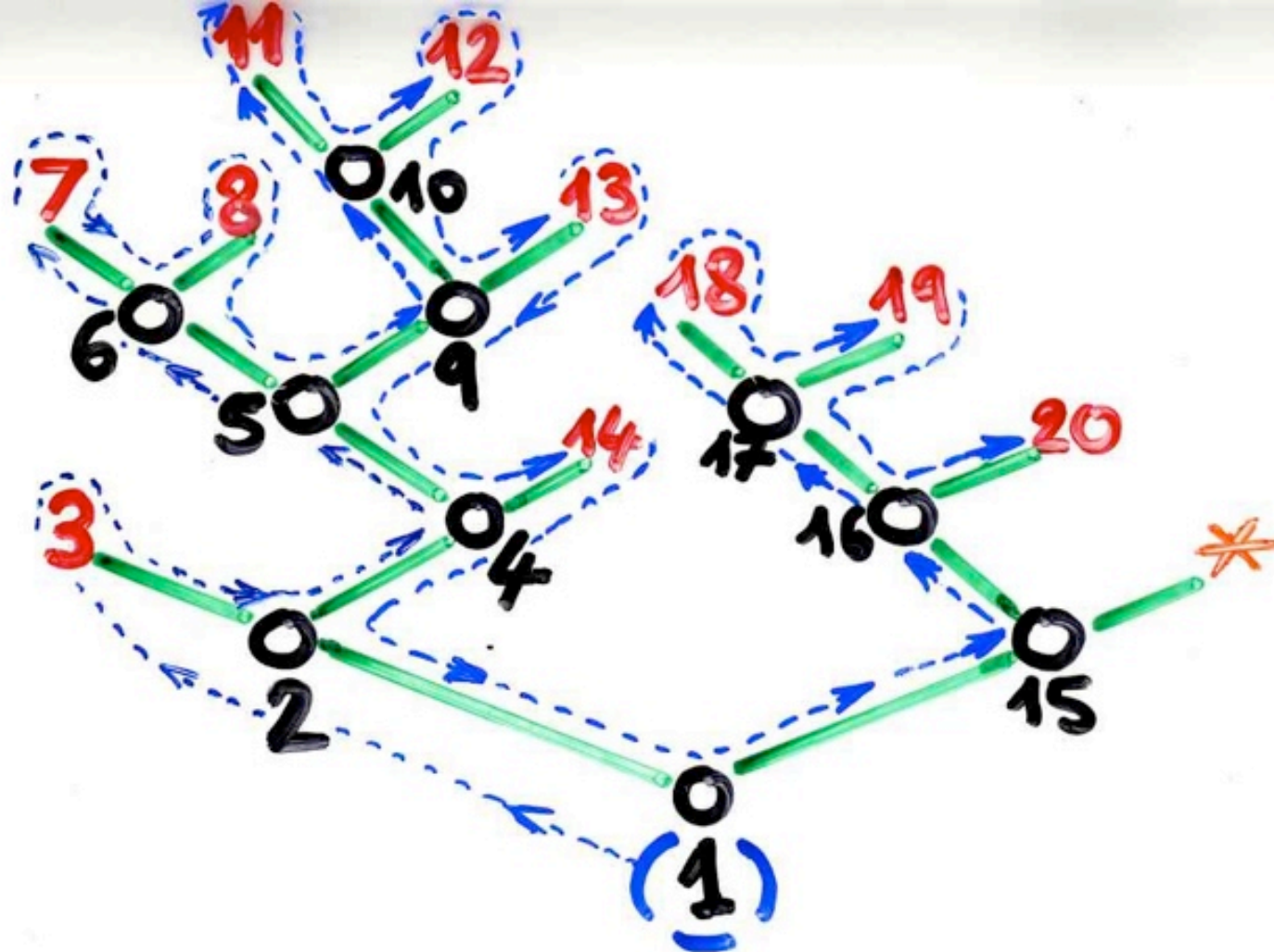


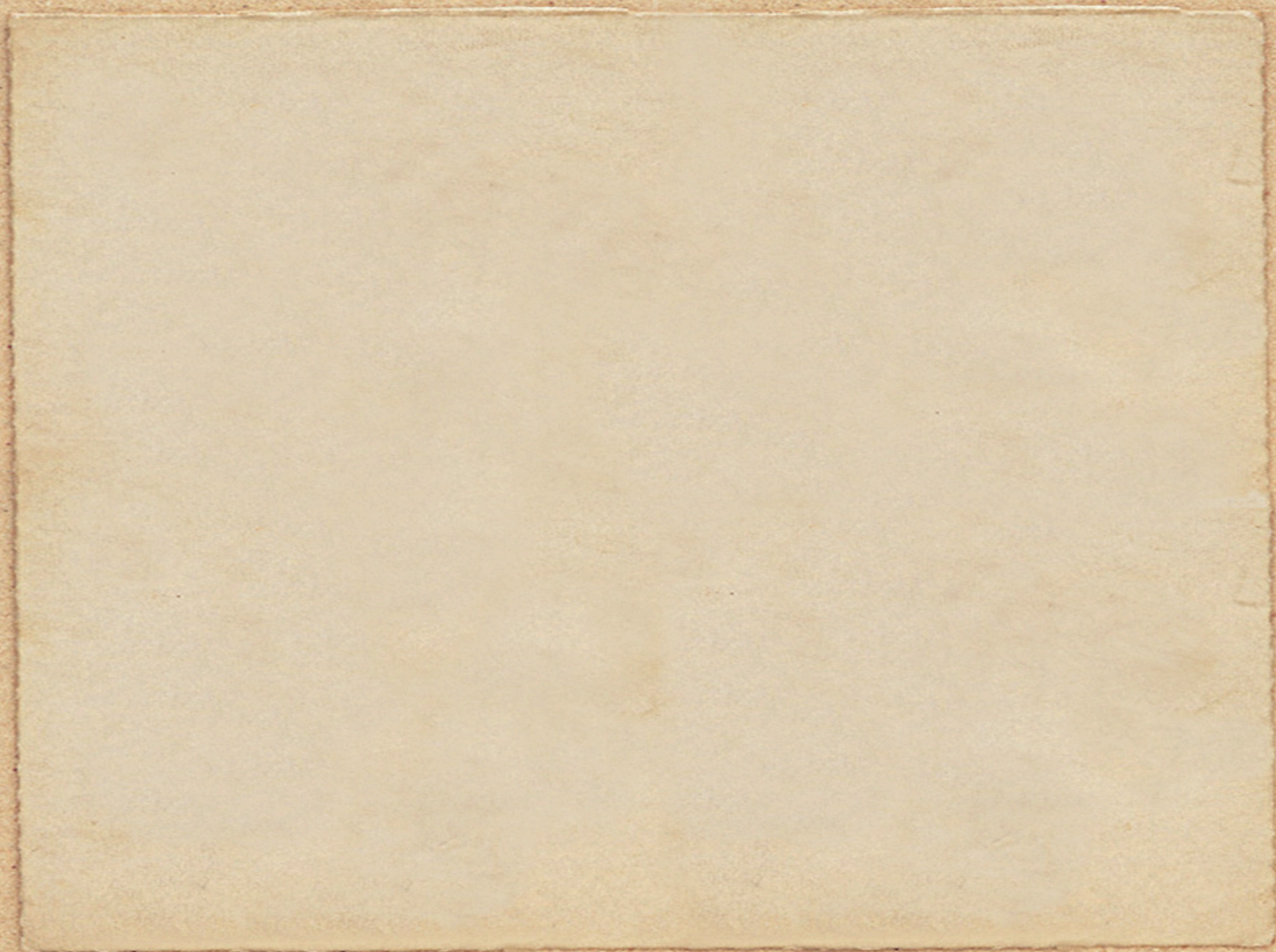


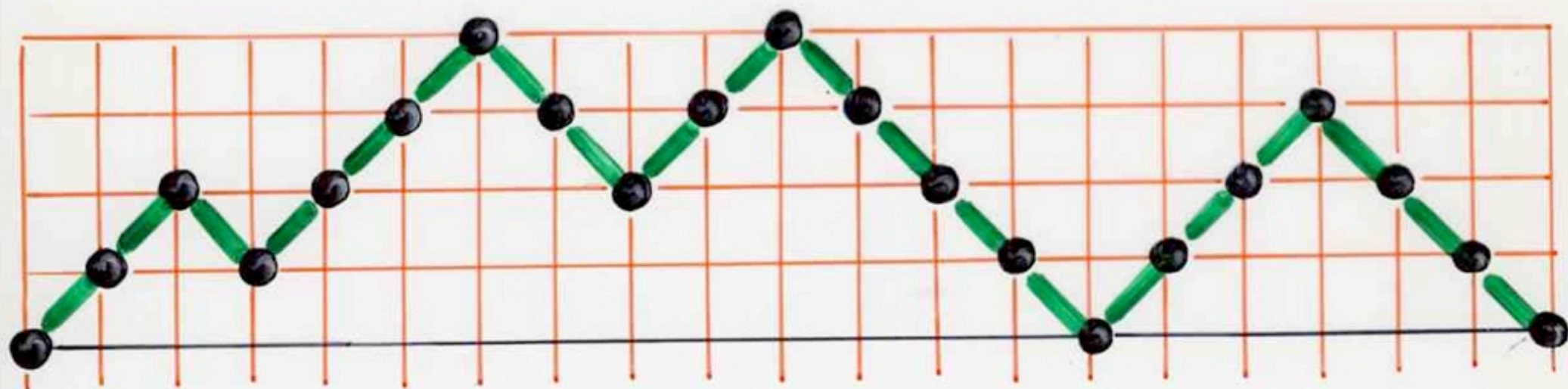


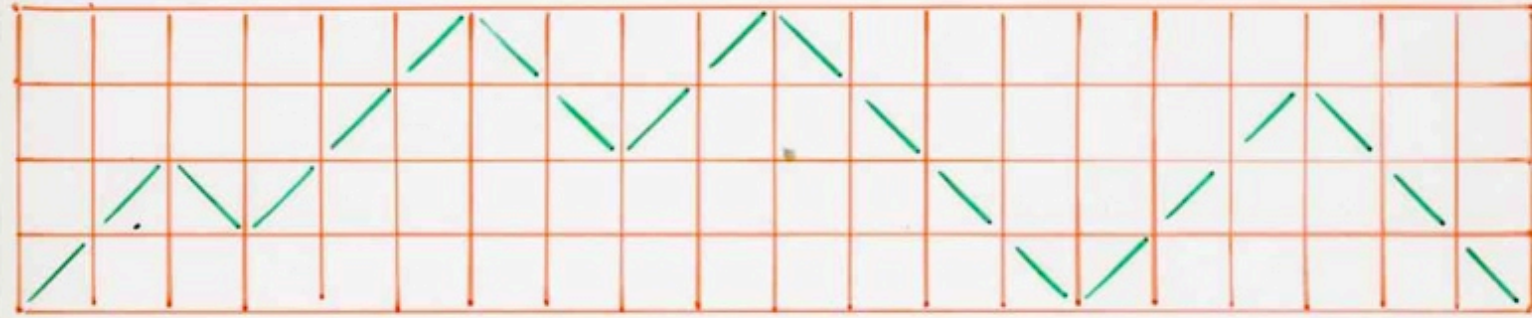


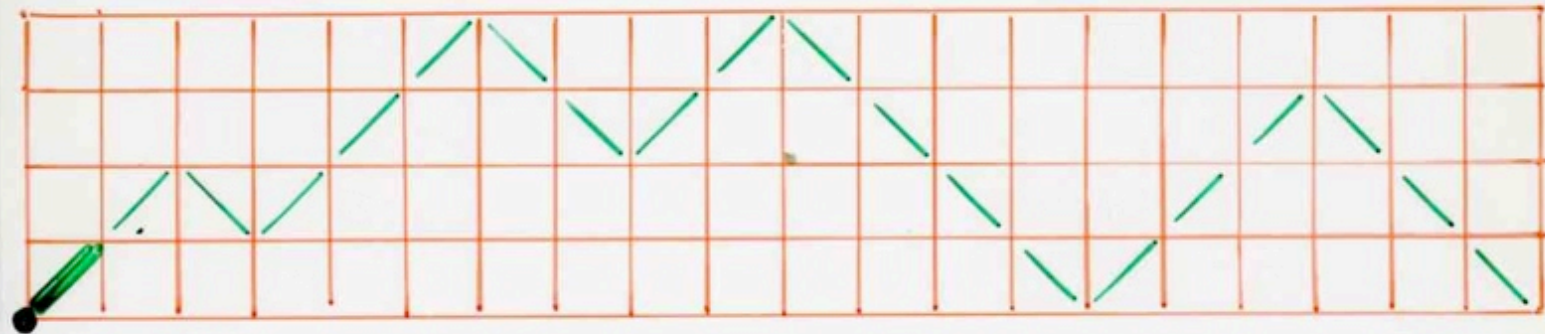


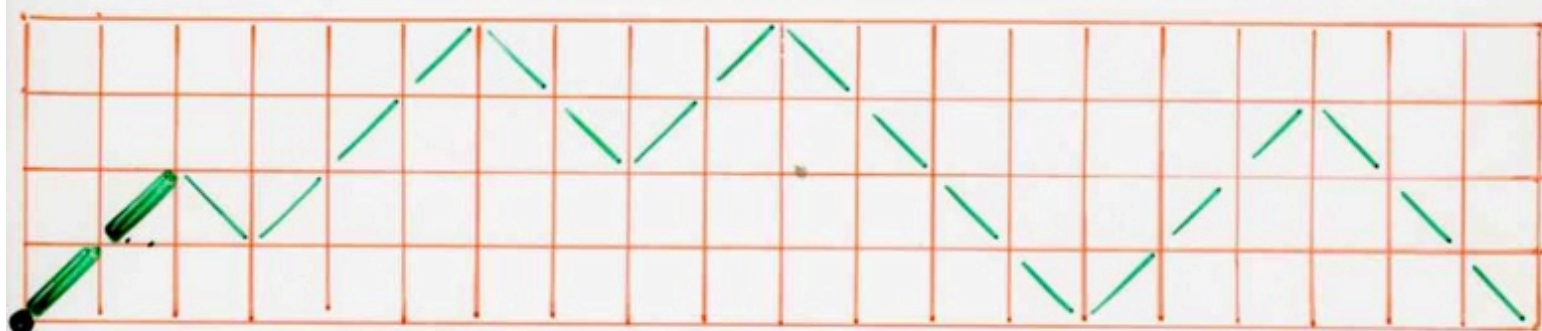
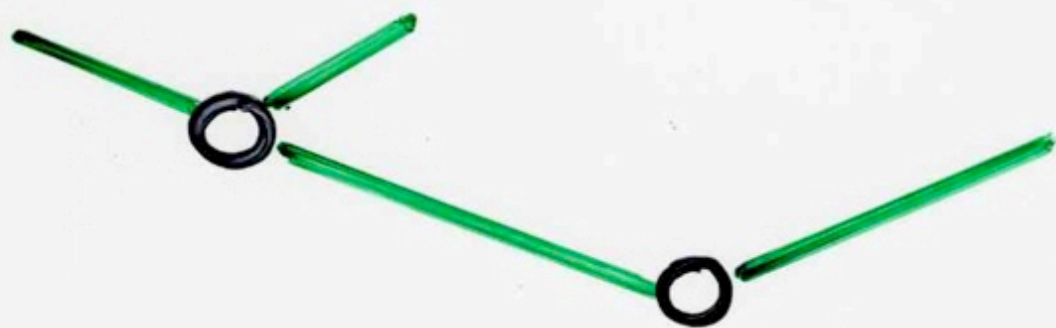


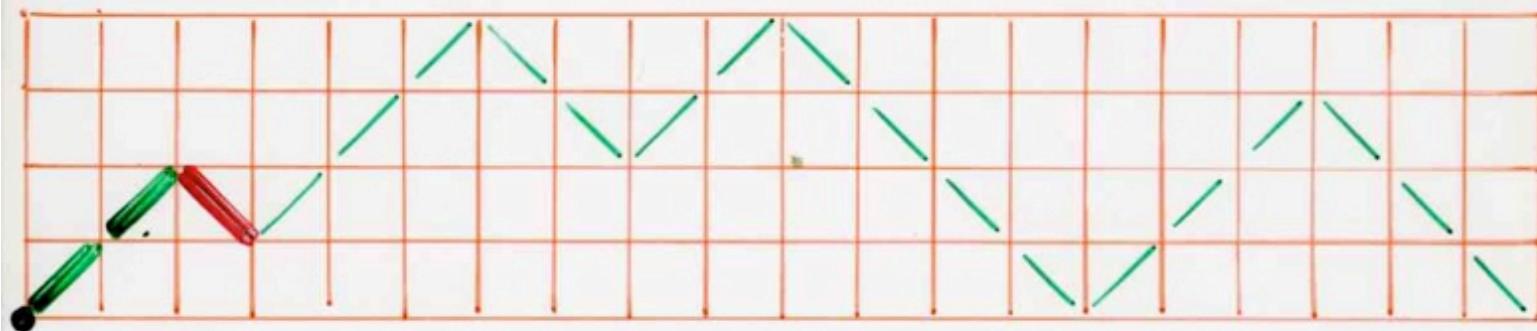
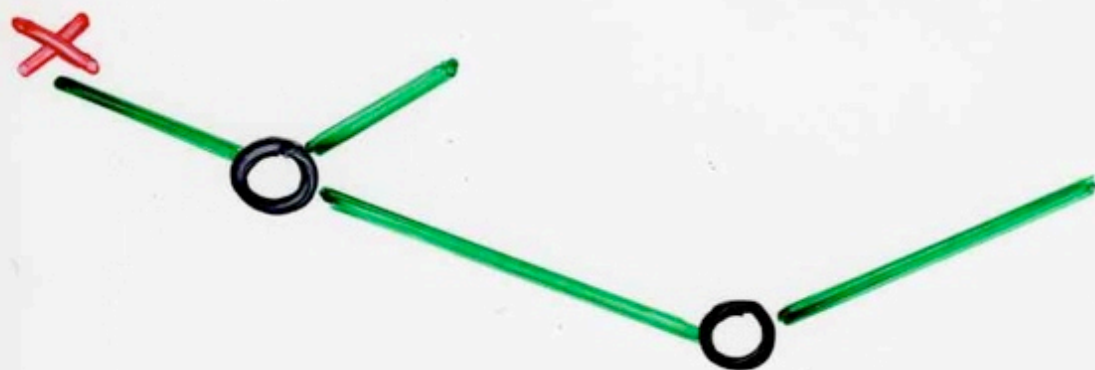


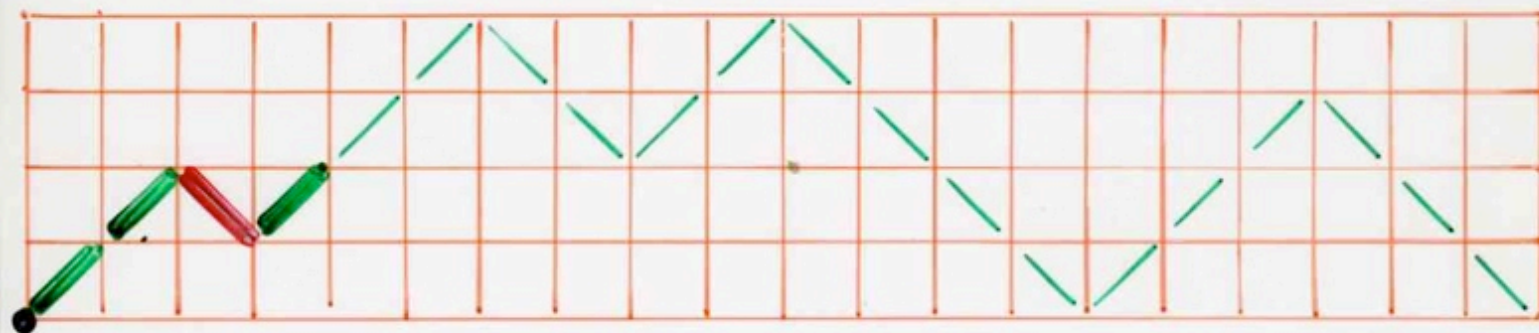
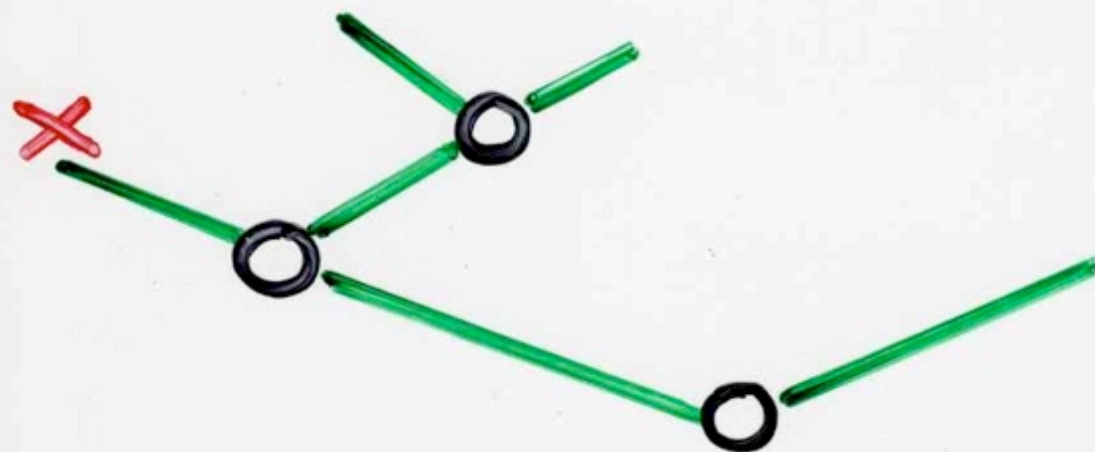


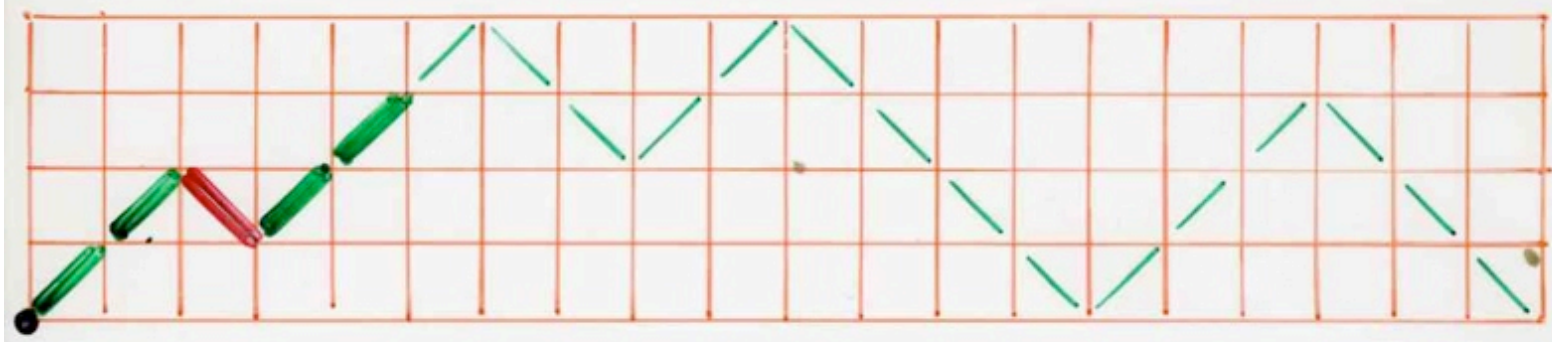
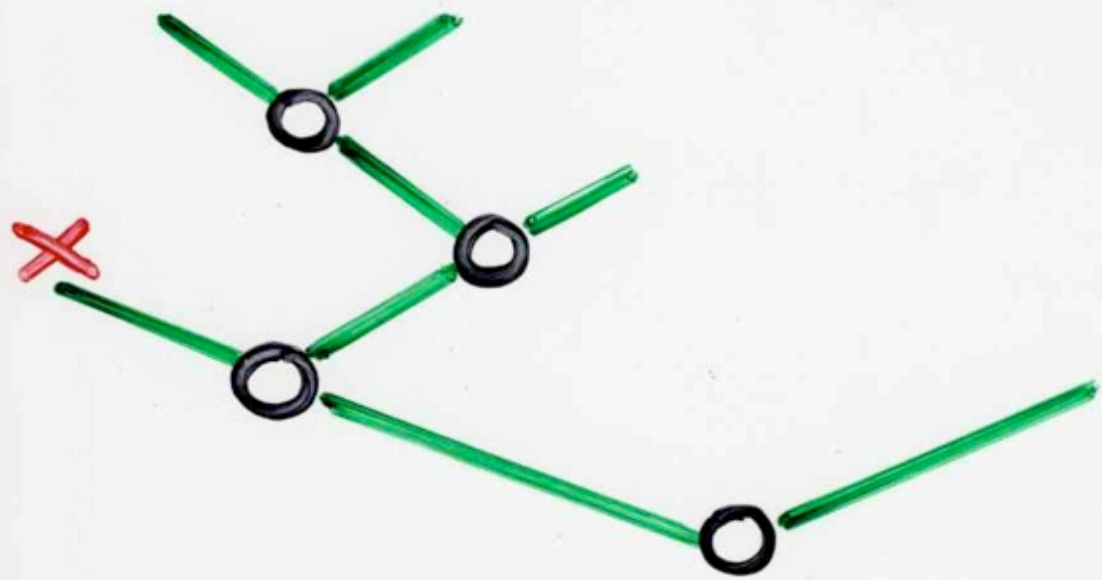


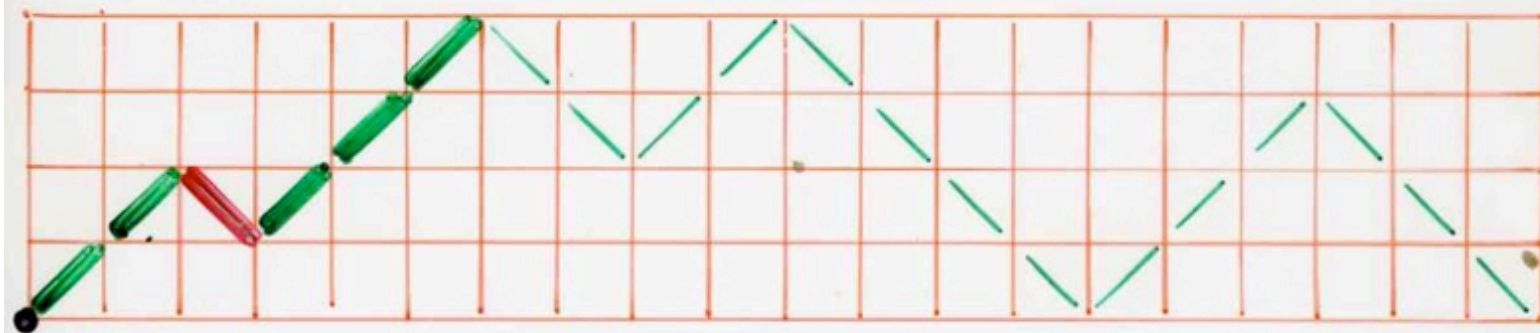
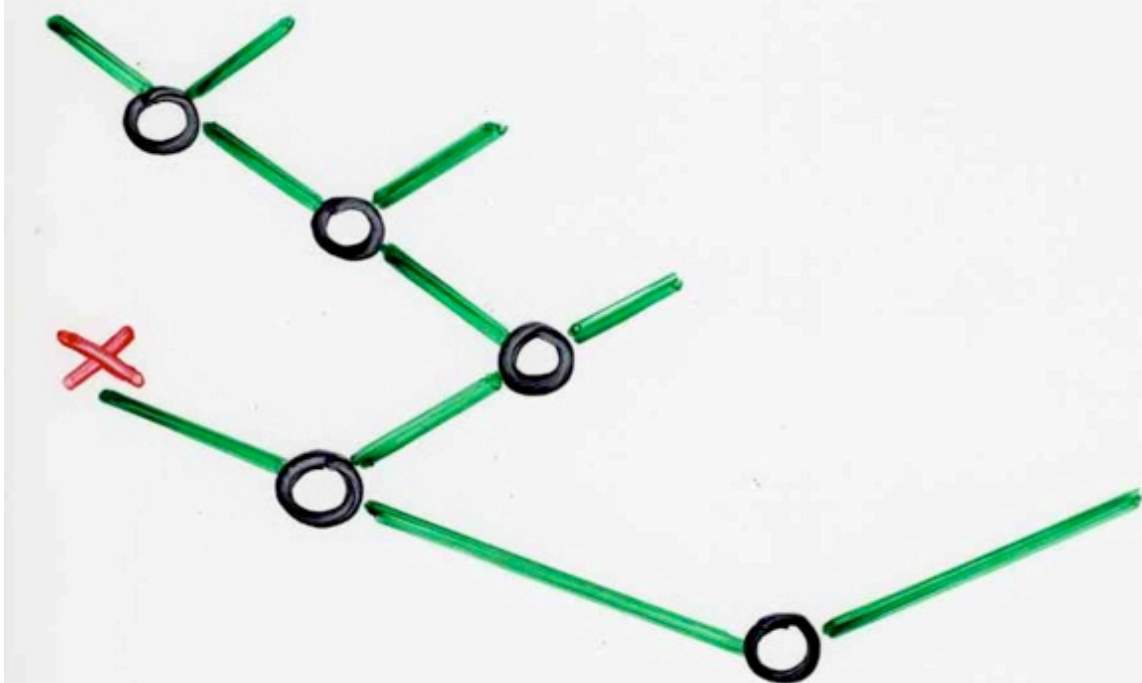


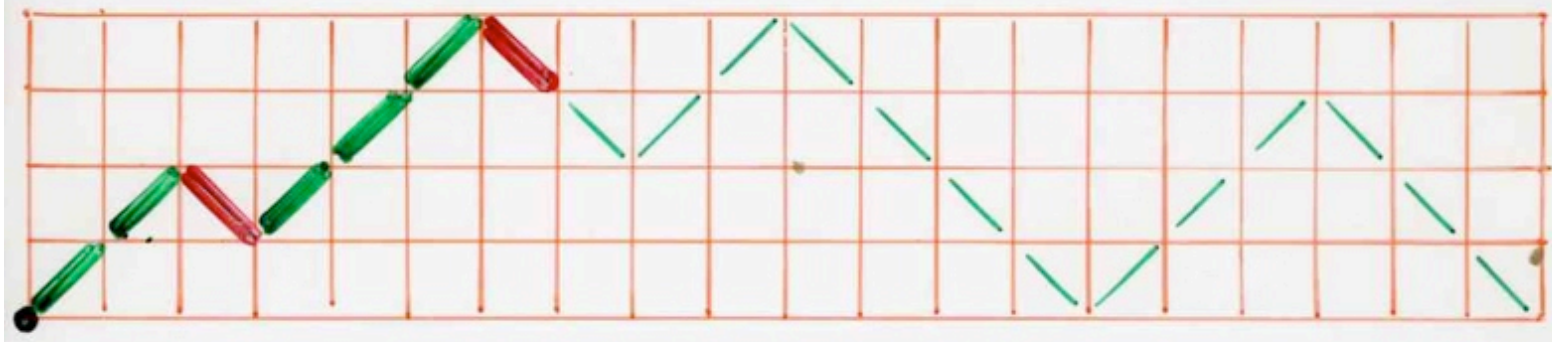
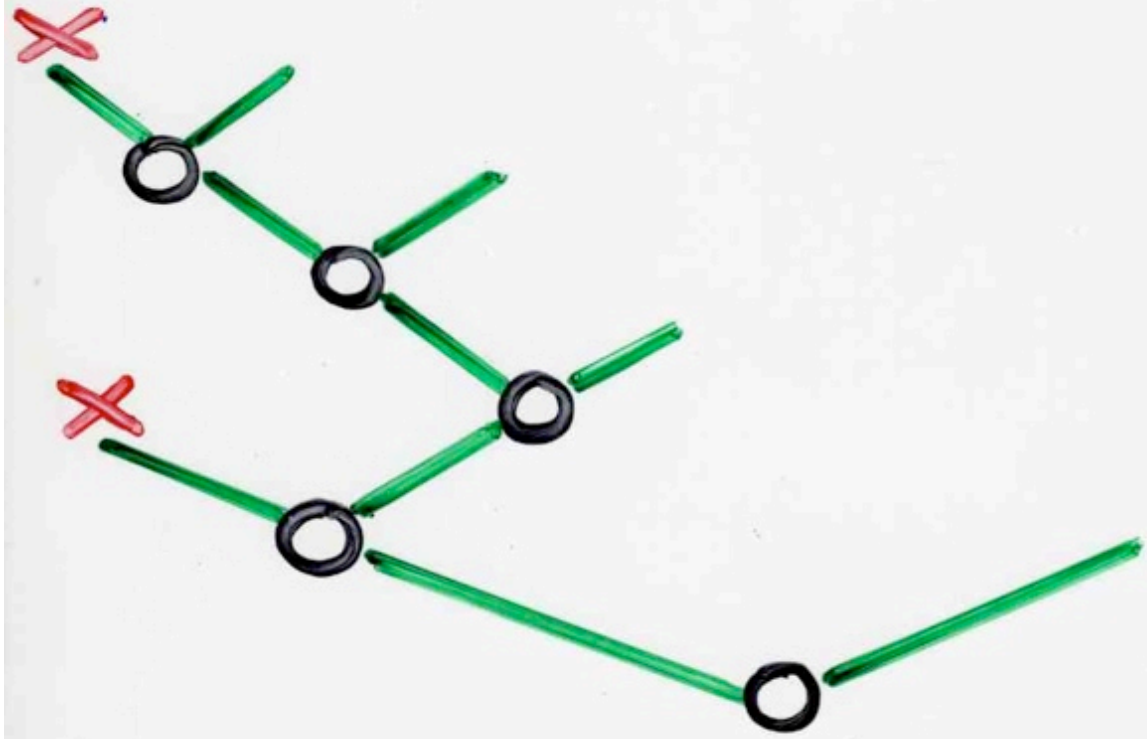


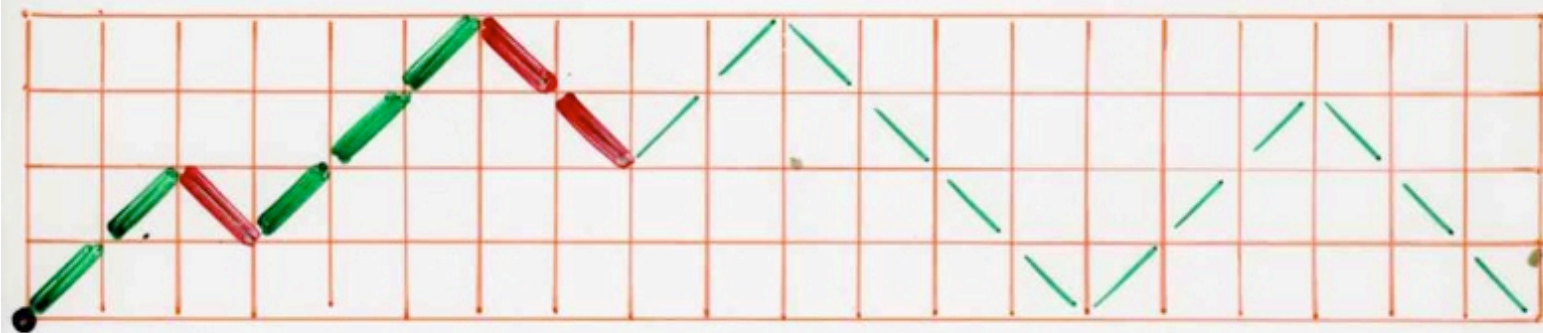
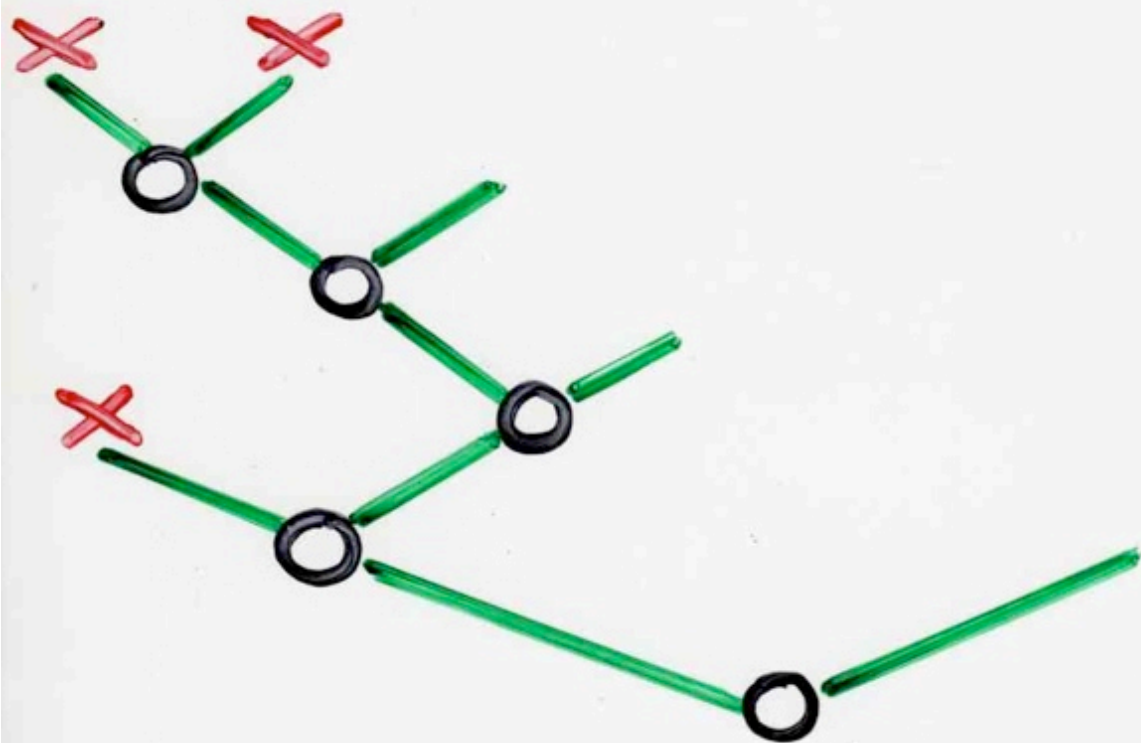


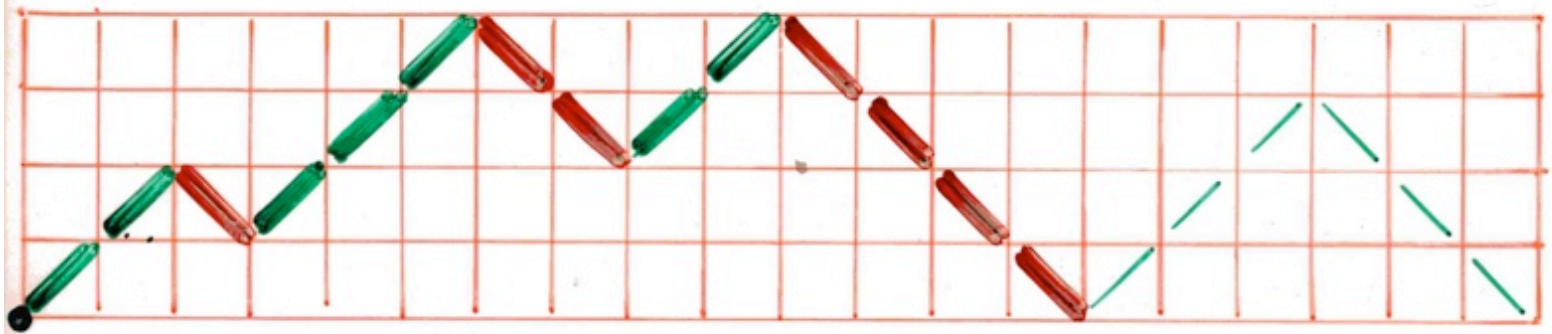
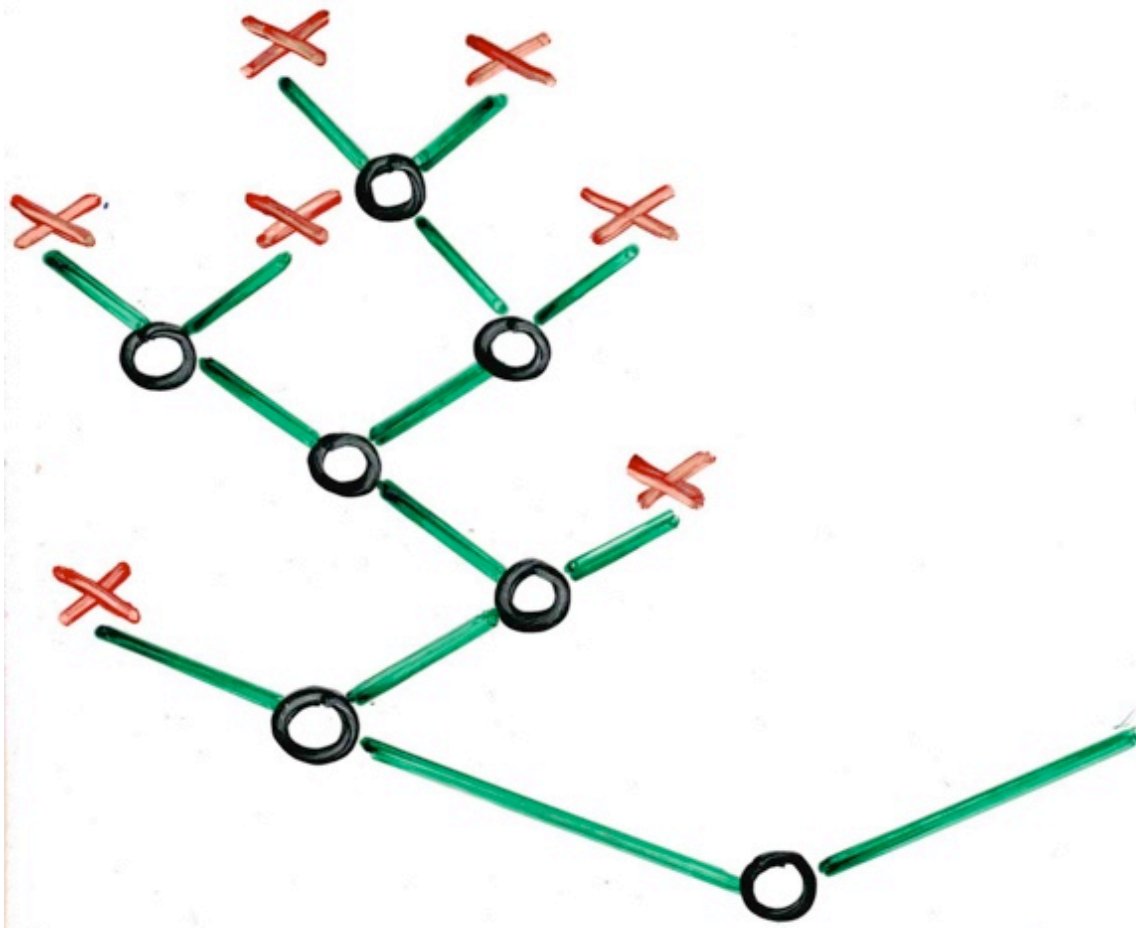


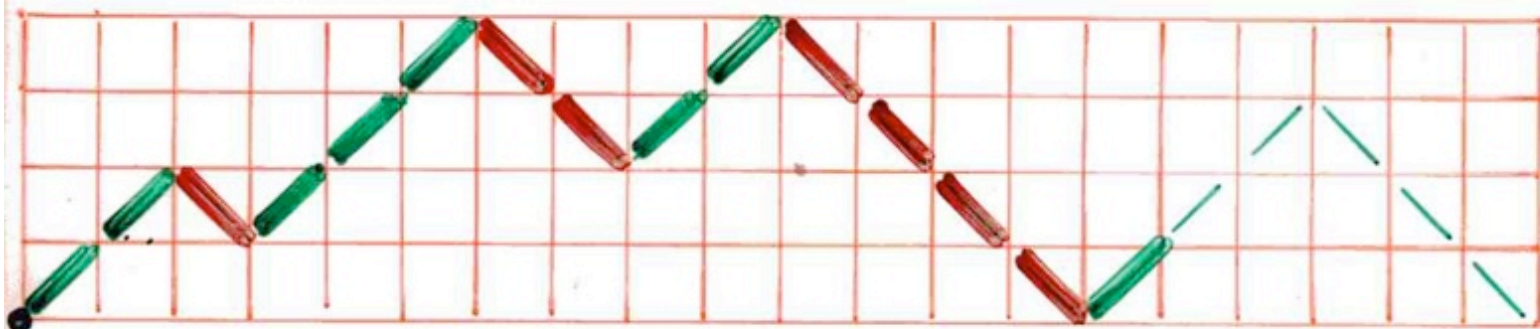
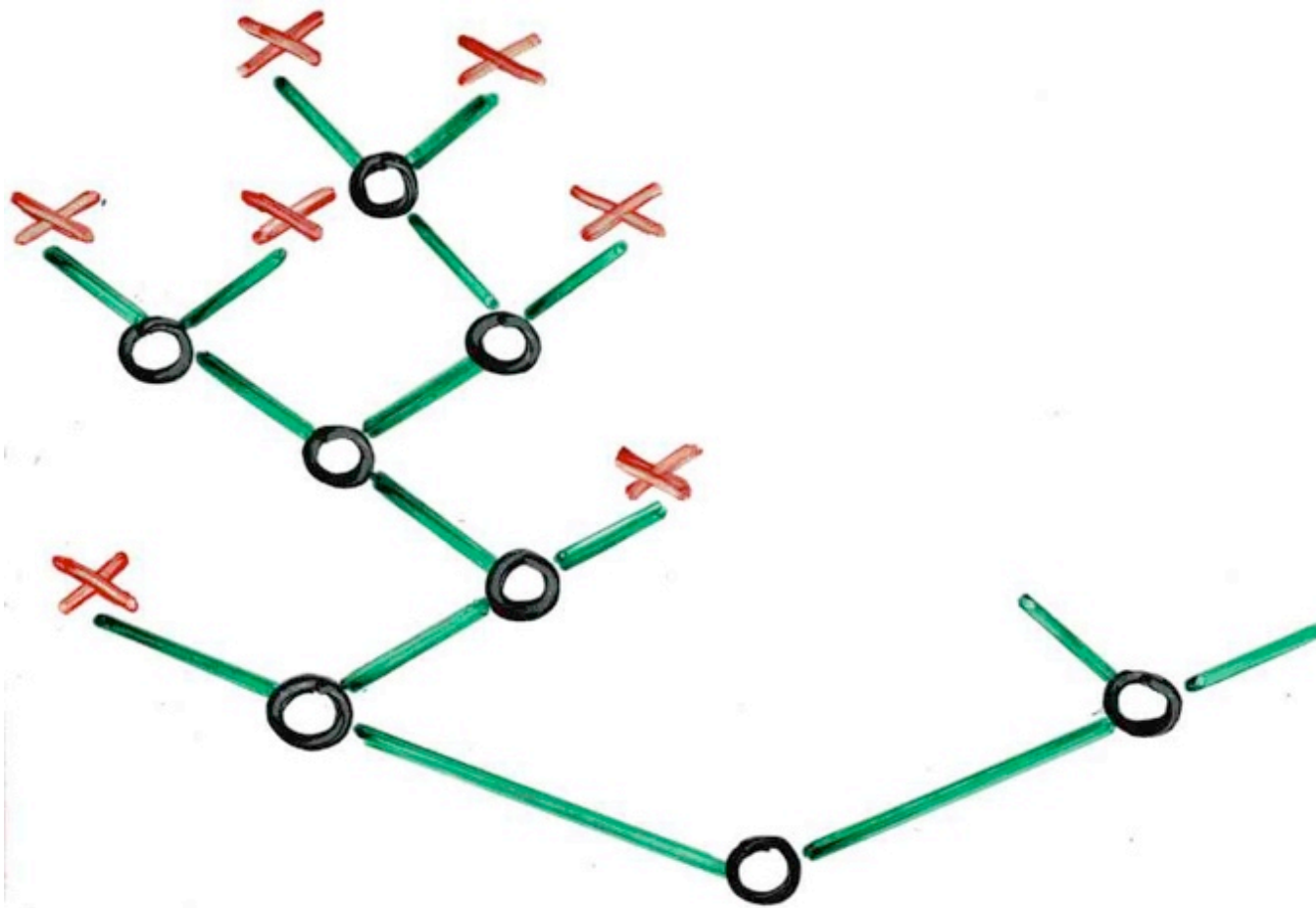


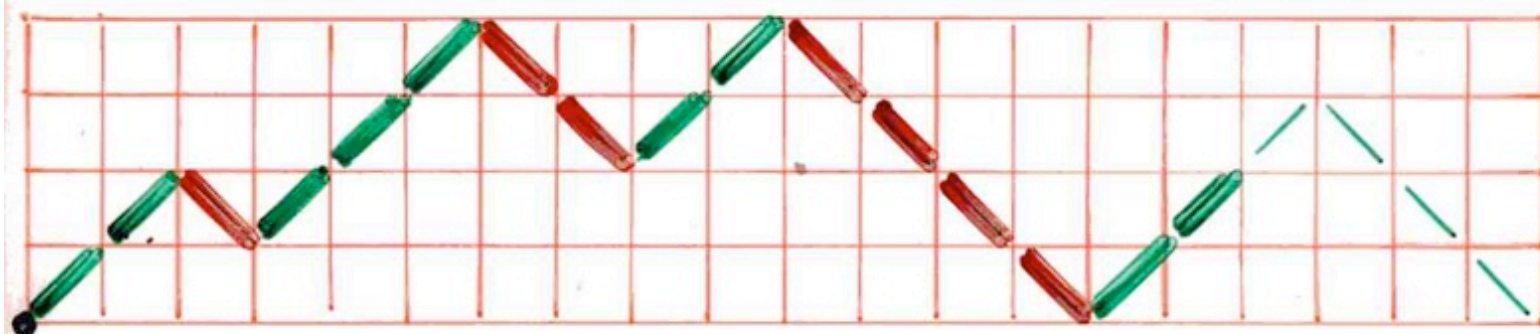
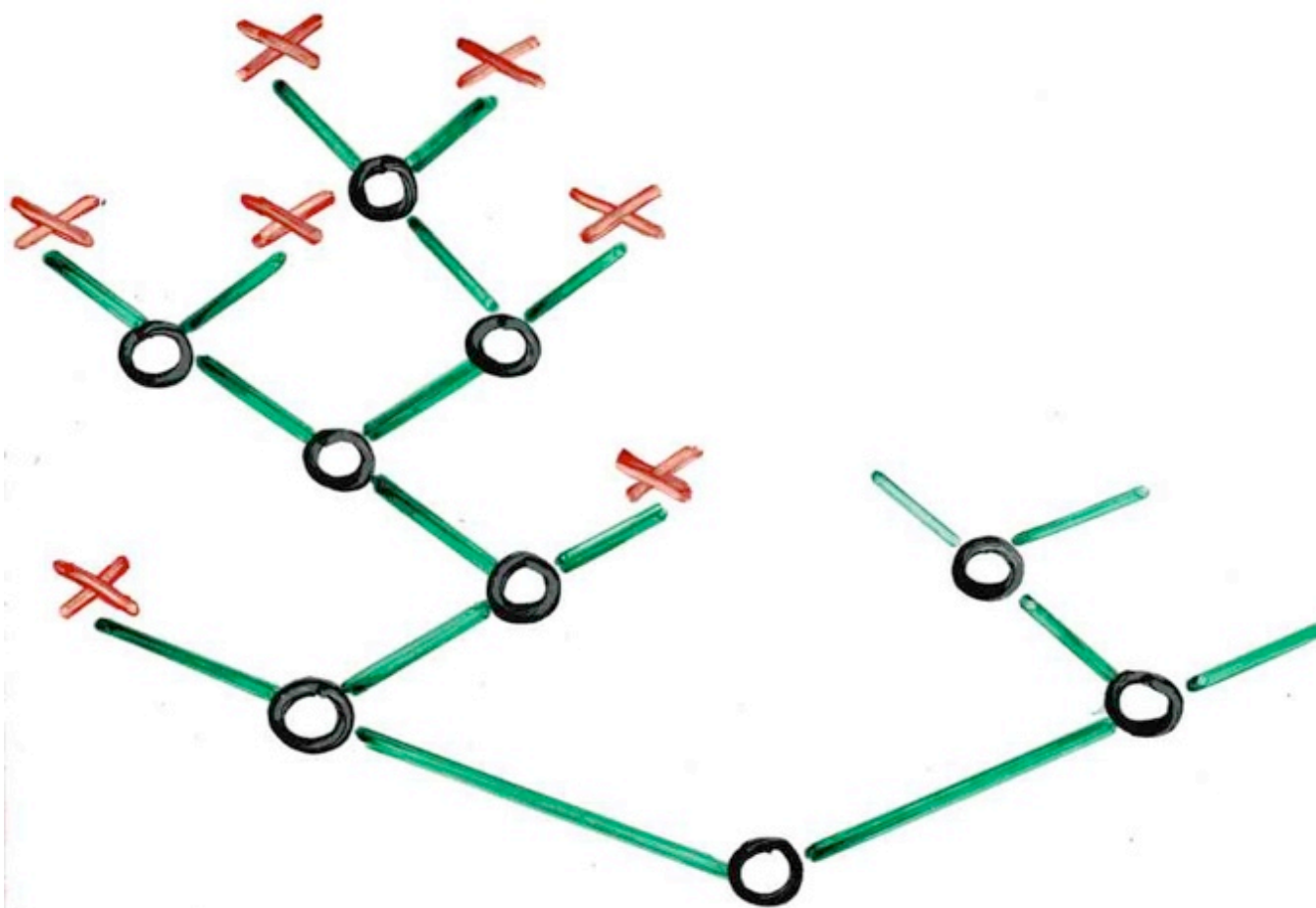


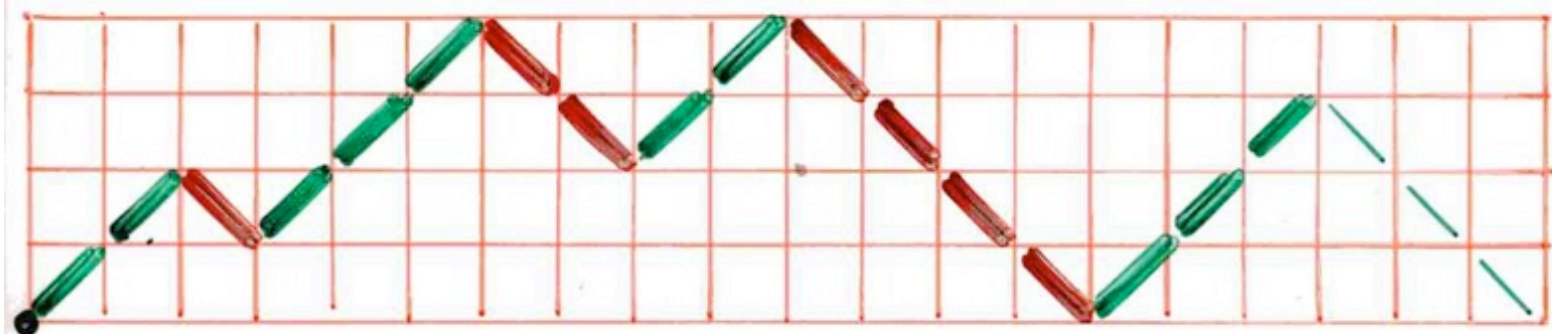
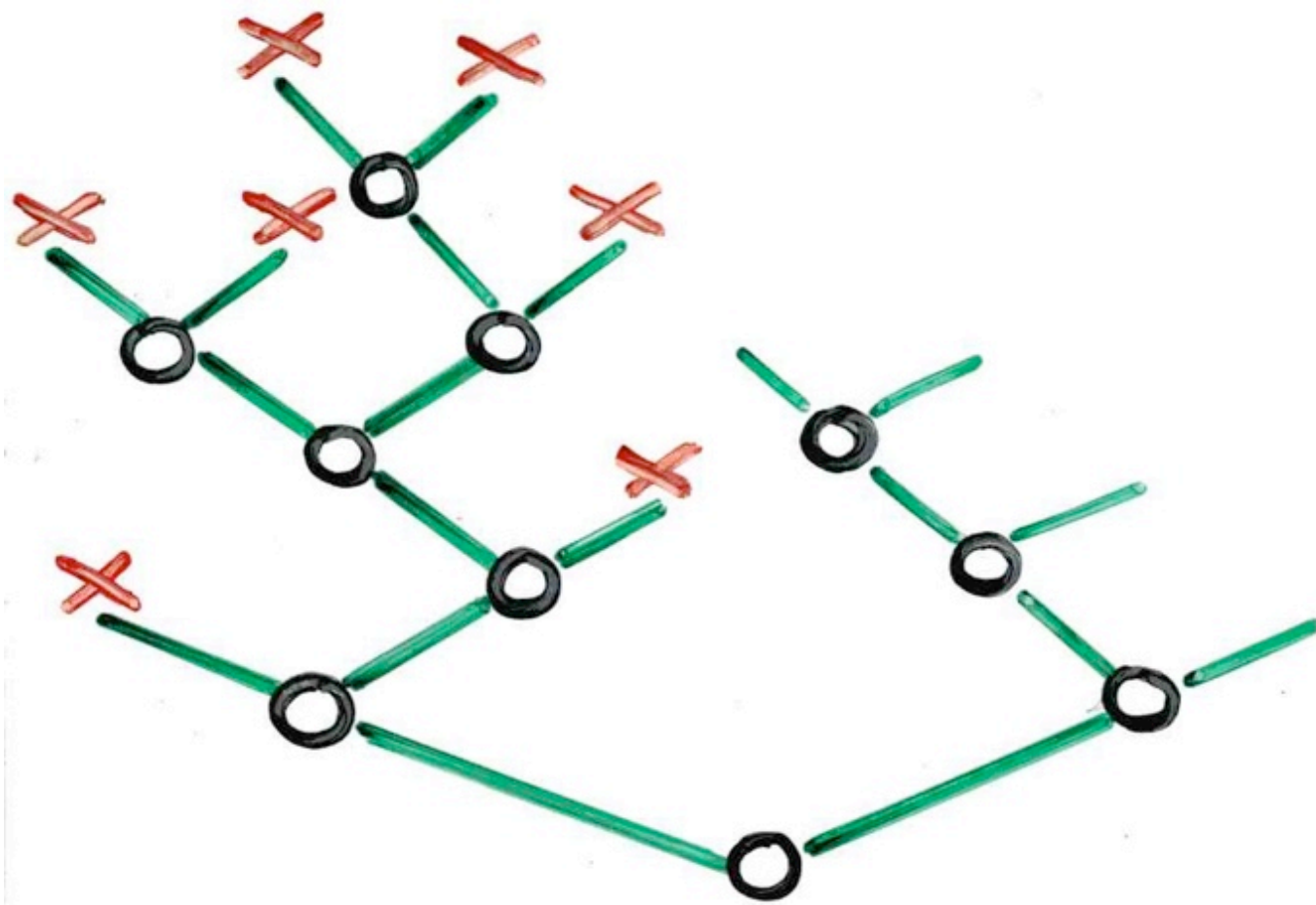


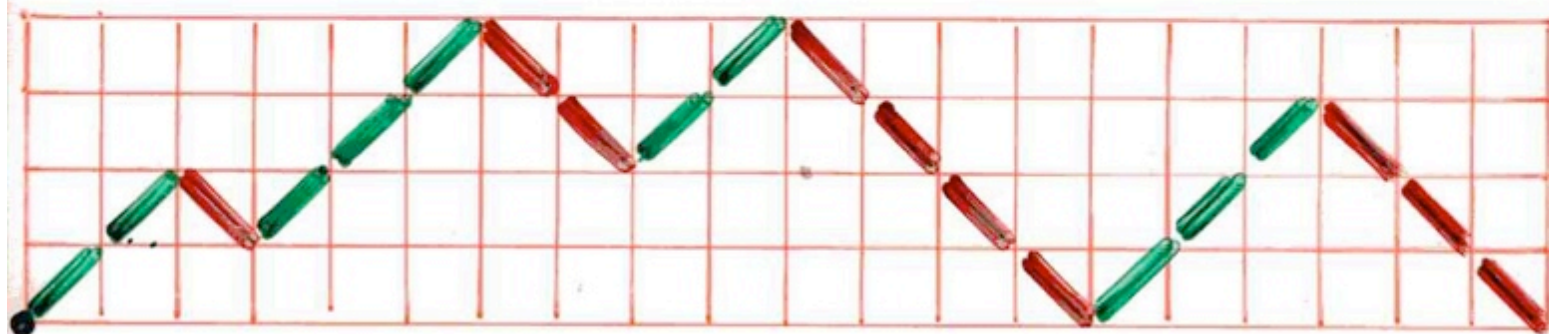
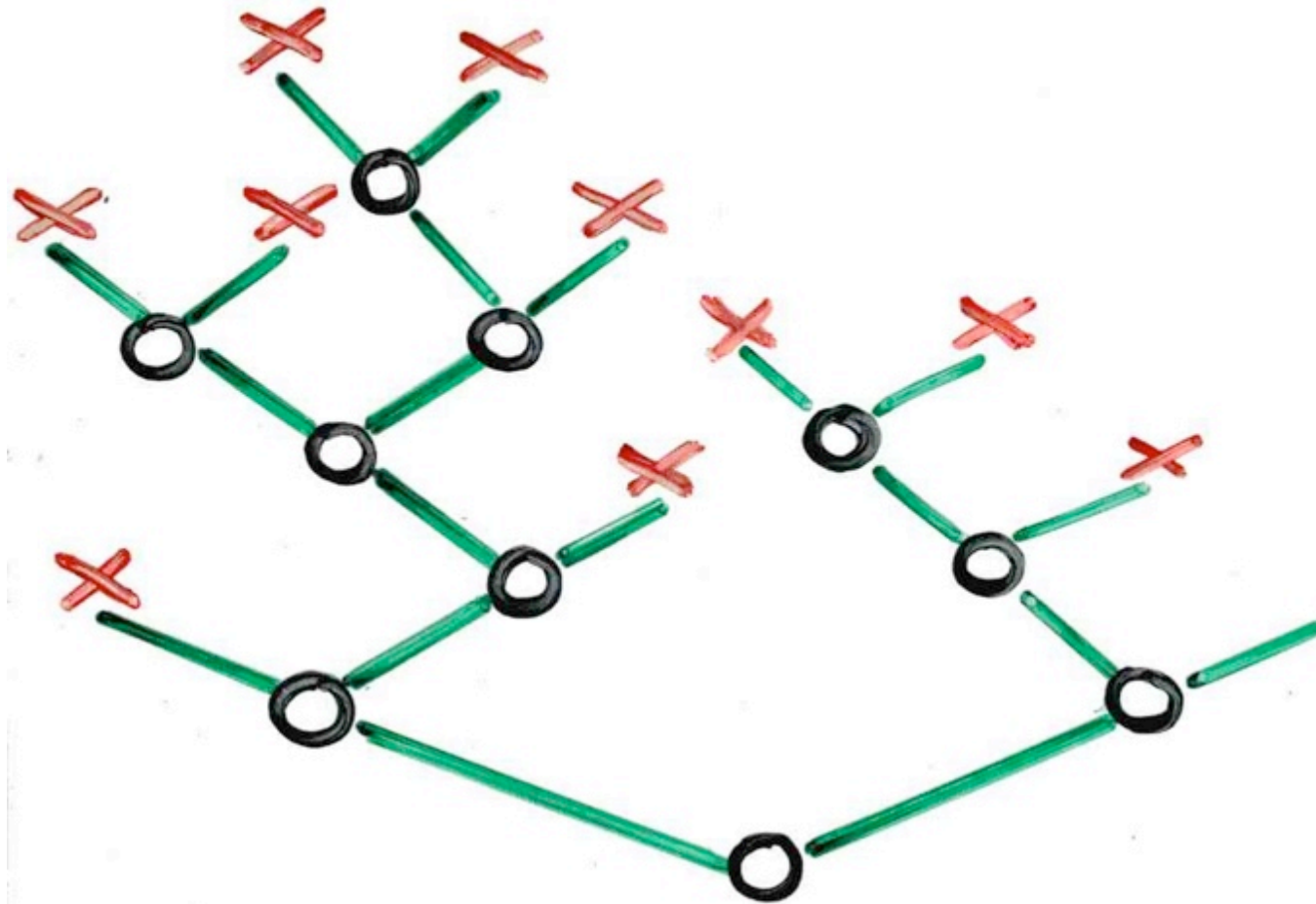


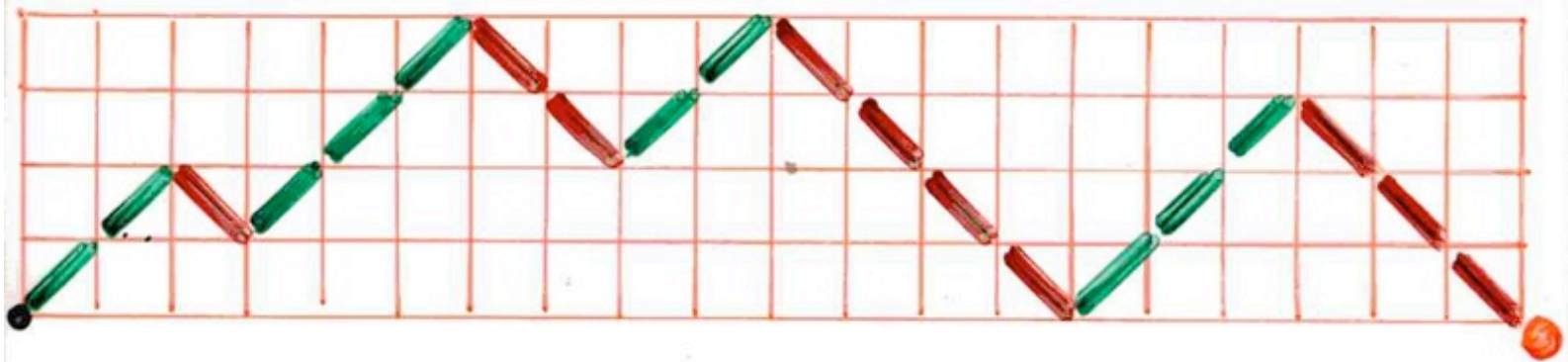
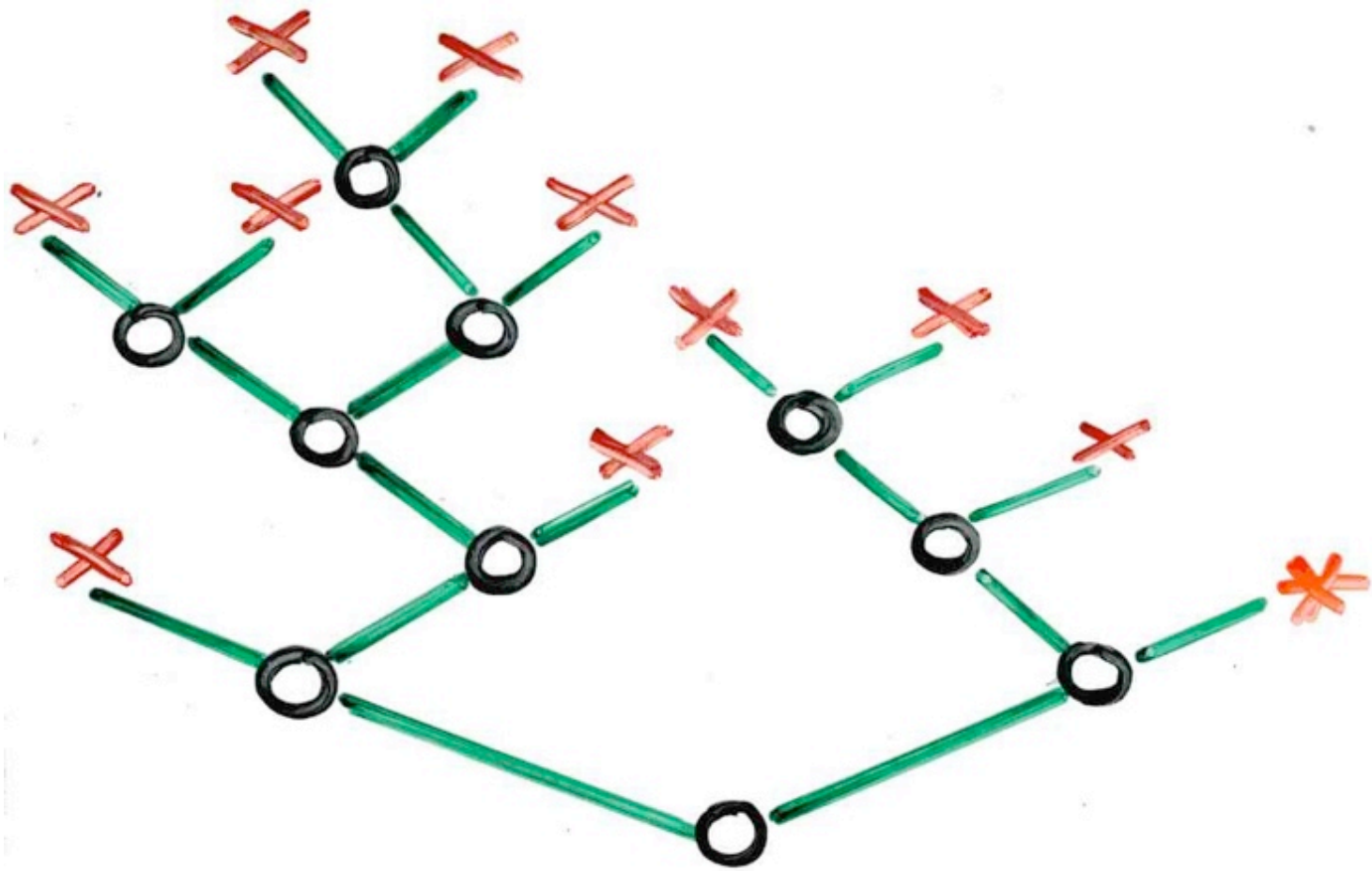


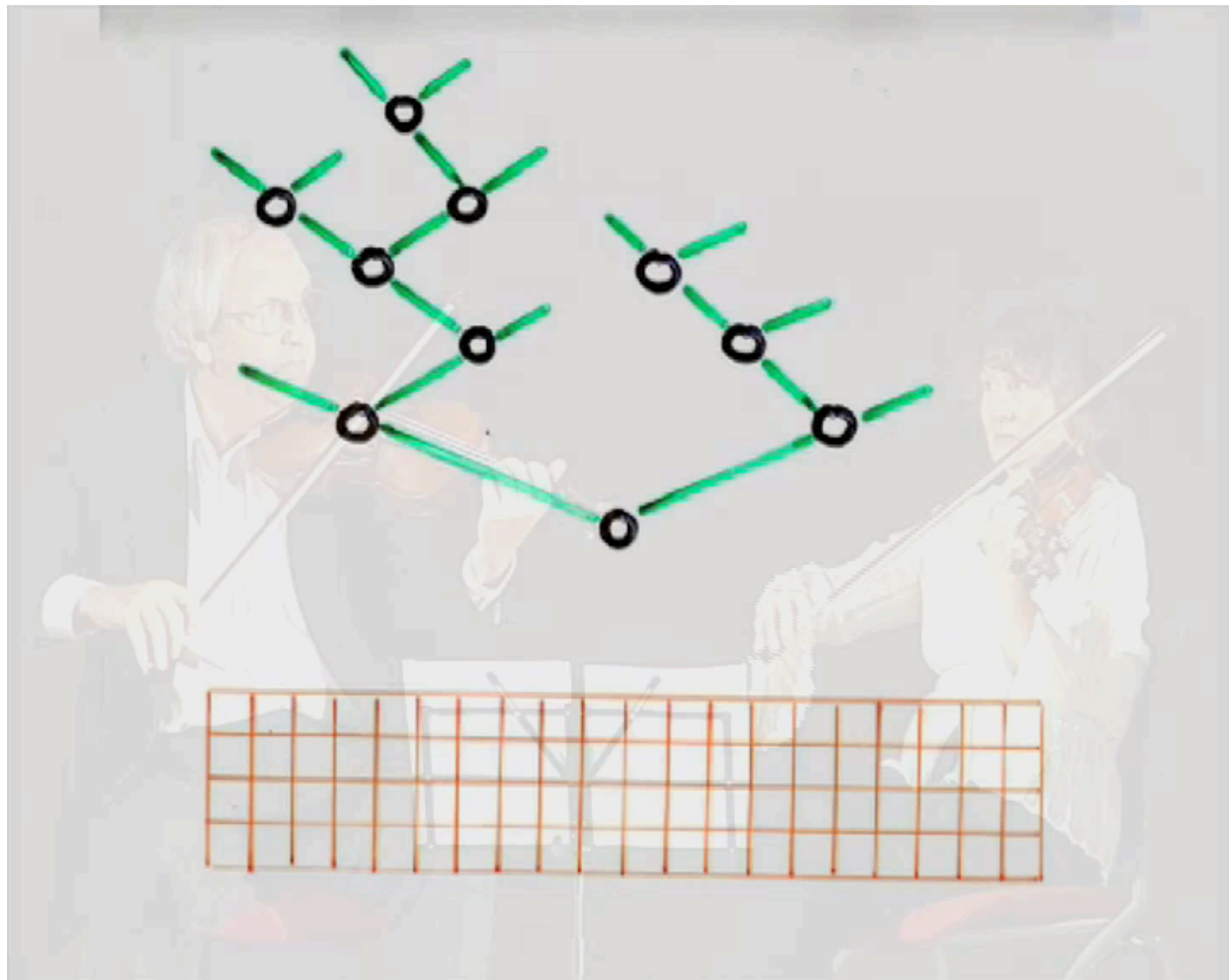








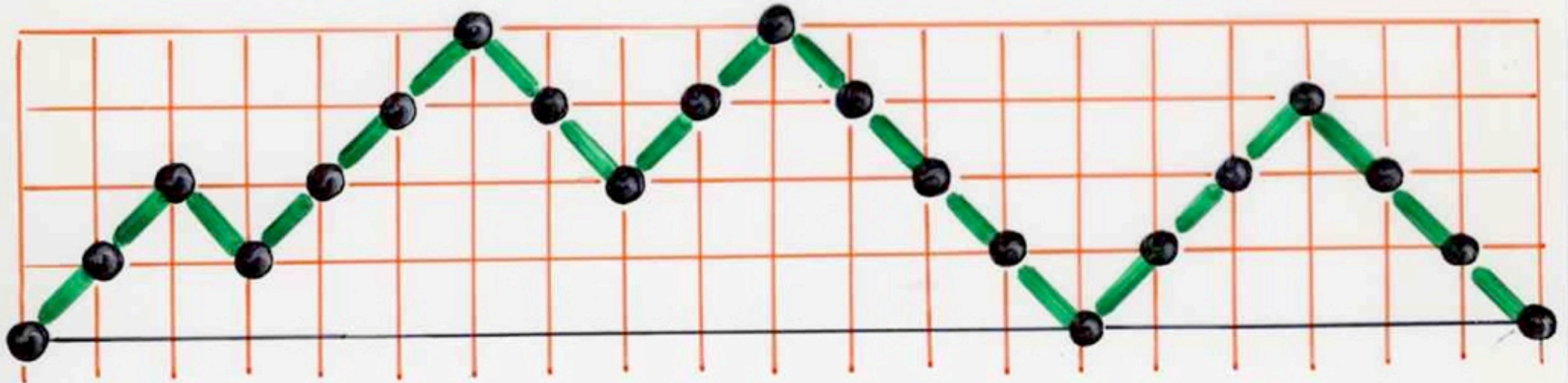




hauteur logarithmique

Dyck path
Height

w
 $h(w)$



Dyck path

Height

w
 $h(w)$

logarithmic height $lh(w)$

$$= \lfloor \log_2(1+h(w)) \rfloor$$

$$lh(w) = k$$

$$\Leftrightarrow 2^k - 1 \leq h(w) < 2^{k+1} - 1$$

(complete)

binary trees

n (internal) vertices

Strahler nb = k

Franson

(1984)

Dyck paths

length $2n$

log. height

$lh(w) = k$

$$S_1 = 1$$

$$S_2 = \frac{t}{1 - 2t}$$

$$S_3 = \frac{t^3}{1 - 6t + 10t^2 - 4t^3}$$

$$S_4 = \frac{t^7}{1 - 14t + 78t^2 - 220t^3 + 330t^4 - 252t^5 + 84t^6 - 8t^7}$$



$$x^3$$



$$-x$$

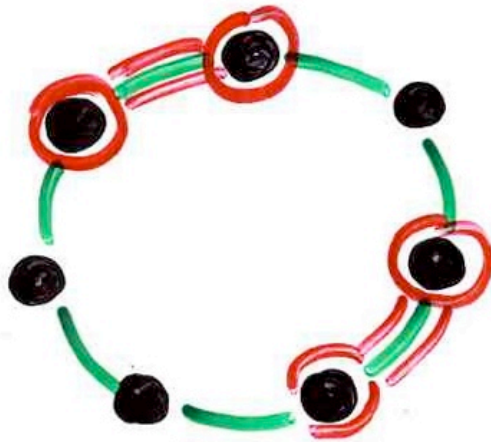


$$-x$$

$$F_3(x) = x^3 - 2x$$

$$\sin(4\theta) = \sin \theta [8 \cos^3 \theta - 4 \cos \theta]$$

$$\cos(n\theta) = T_n(\cos \theta)$$



Nombre de **Strahler** **moyen**
d'un arbre linéaire ayant n sommets

$$A_n =$$

Nombre de **Strahler** **moyen**
d'un arbre binaire ayant n sommets

$$A_n = \log_4 n +$$

Nombre de **Strahler** **moyen**
d'un arbre binaire ayant n sommets

$$A_n = \log_4 n + \underbrace{\mathcal{P}}_{\text{périodique}}(\log_4 n) + \dots$$

retour aux
«matrices de ramification»

ou

«matrices de formes»

Génération de l'arbre combinatoire

choix de la matrice de ramification

Génération de l'arbre géométrique

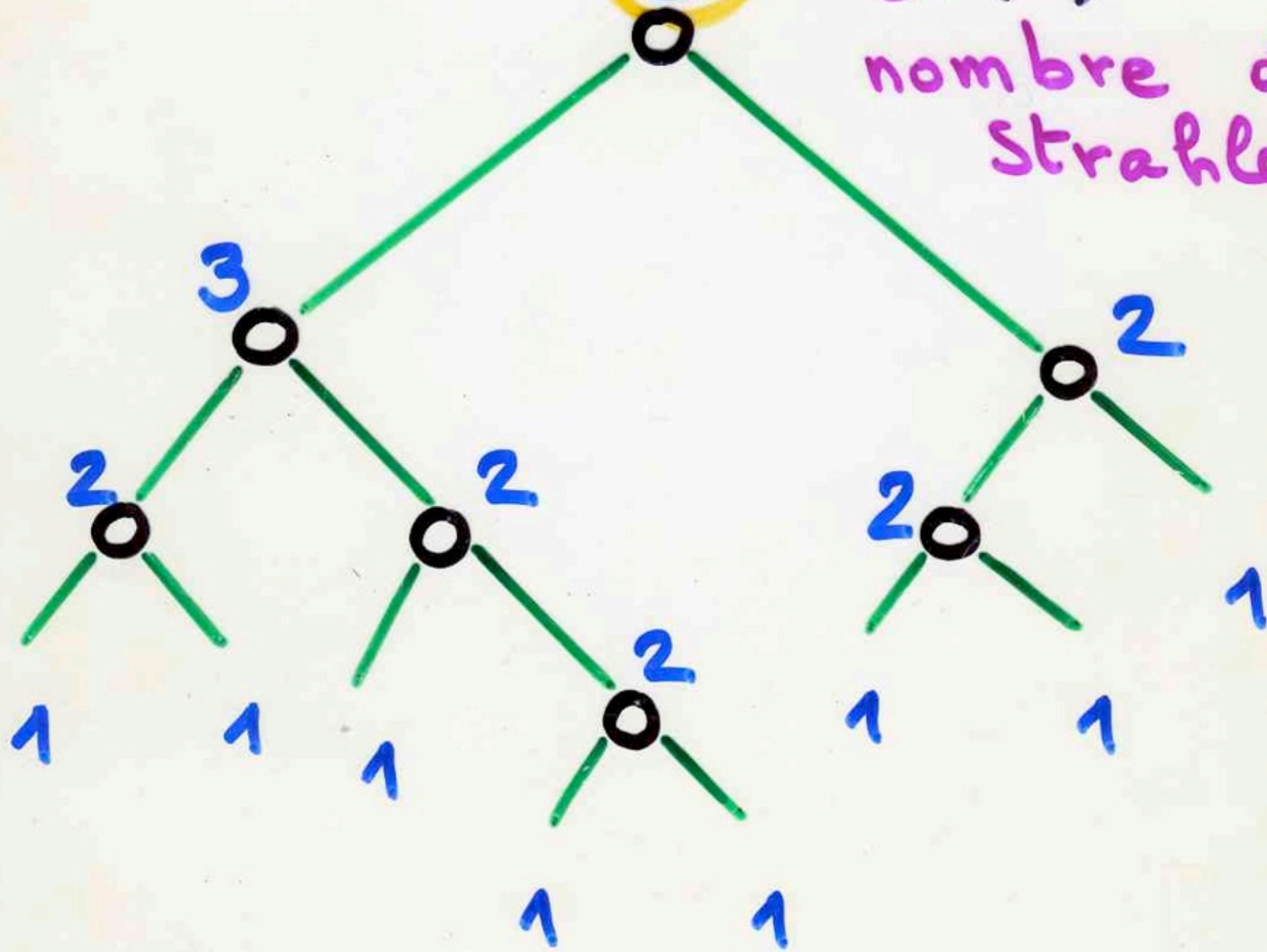
- largeur $w(k)$
 - longueur $L(k)$
- } fonction de l'ordre
- angle de déviation
 - angle de branchement
- } fonction du biordre

dessin en 2D

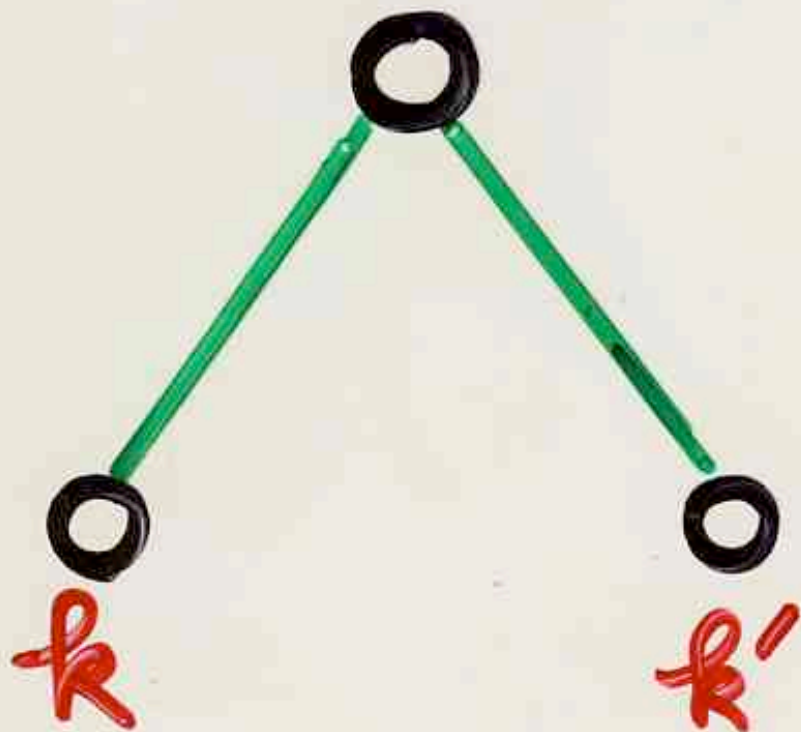


$= St(B)$

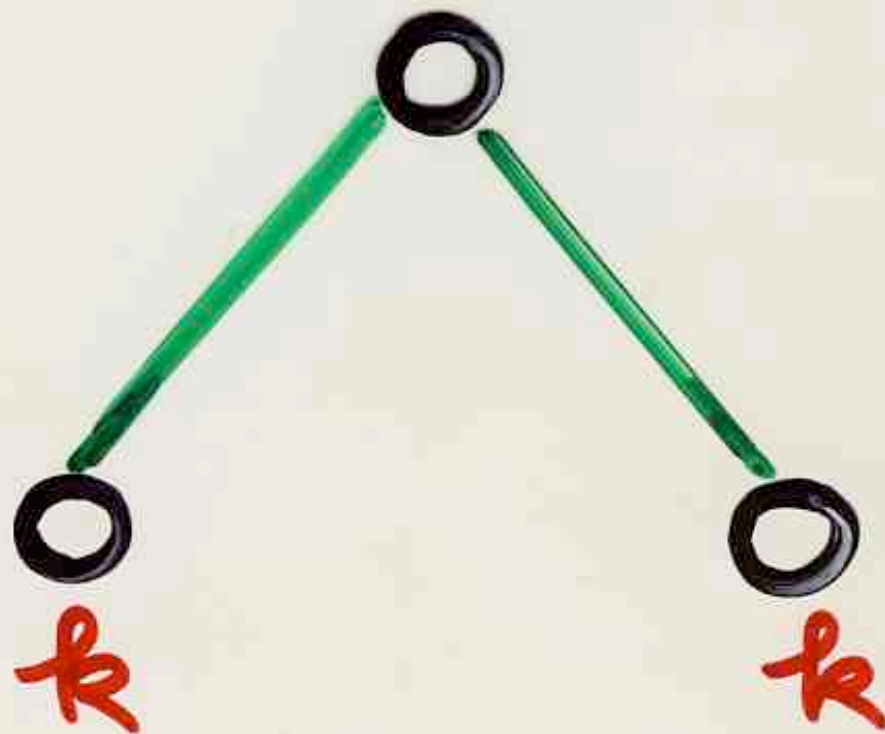
nombre de
Strahler



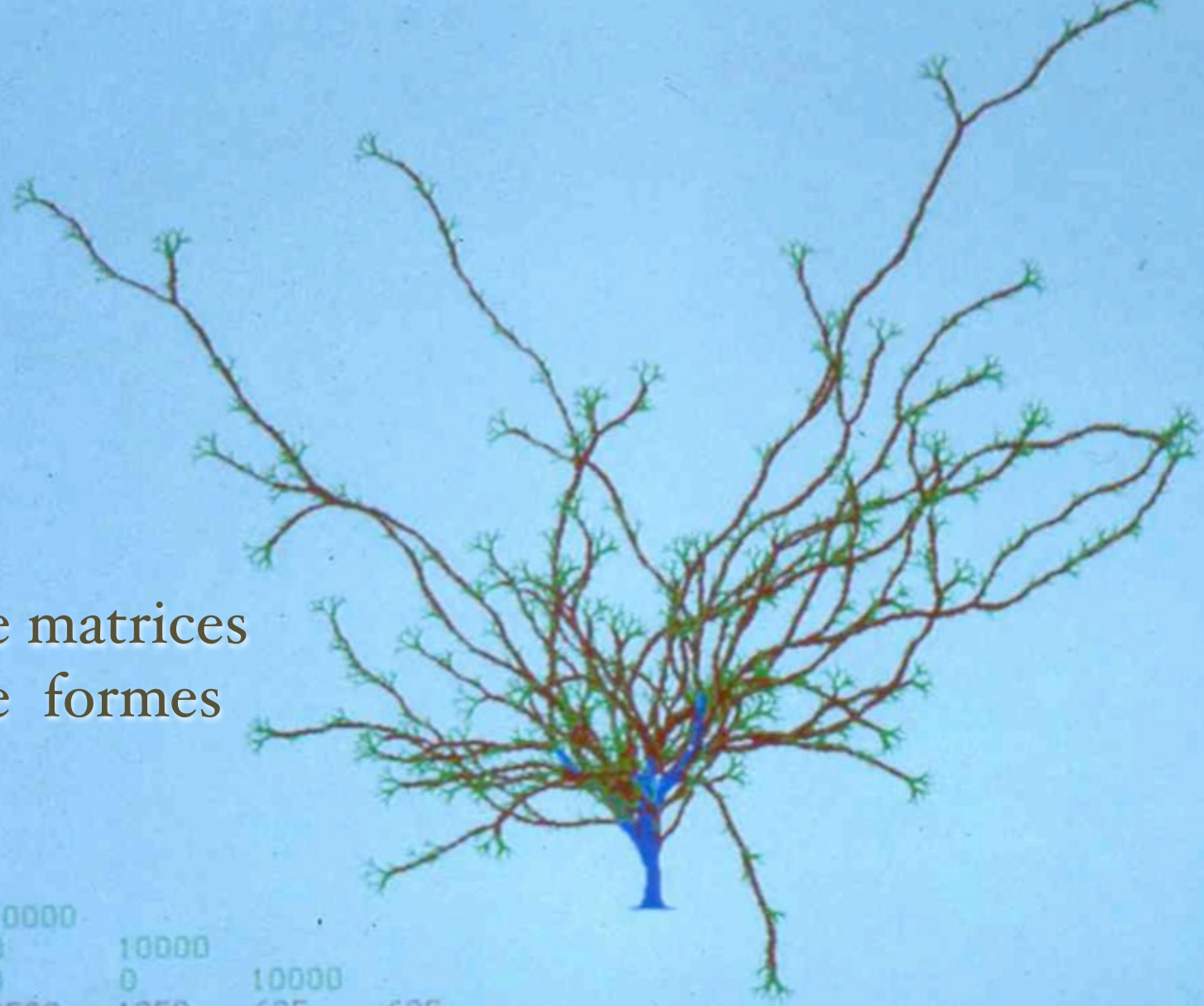
$\max(k, k')$



$k+1$



mélange de matrices
mélange de formes



2 : 0	10000										
3 : 0	0	10000									
4 : 0	0	0	10000								
5 : 5000	2500	1250	625	625							
6 : 5000	2500	1250	625	313	312						
7 : 125	250	500	1000	2000	3000	3125					
8 : 63	125	250	500	1000	2000	3000	3062				
9 : 31	63	125	250	500	1000	2000	3000	3031			
10 : 15	31	63	125	250	500	1000	2000	3000	3016		
11 : 7	15	31	63	250	125	500	1000	2000	3000	3009	

● Classification of Galactograms with ramification matrices

P. Bakic, M. Albert, A. Maidment
(2003)

Digital mammography

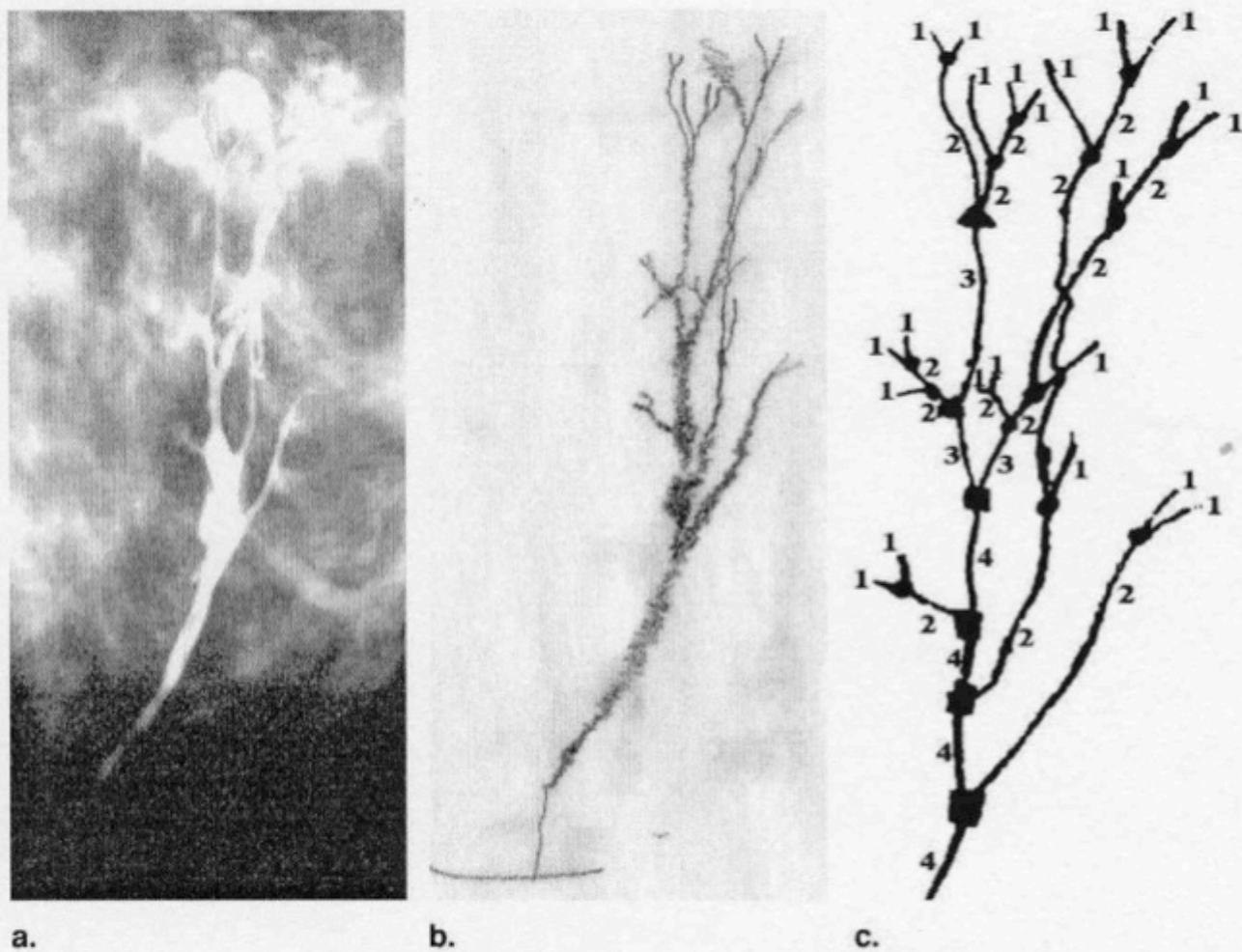


Figure 1. Segmentation of a ductal tree, showing (a) part of a galactogram with a contrast-enhanced ductal network, (b) the manually traced network of larger ducts from the contrast-enhanced portion of the galactogram, (c) numeric labeling of branches in the ductal network, and (d) the R matrix computed from the branching pattern. The dots, triangles, and squares denote branching points of different levels of the tree.

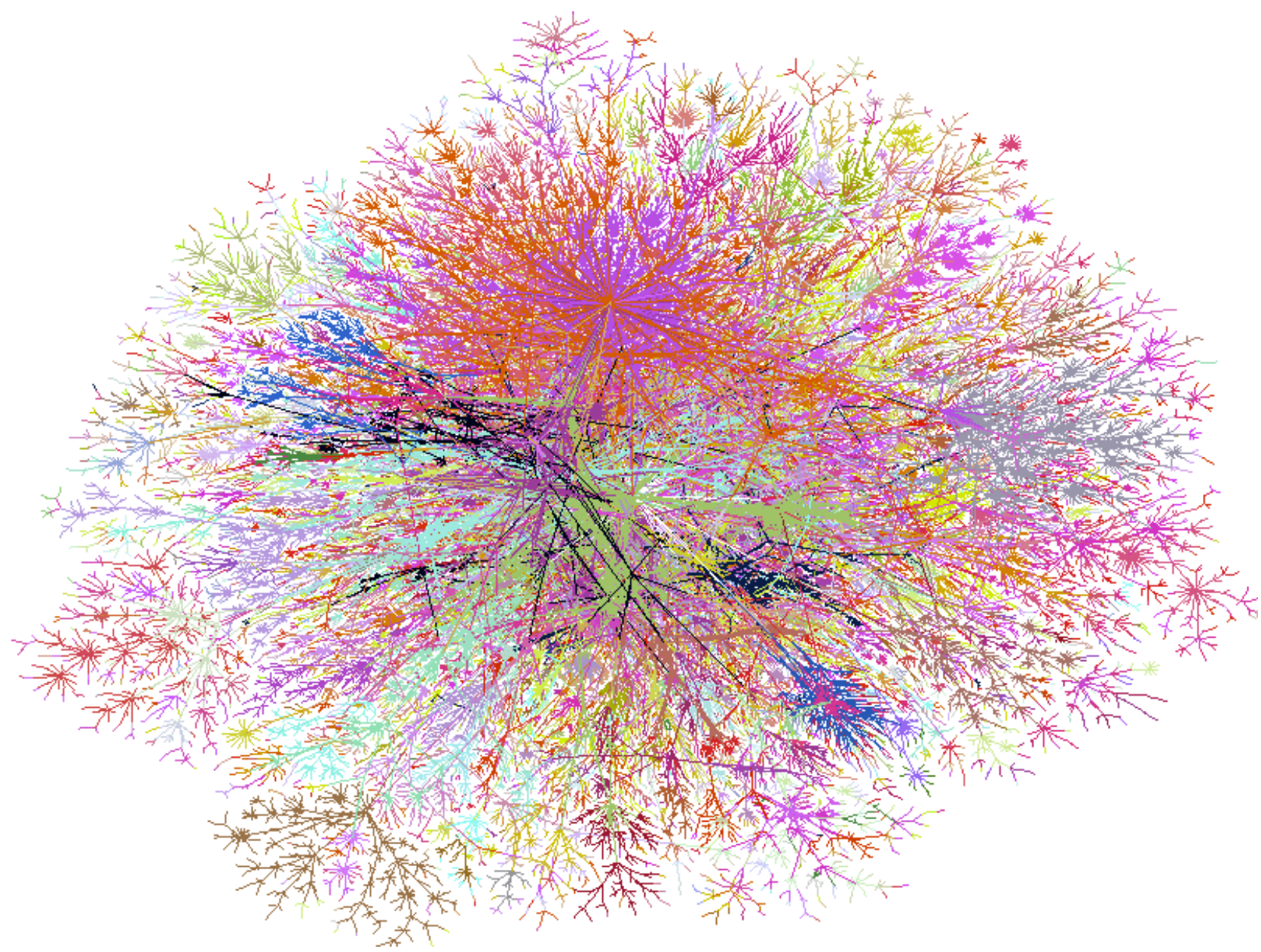
$$R = \begin{bmatrix} r_{2,1} & r_{2,2} & . & . \\ r_{3,1} & r_{3,2} & r_{3,3} & . \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{bmatrix} = \begin{bmatrix} 0.43 & 0.57 & . & . \\ 0 & 0.33 & 0.67 & . \\ 0 & 0.75 & 0 & 0.25 \end{bmatrix}$$

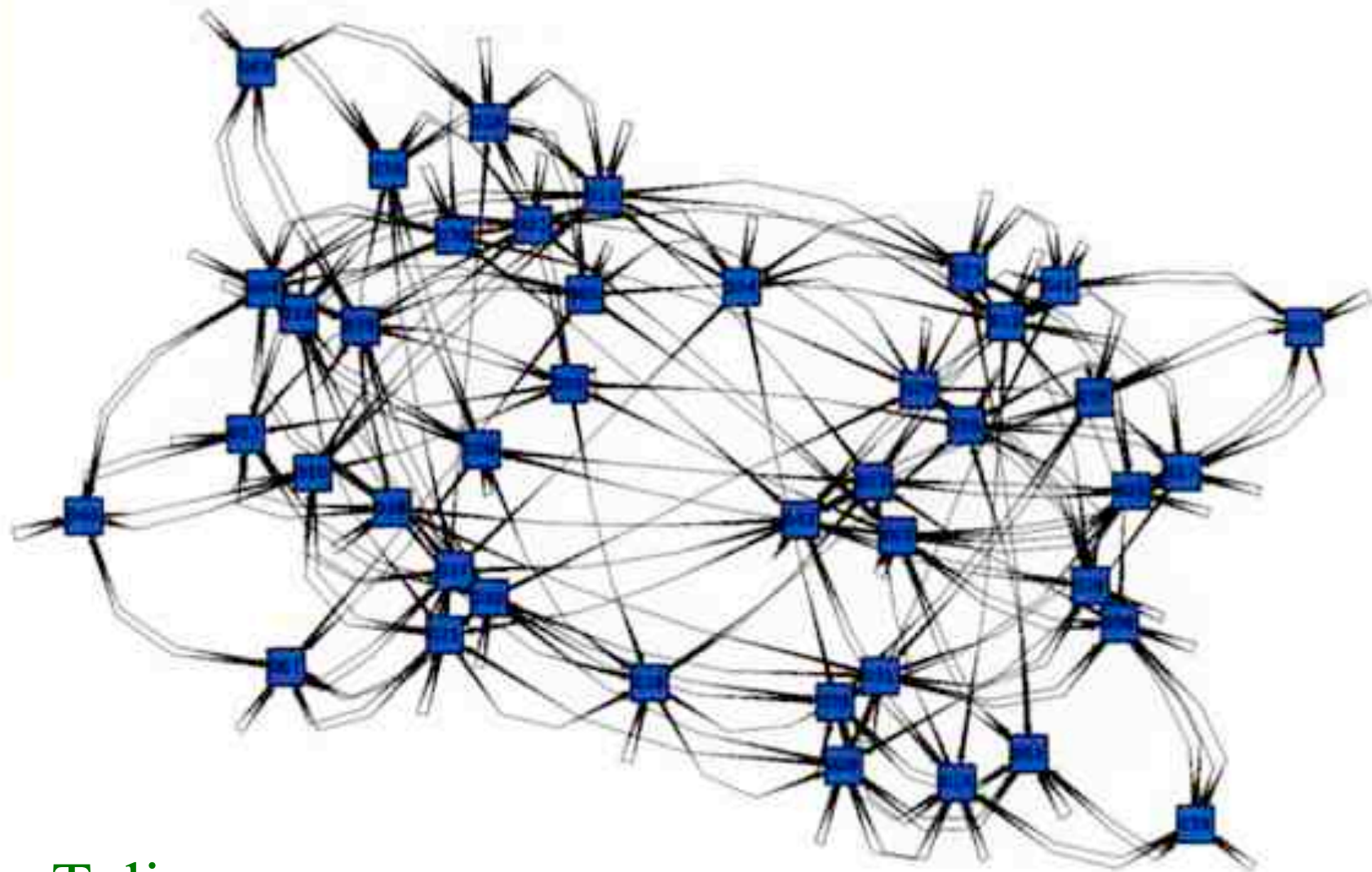
d.

- Visualisation de l'information
trois grands graphes

D. Auber, M. Delest
Y. Chénicota, G. Mélançon, J.M. Fedou

analyse de Horton -
Strahler

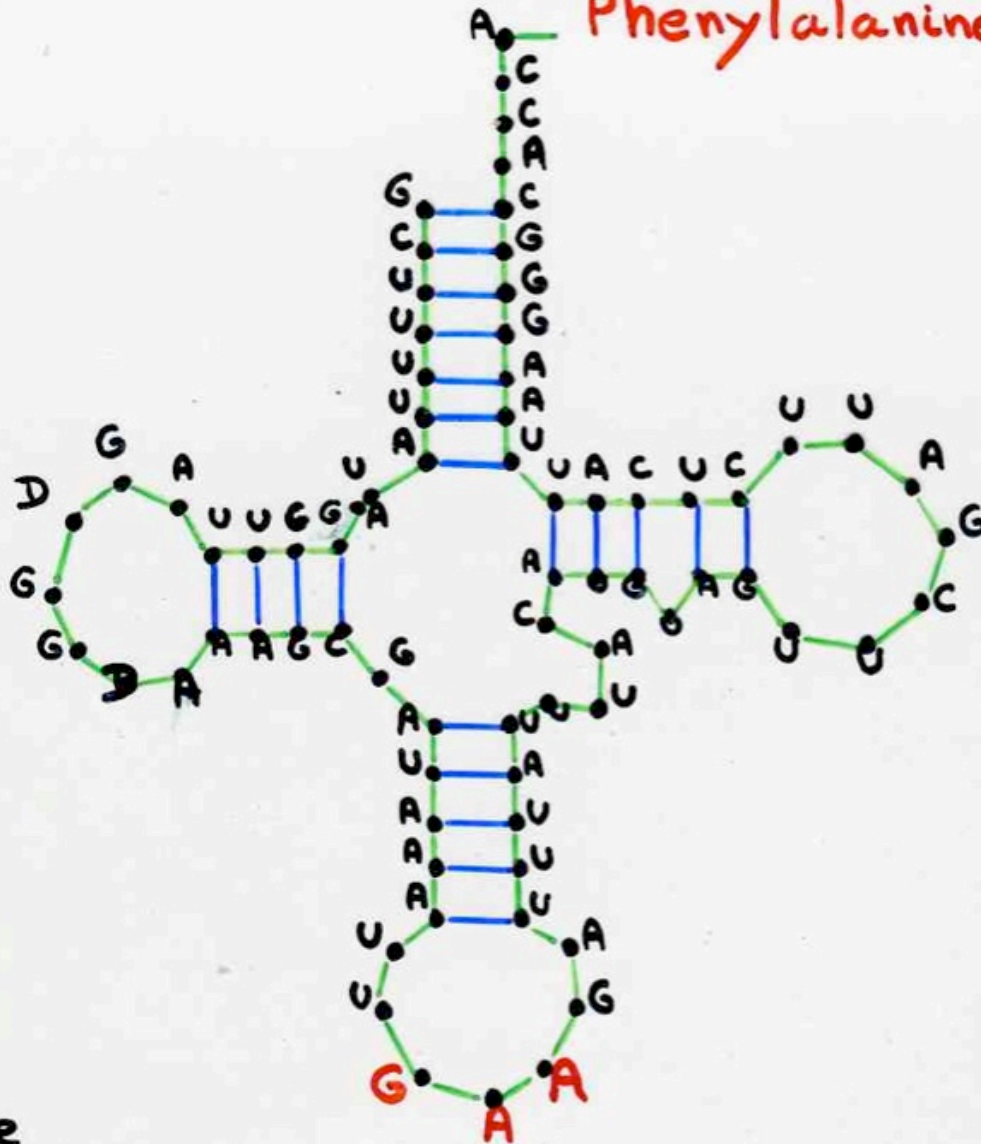




Tulip
D. Auber

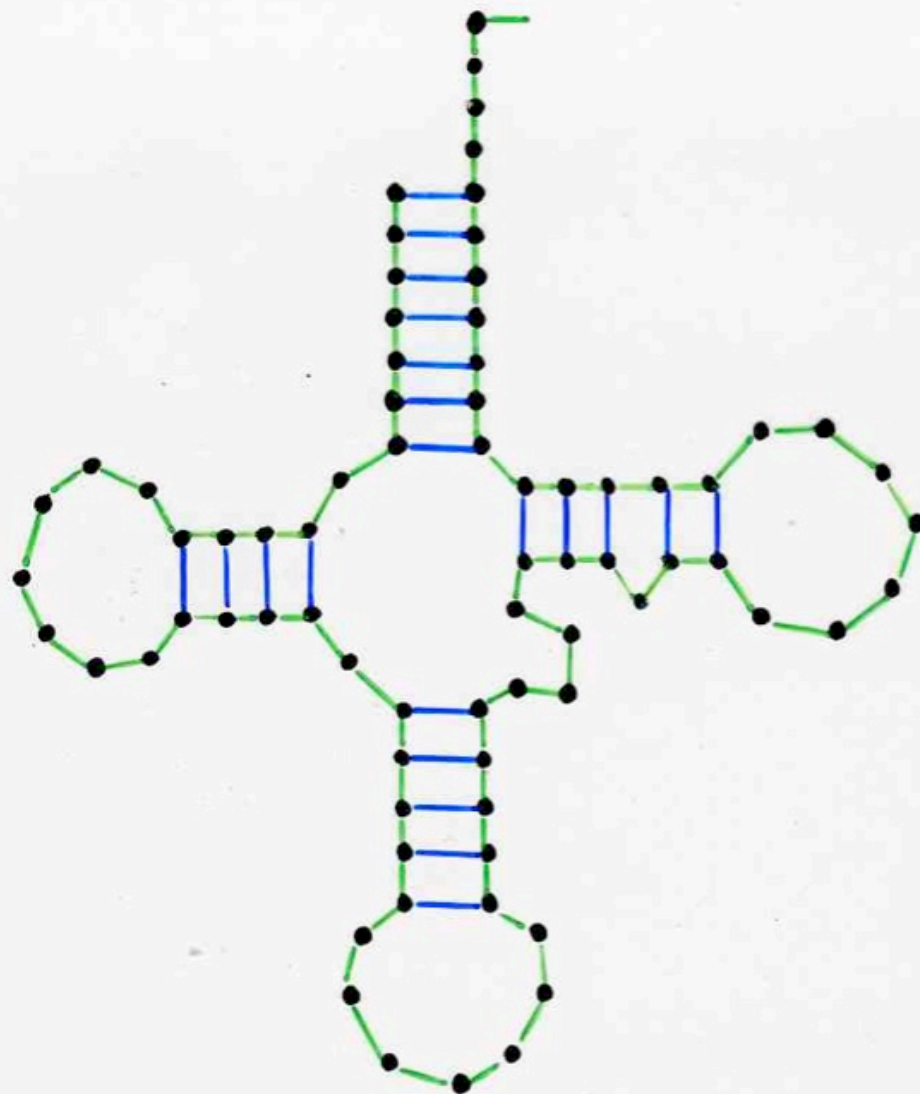
des arbres dans les molécules ...

Phenylalanine

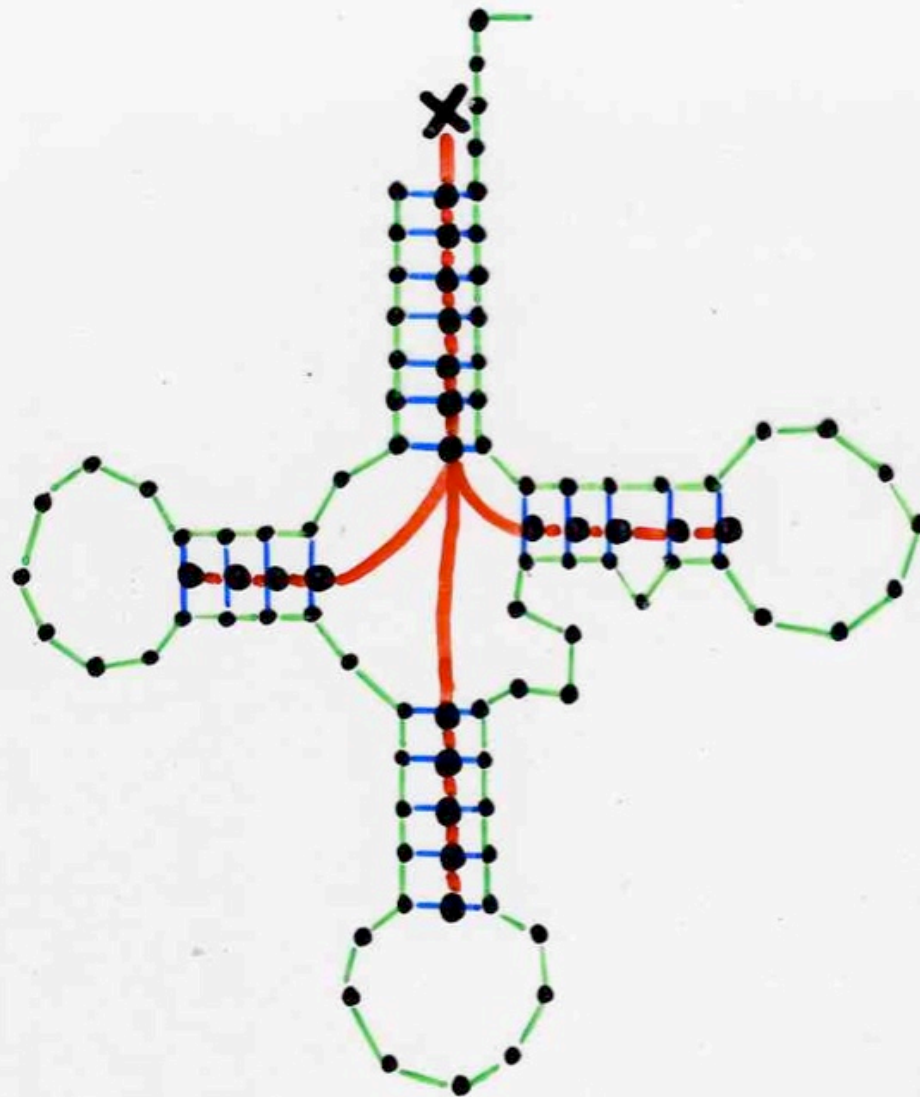


A denine
U racyle
G uanine
C ytosine

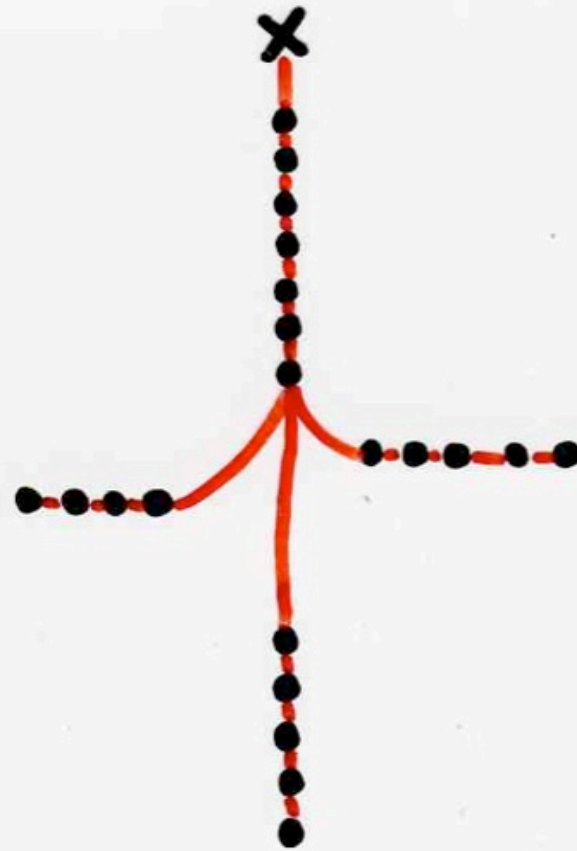
tARN^{Phe}



tARN^{Phe}



tARN^{Phe}



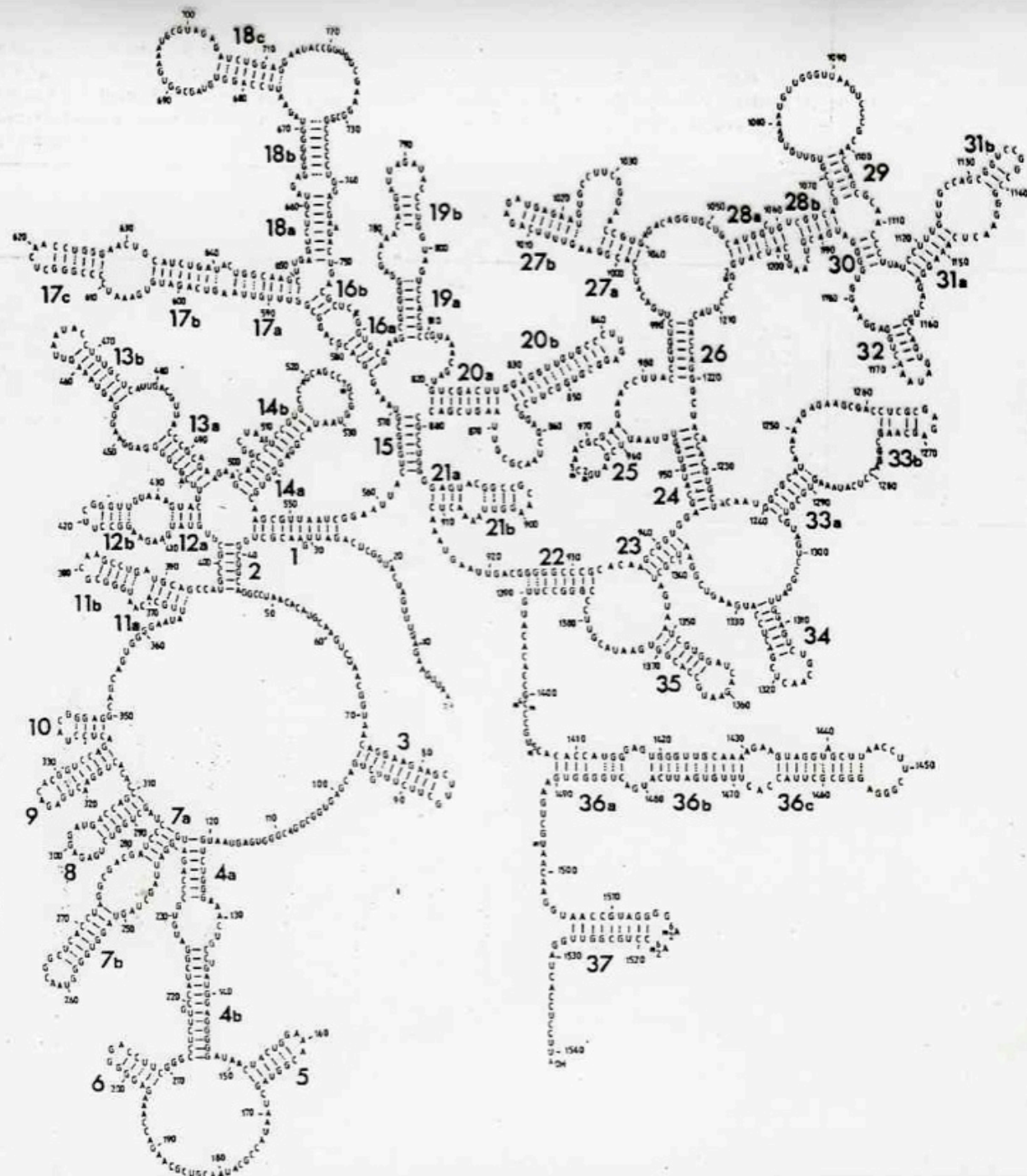


Fig. 1. Secondary structure model of the 16S RNA from *E. coli*. This model has been fully described elsewhere [18]. The various secondary structure motifs are numbered for reference. Base-pairings 2 and 23 are included in this up-dated scheme and slight modifications have been introduced into helices 18b and 33b

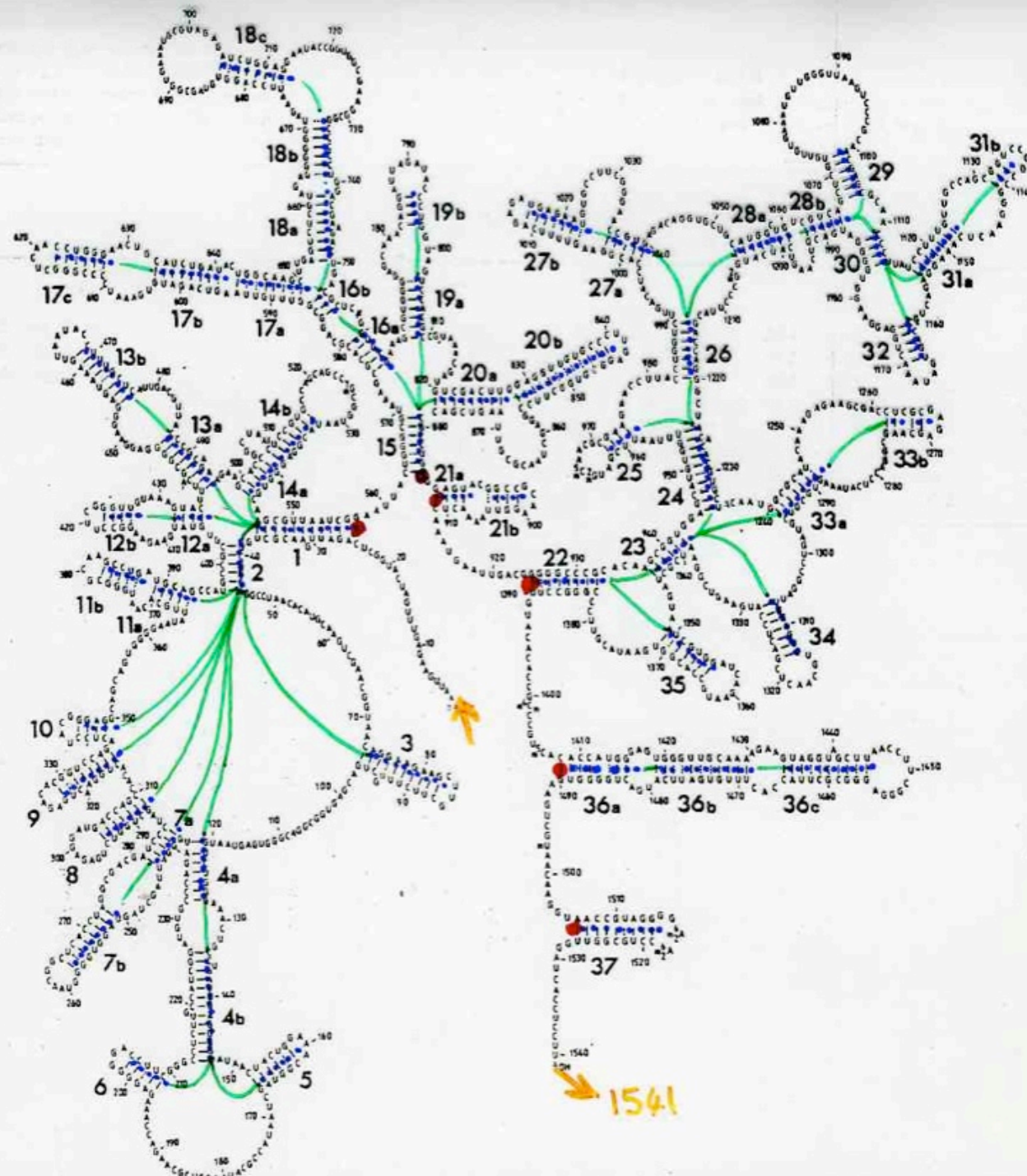
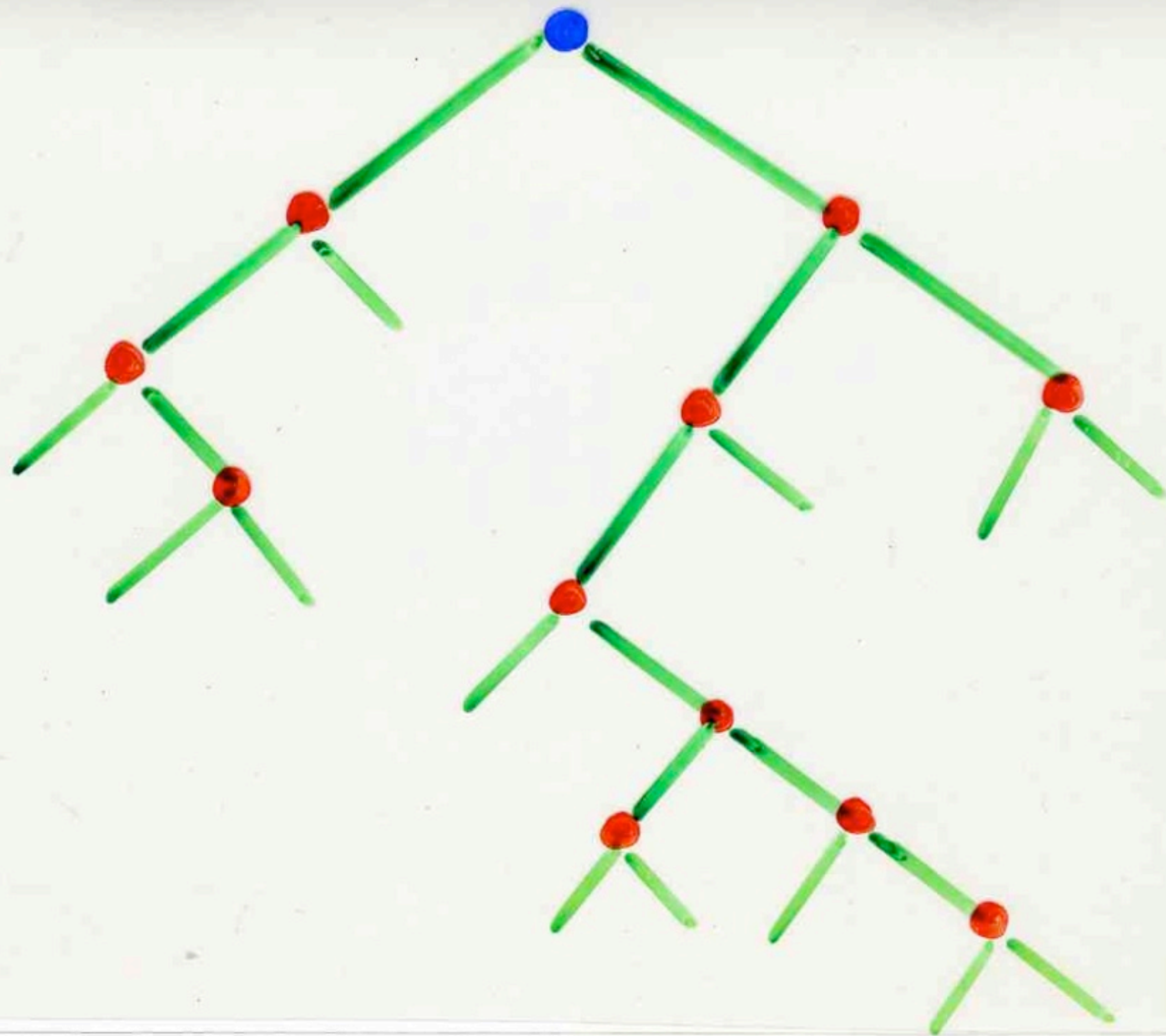
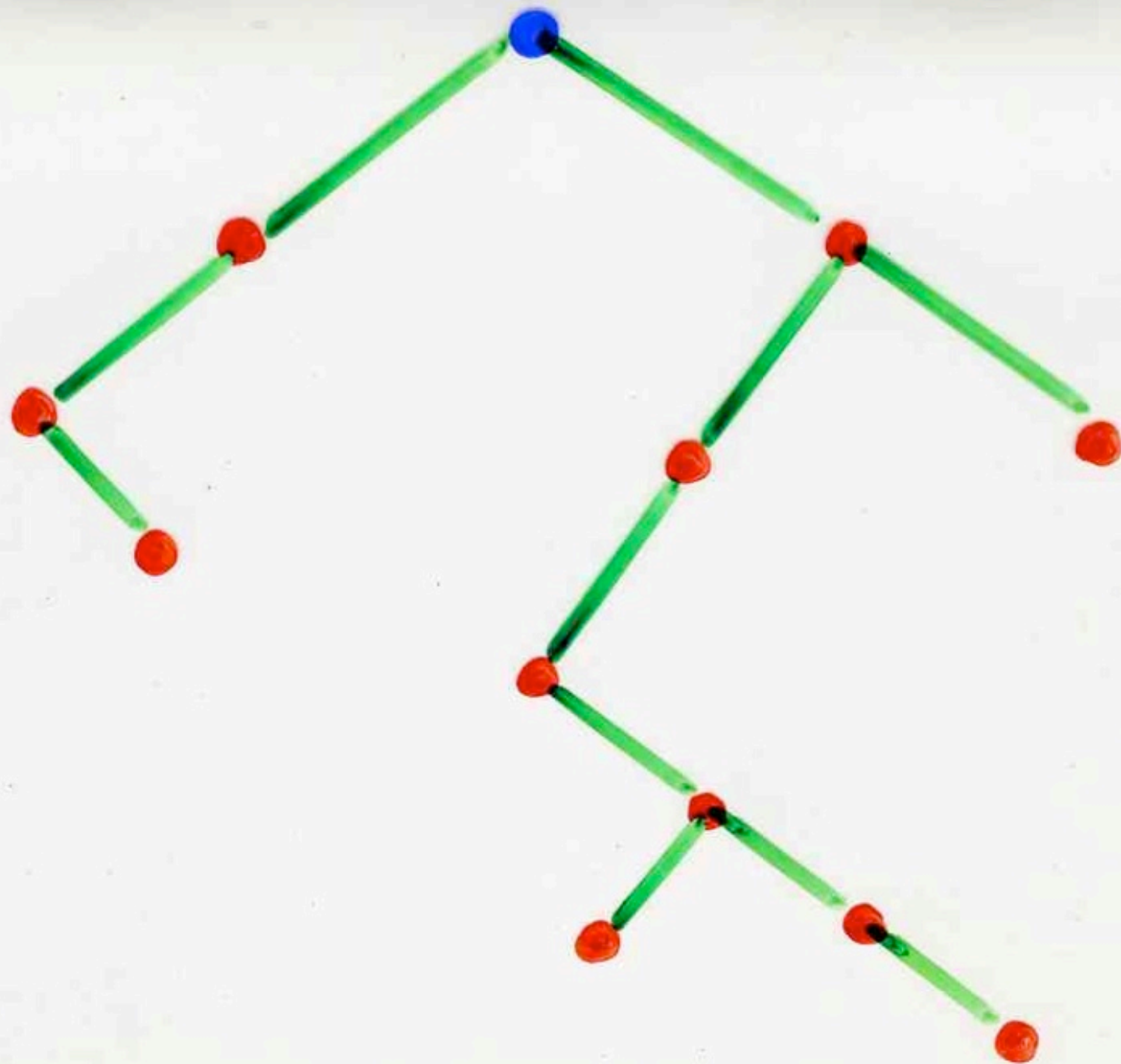
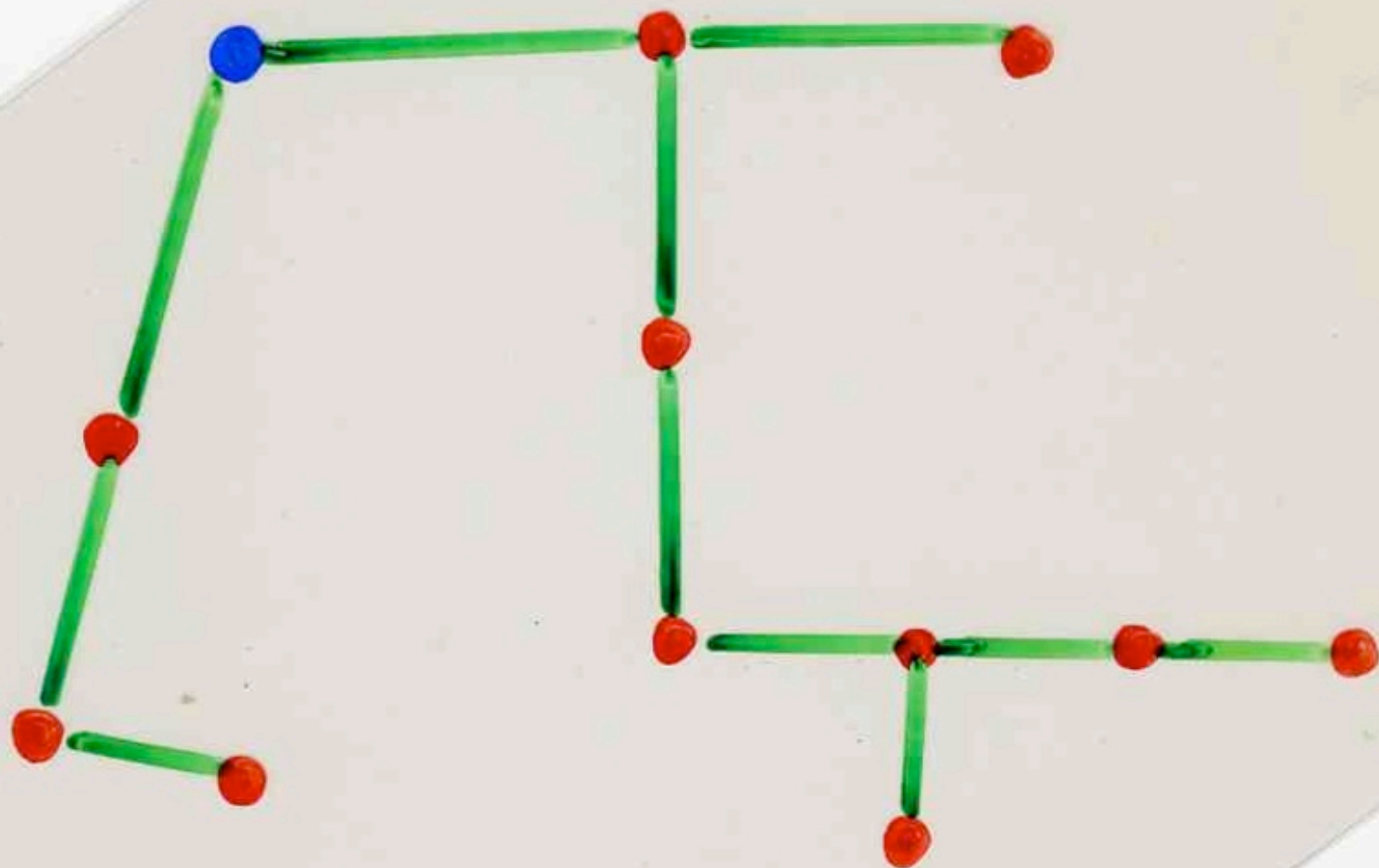
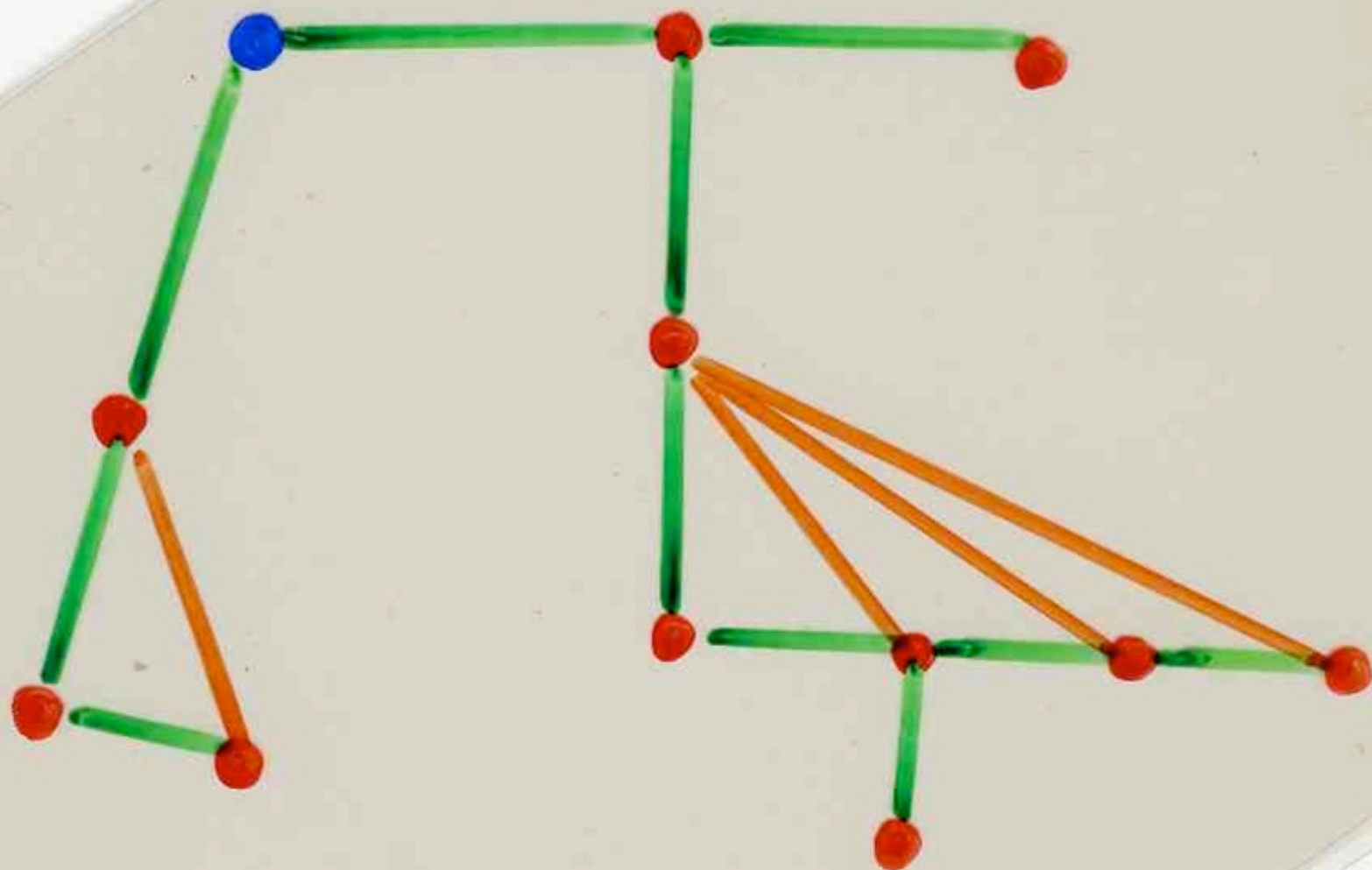


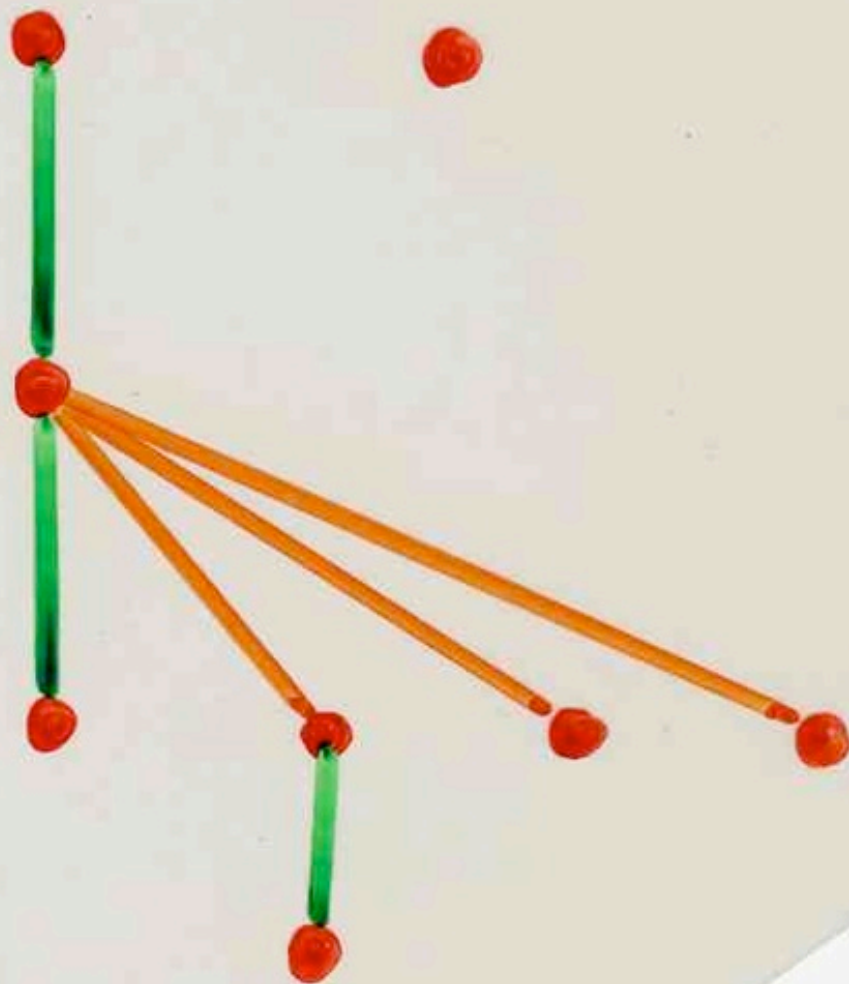
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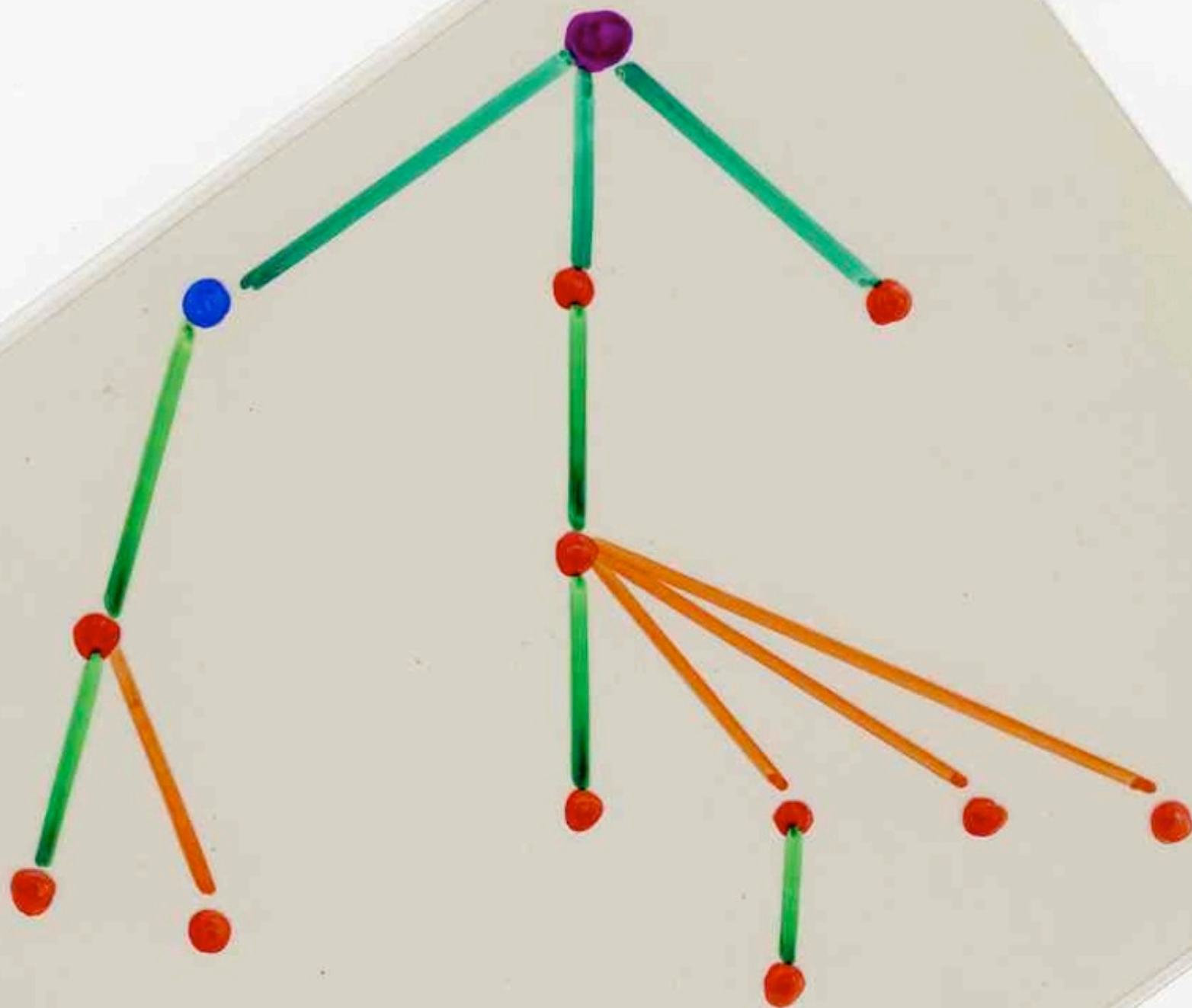






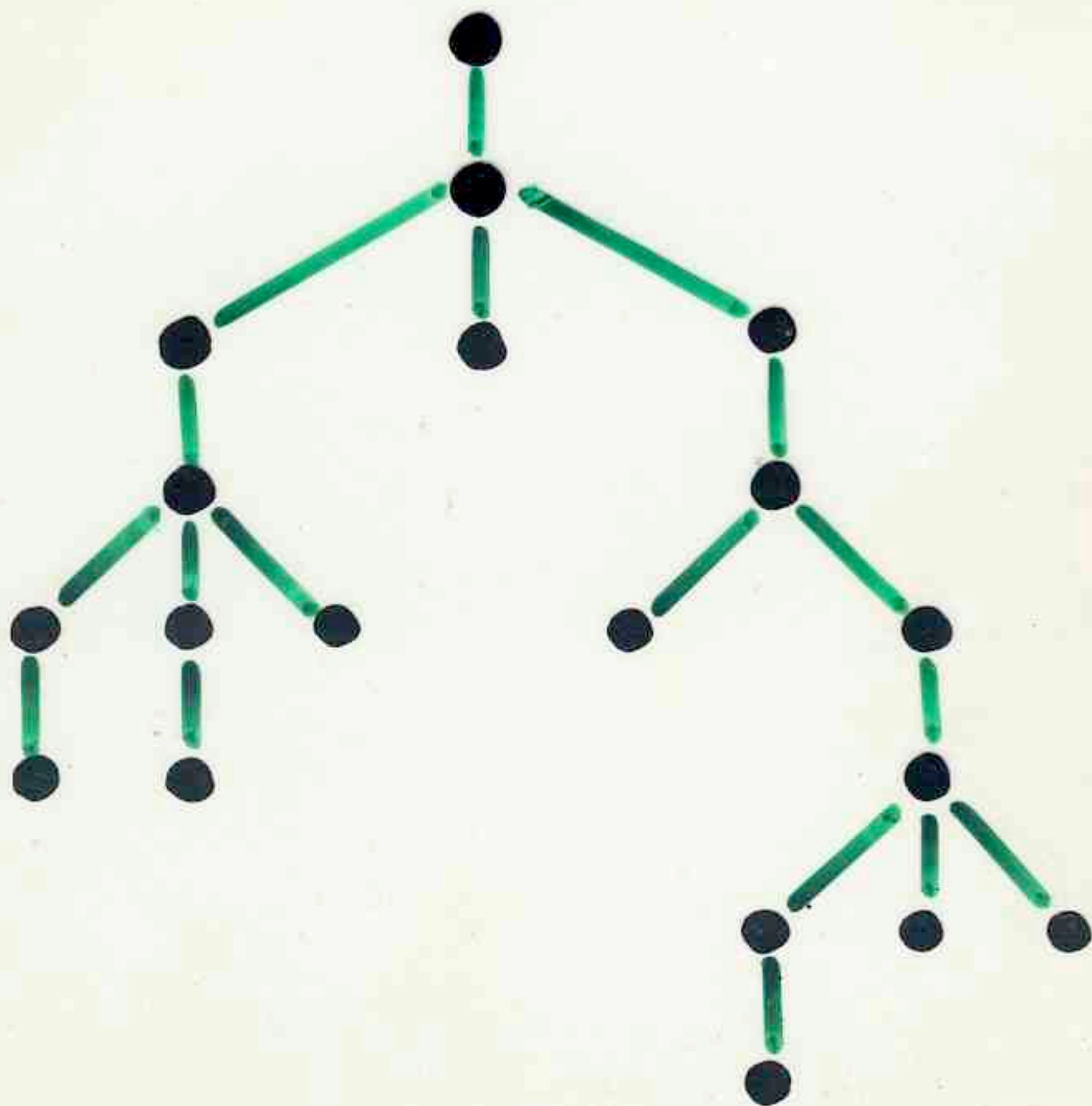


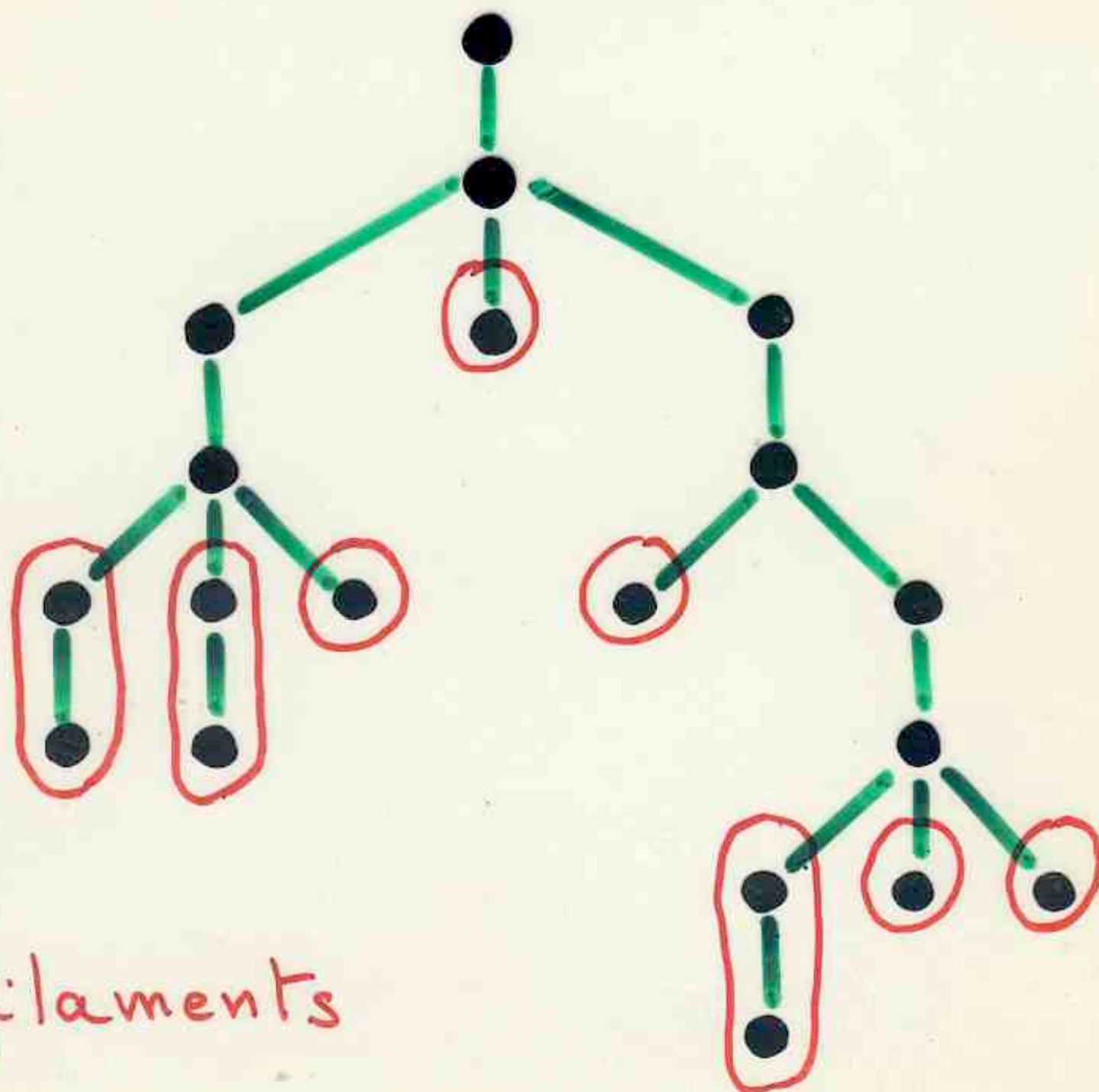




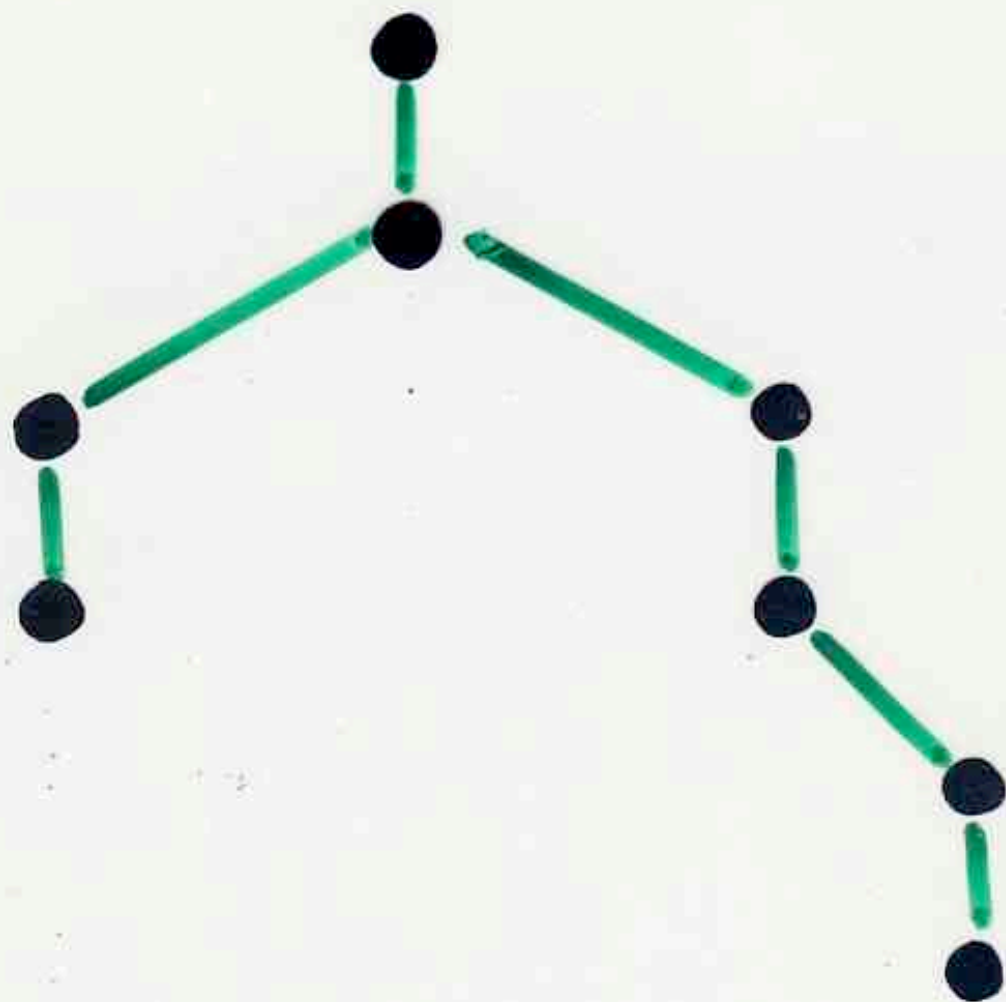
M. Waterman

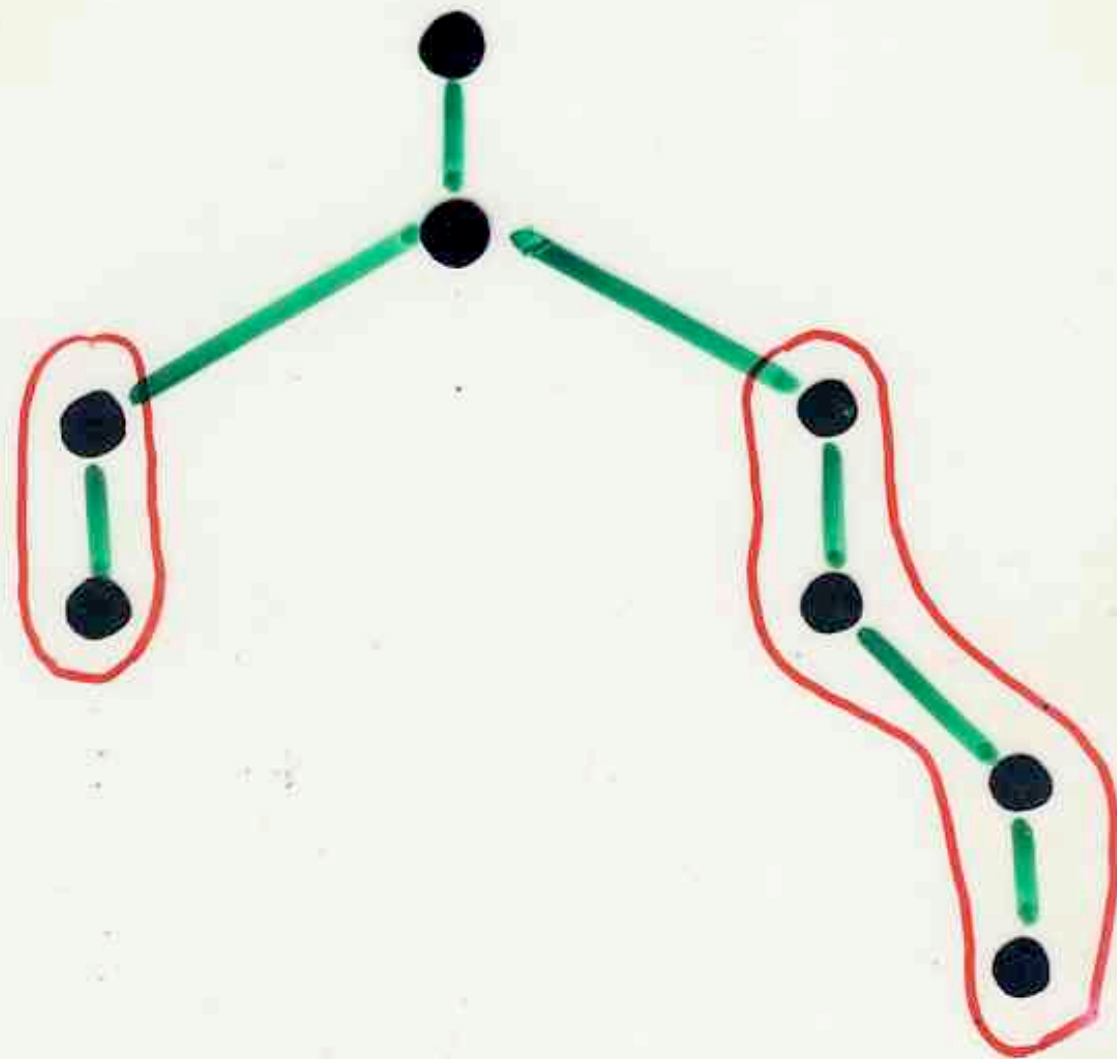
- ordre d'une structure secondaire ARN
- ordre de la forêt d'arbres sous-jacente
- ordre d'un arbre (planaire)





filaments











$F_{n,k}$ = nombre de
forêts d'arbres
ayant n sommets
et d'ordre k

$F_{n,k}$ = nombre de
forêts d'arbres
ayant n sommets
et d'ordre k

= $S_{n,k}$

Vauchassade de Chaumont
X. V. (1985) (2001)

D. Zeilberger (1985)

Logarithmic
Height

$$\lfloor \log_2(H) \rfloor$$

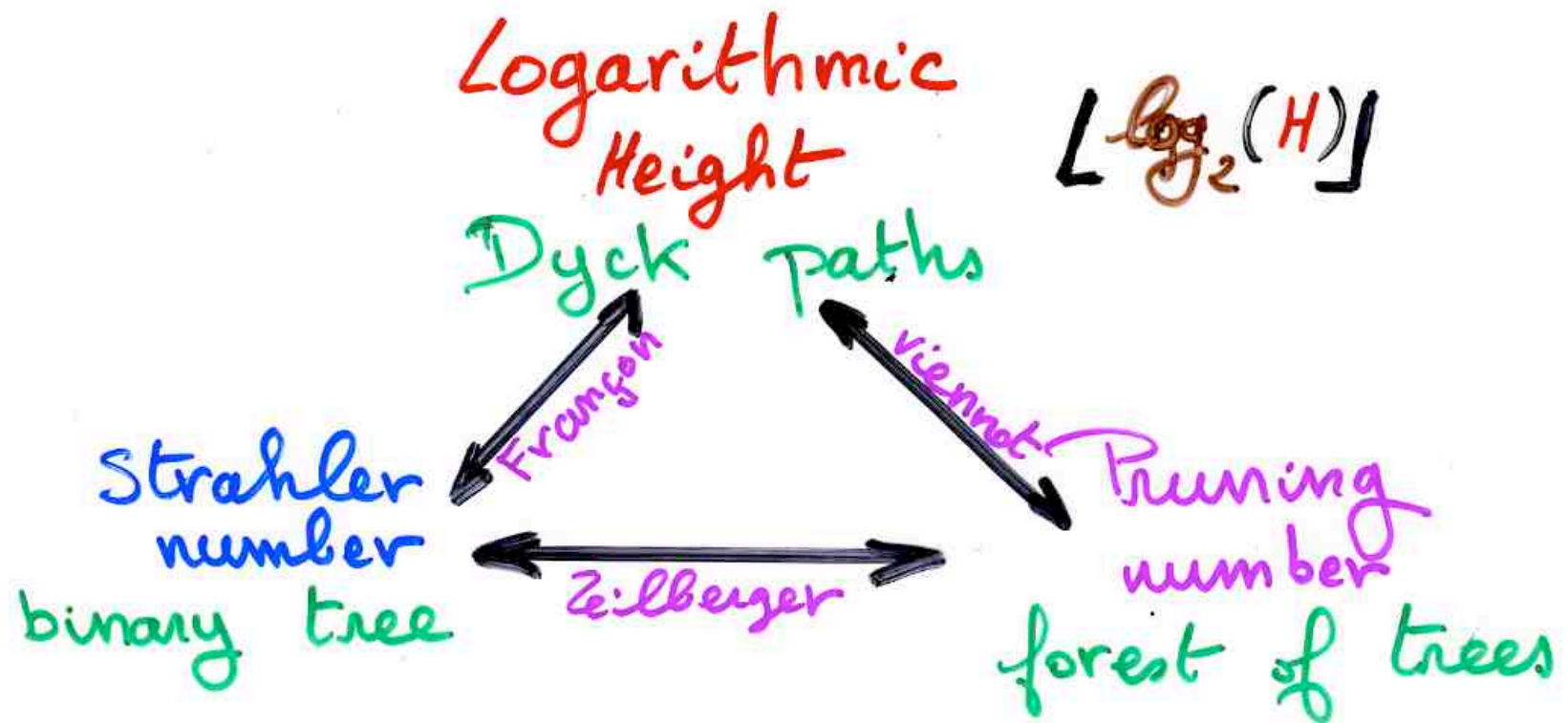
Dyck paths



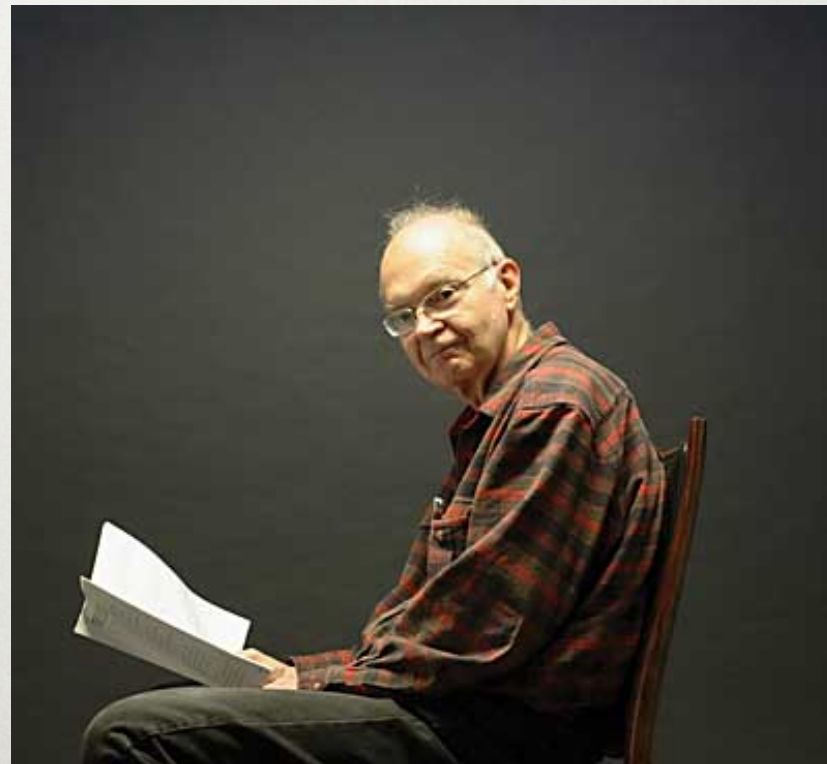
Frangon

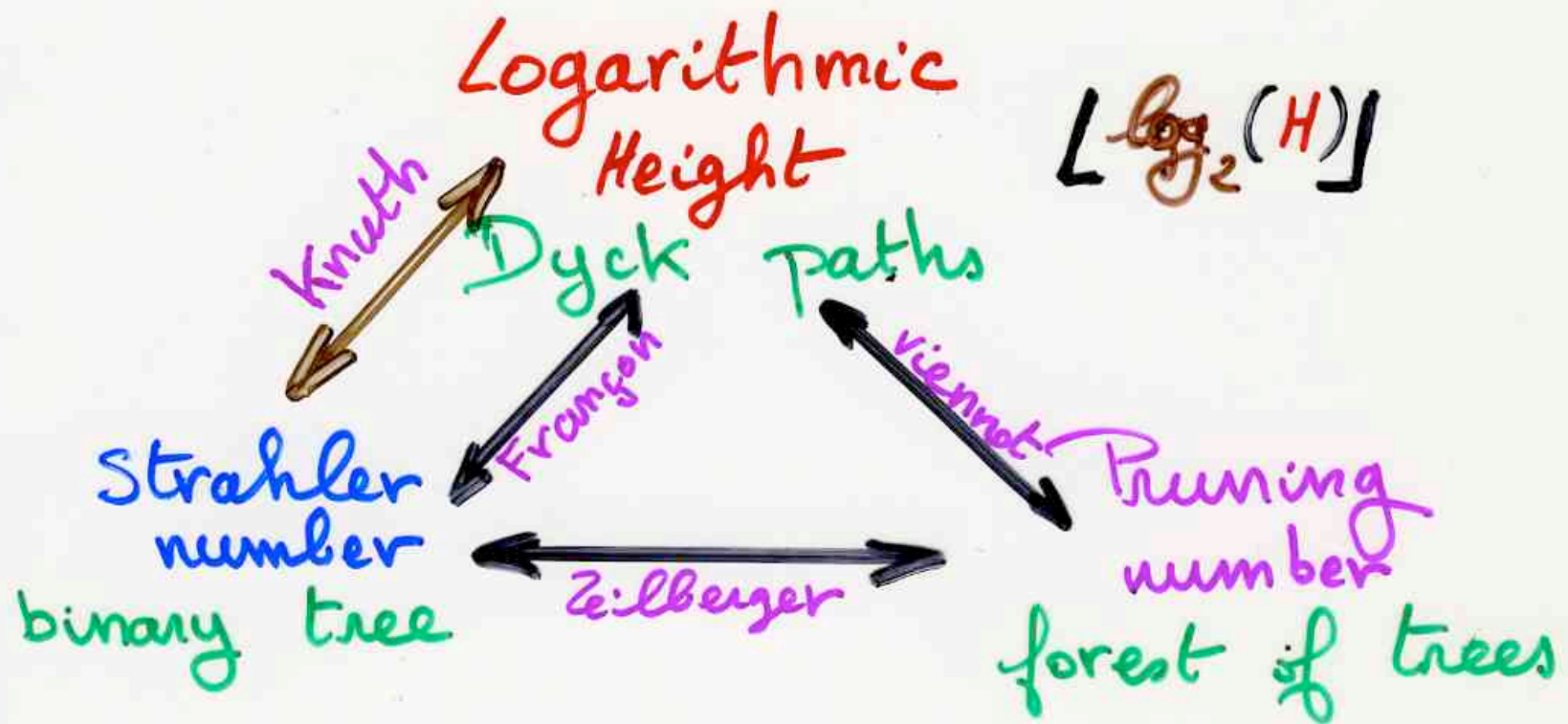
Strahler
number

binary tree



Donald Knuth

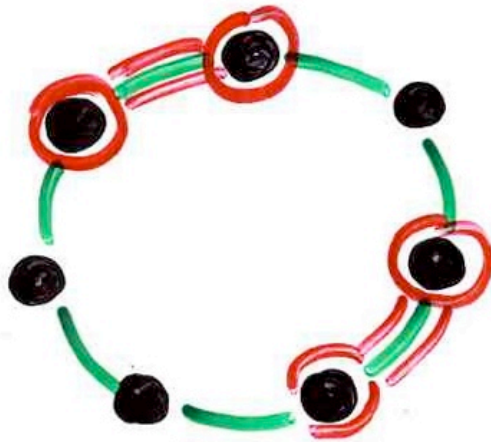


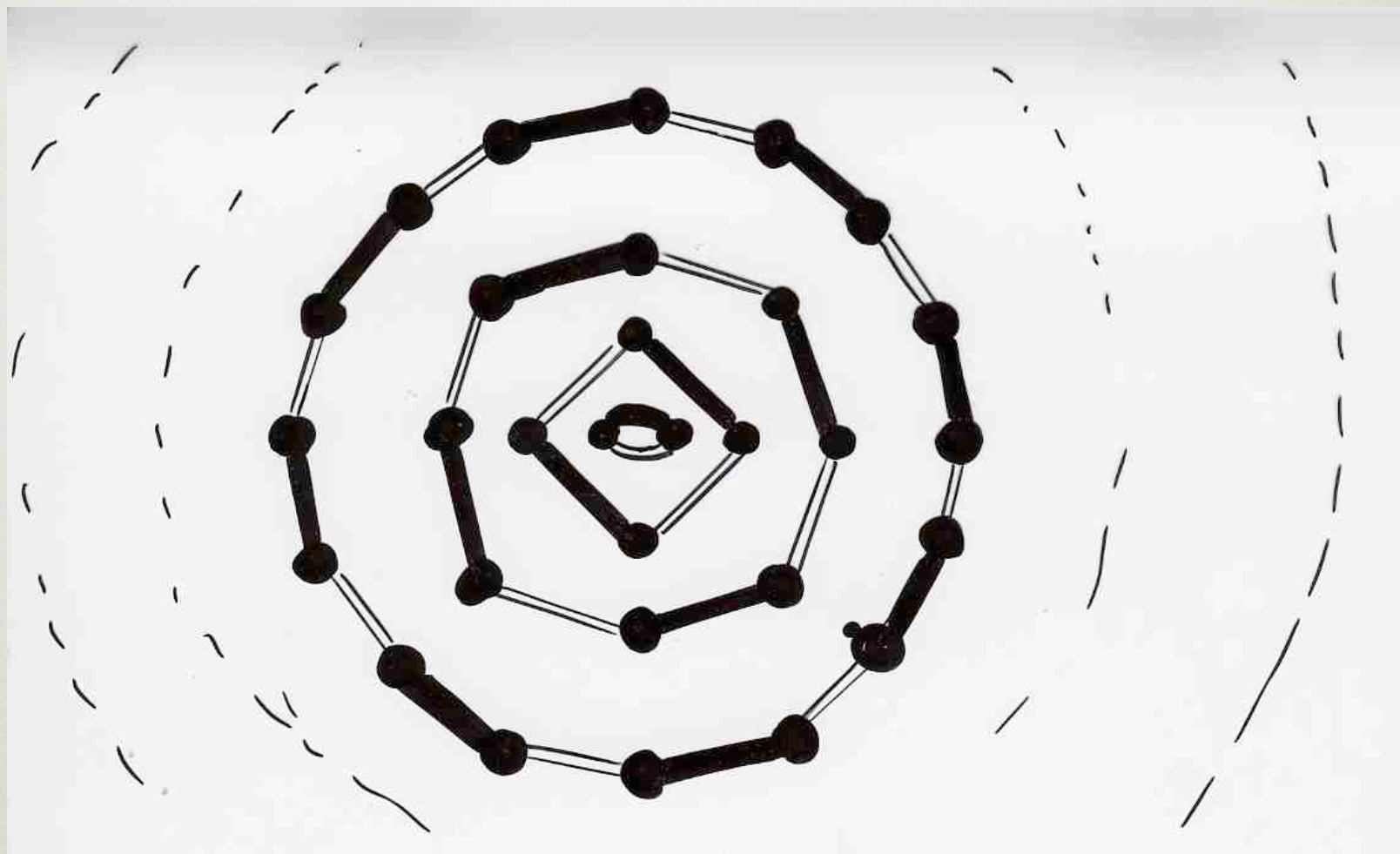


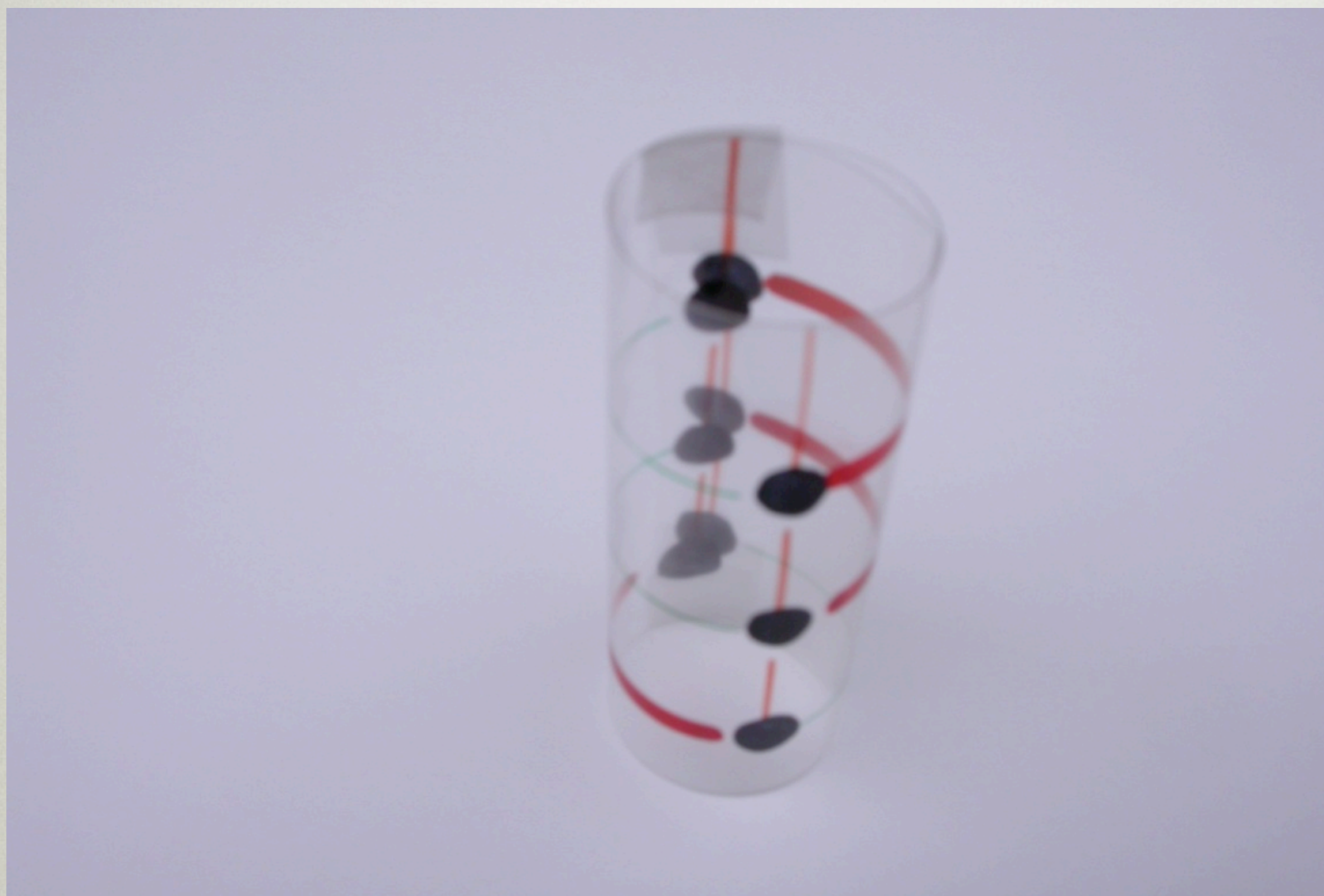
Nombre de **Strahler** **moyen**
d'un arbre binaire ayant n sommets

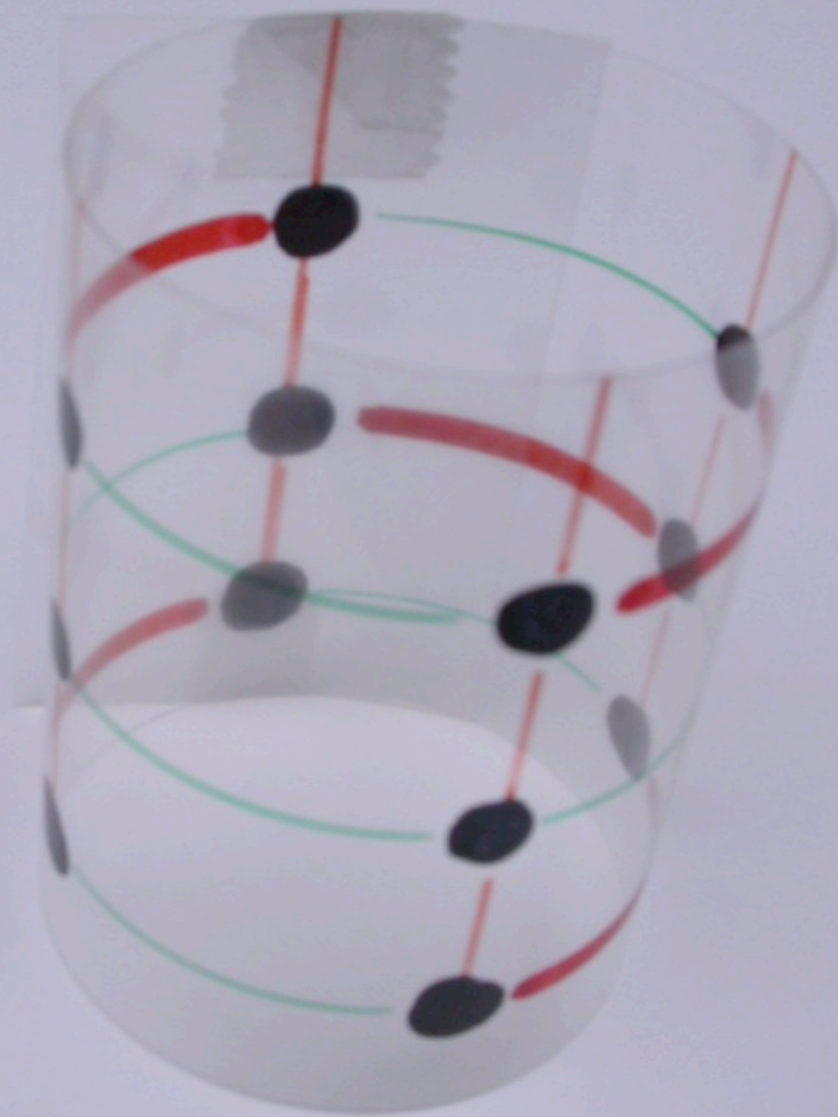
$$A_n = \log_4 n + \underbrace{\mathcal{P}}_{\text{périodique}}(\log_4 n) + \dots$$

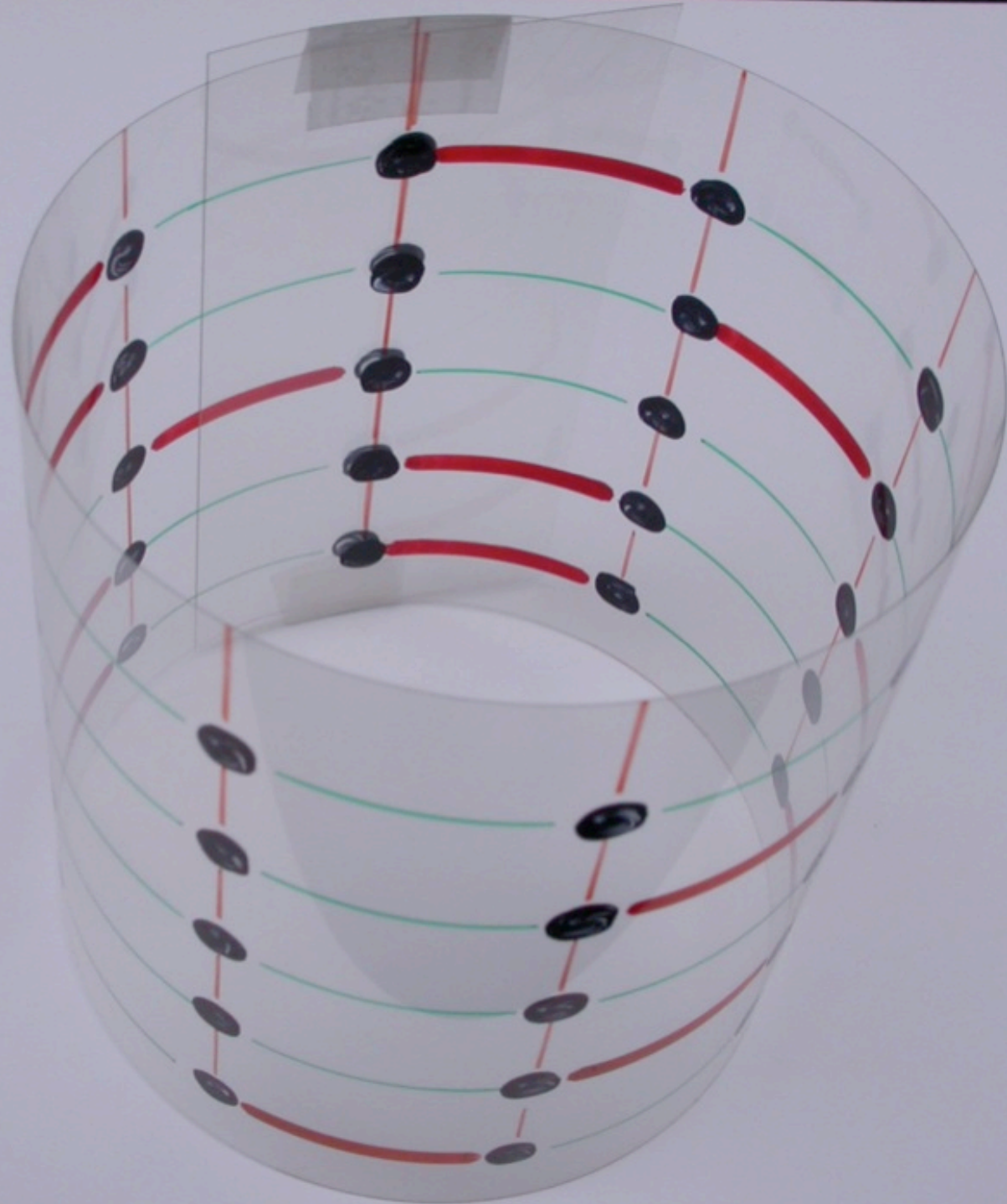
$$\cos(n\theta) = T_n(\cos \theta)$$

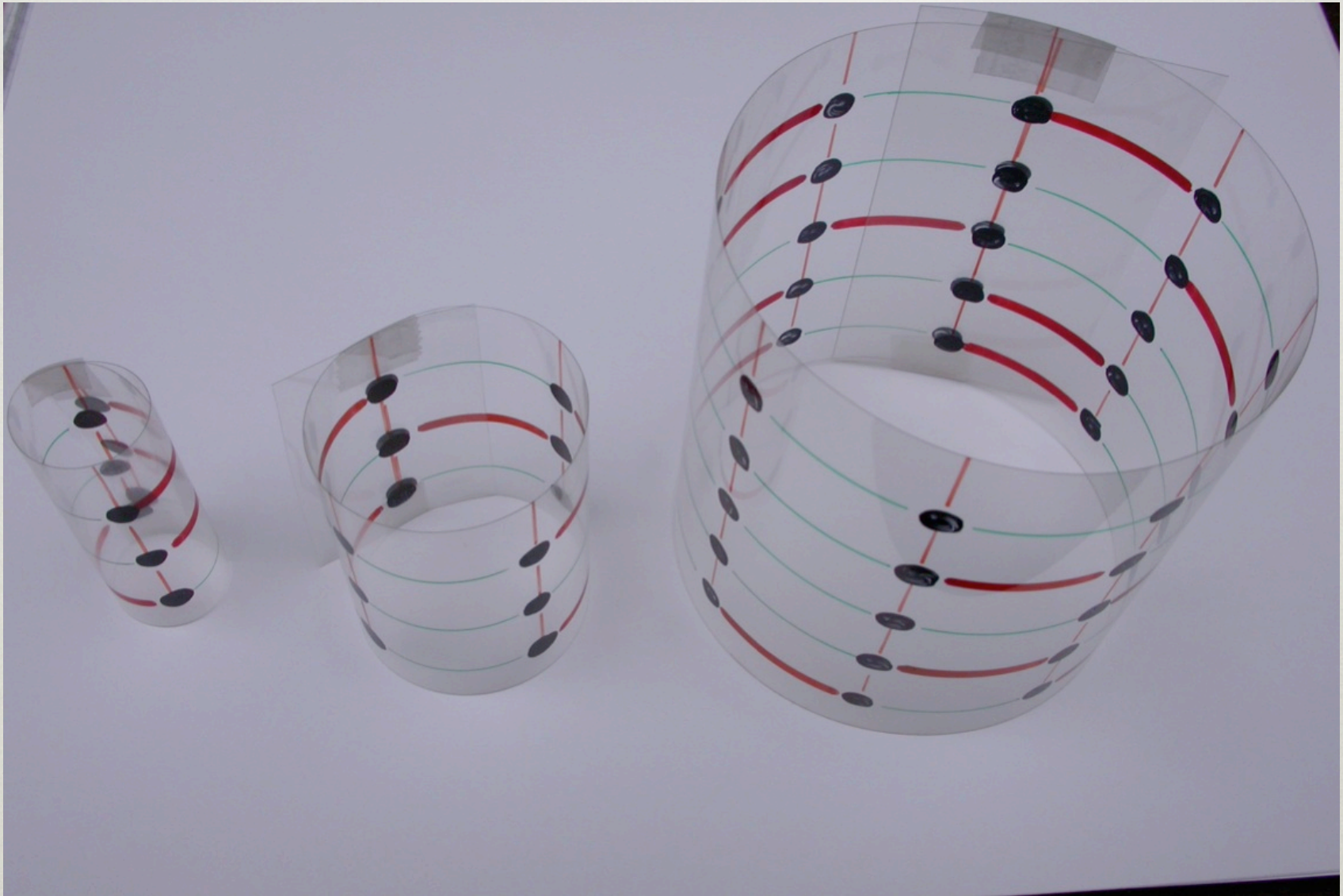


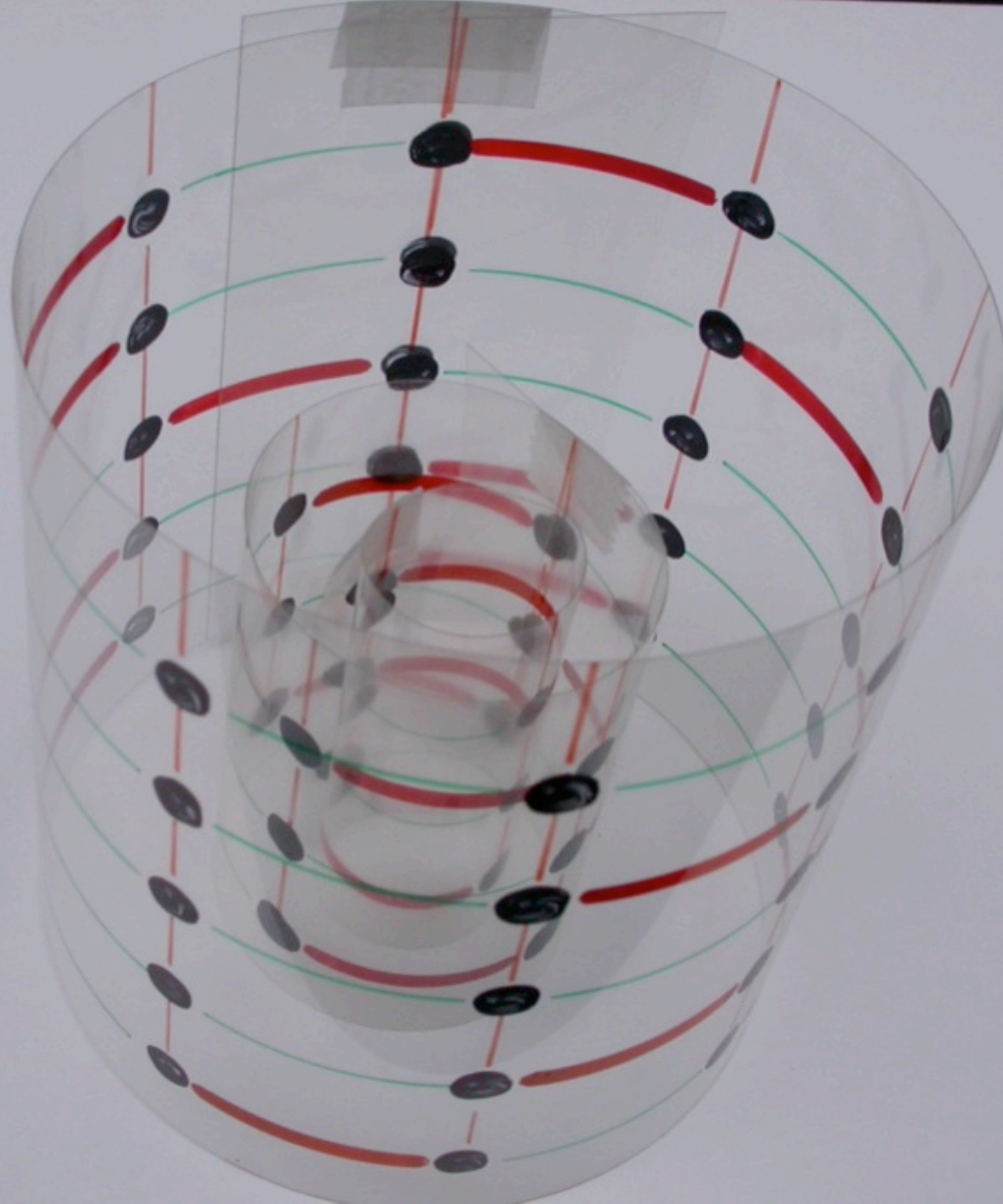


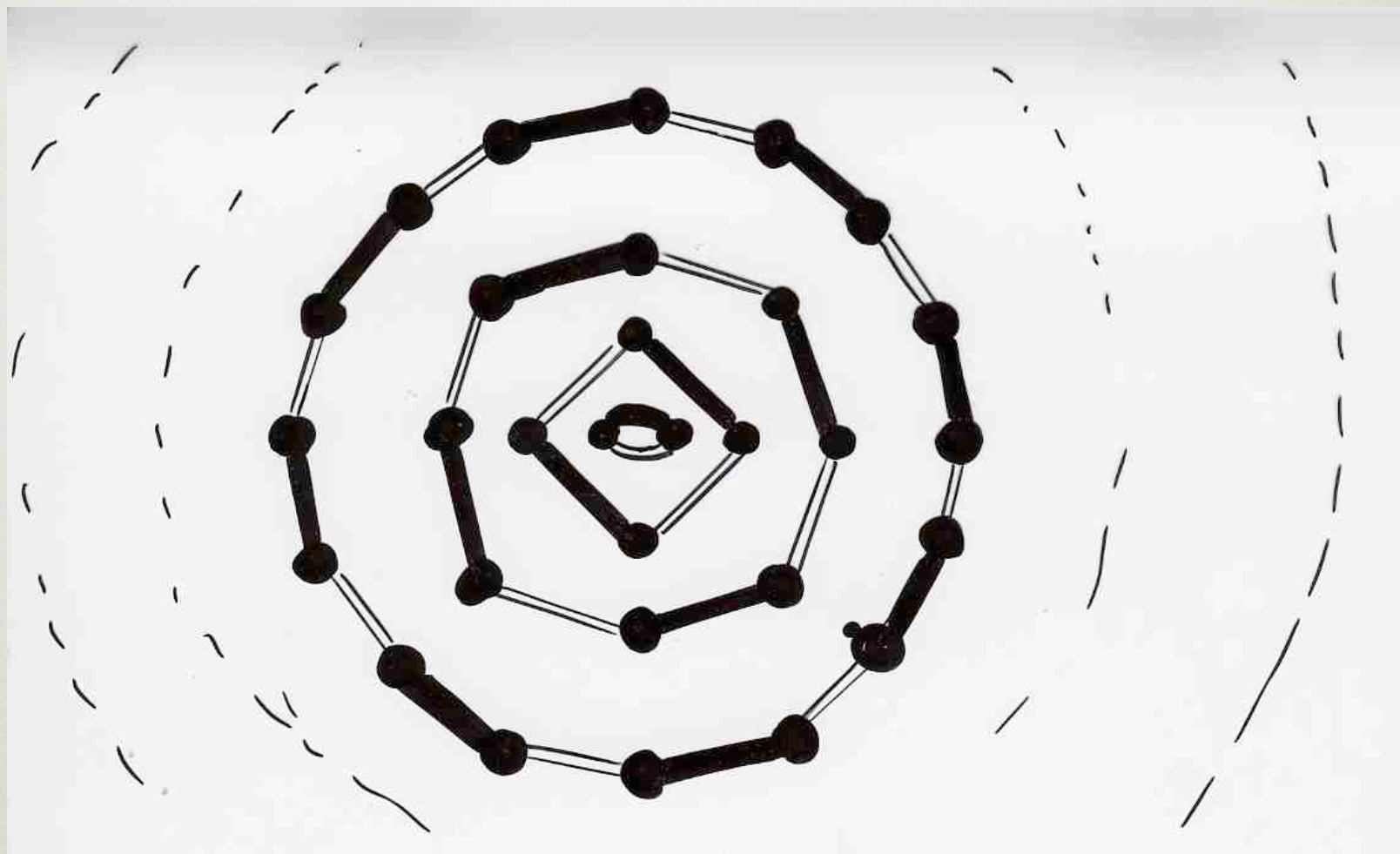










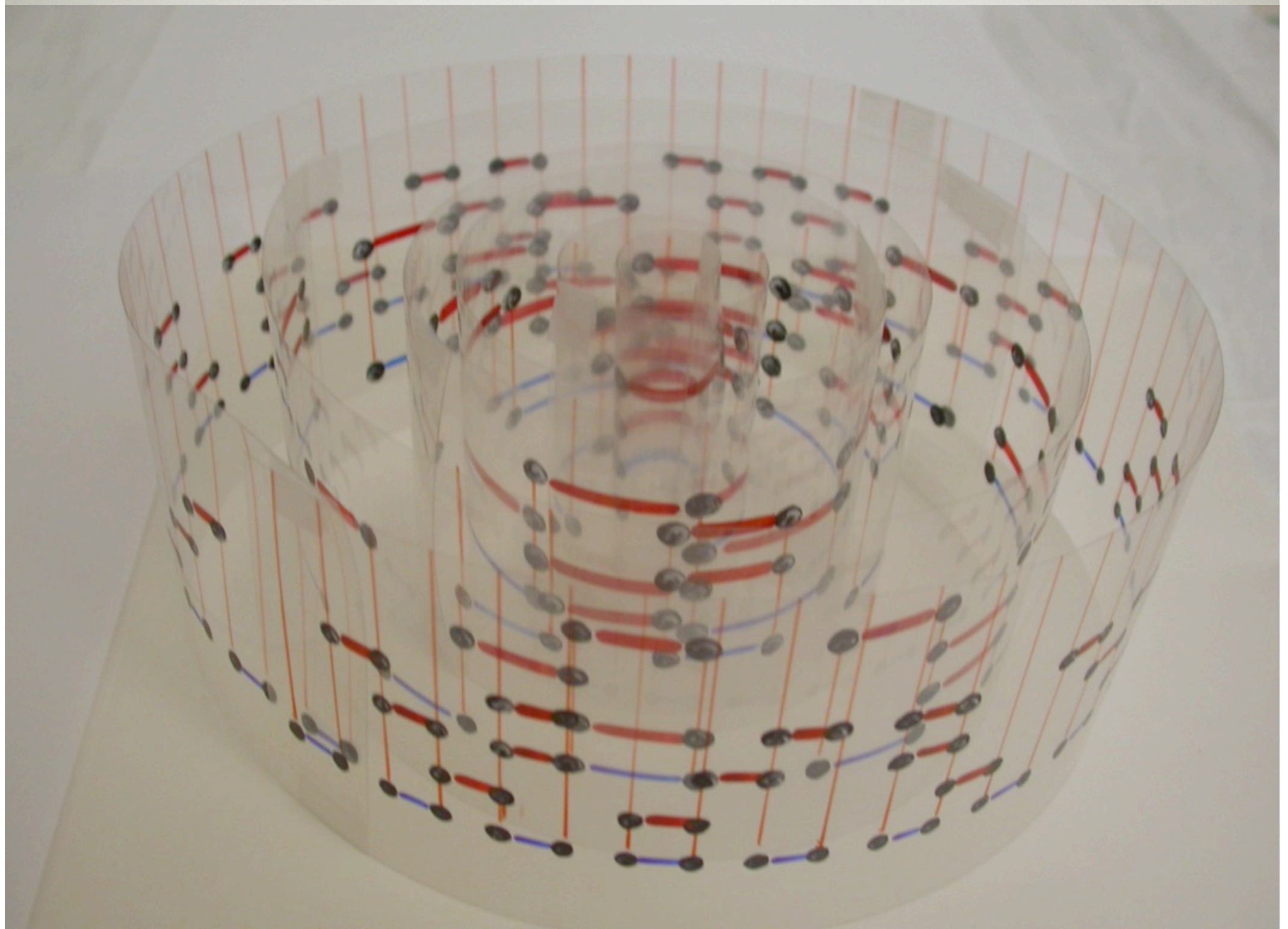


System of Kepler towers

- regular polygons $P_2, P_4, P_8, \dots$
 P_i 2^i edges {  }

- heaps $H_1, \dots, H_k$
 H_i heap of dimers above P_i (= tower)
 $1 \leq i \leq k$

(*) at level 0 H_i contains
 all 2^{i-1} black edges of P_i



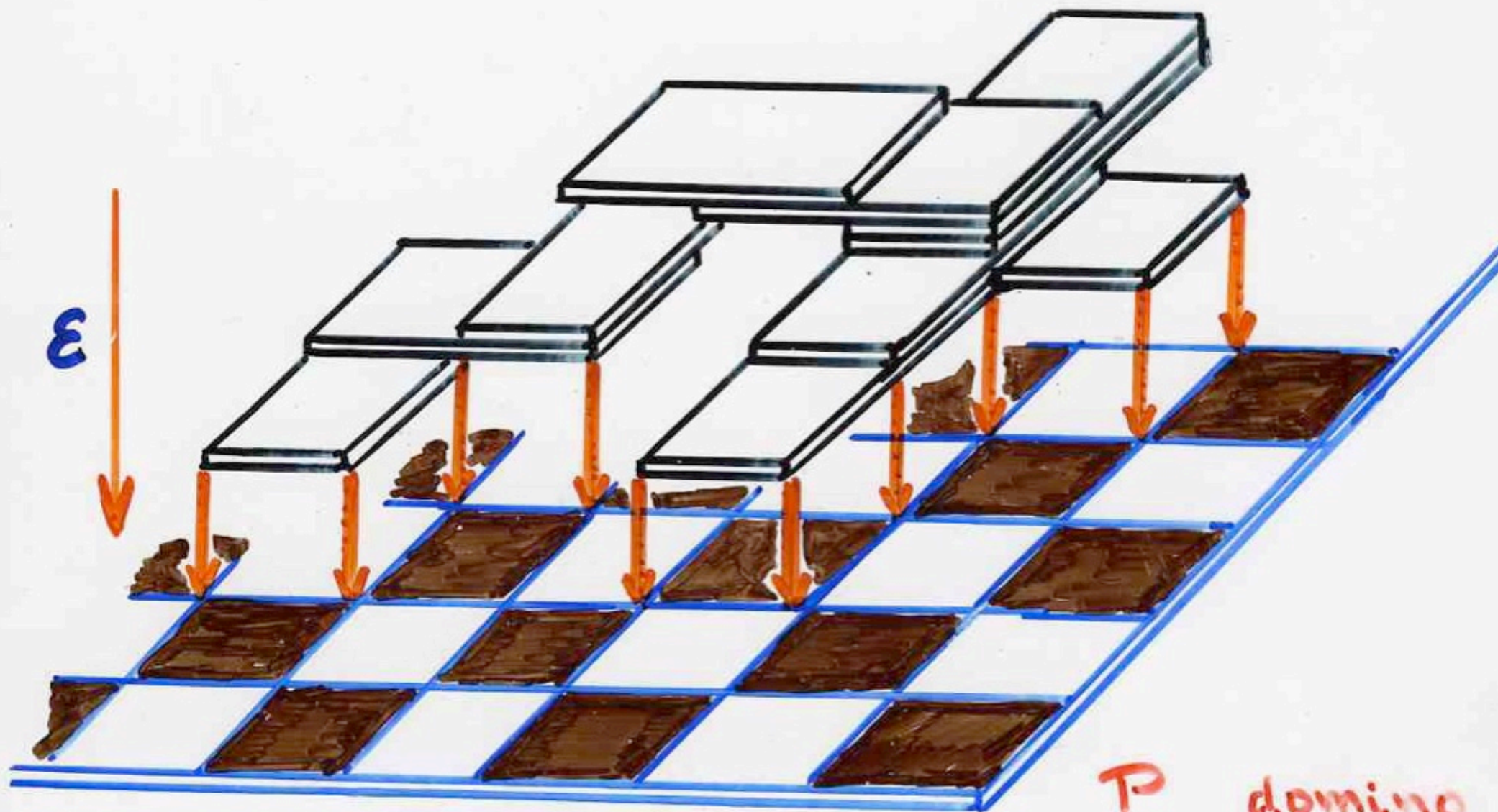
Prop. The number of ^{system} Kepler towers having n ^{dimers} is

Catalan
number

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

The distribution of ^{system} Kepler towers according to the number of ^{towers} is the ^{Strahler} distribution

Des empilements de pièces ...

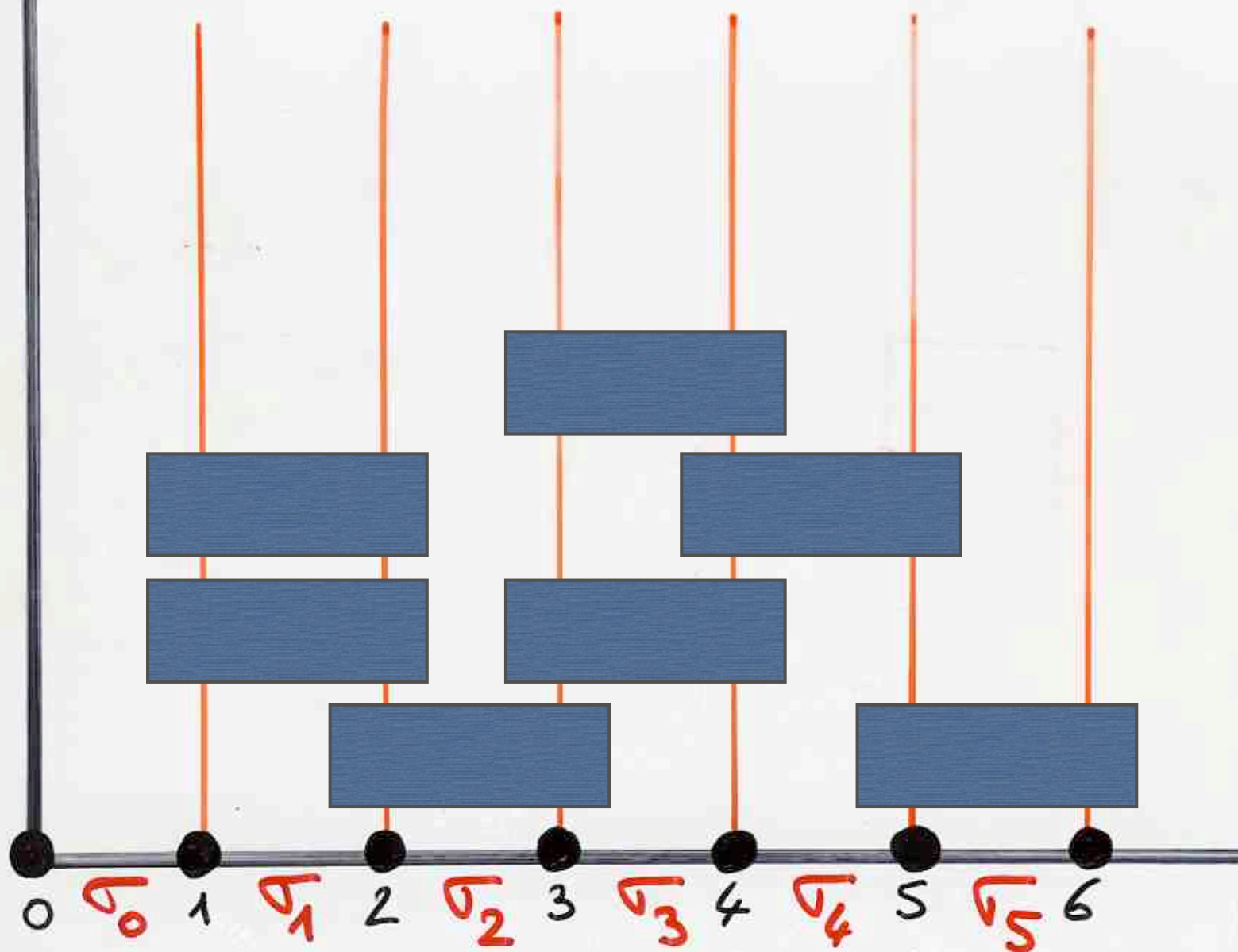


$$B = \mathbb{R} \times \mathbb{R}$$

P domino

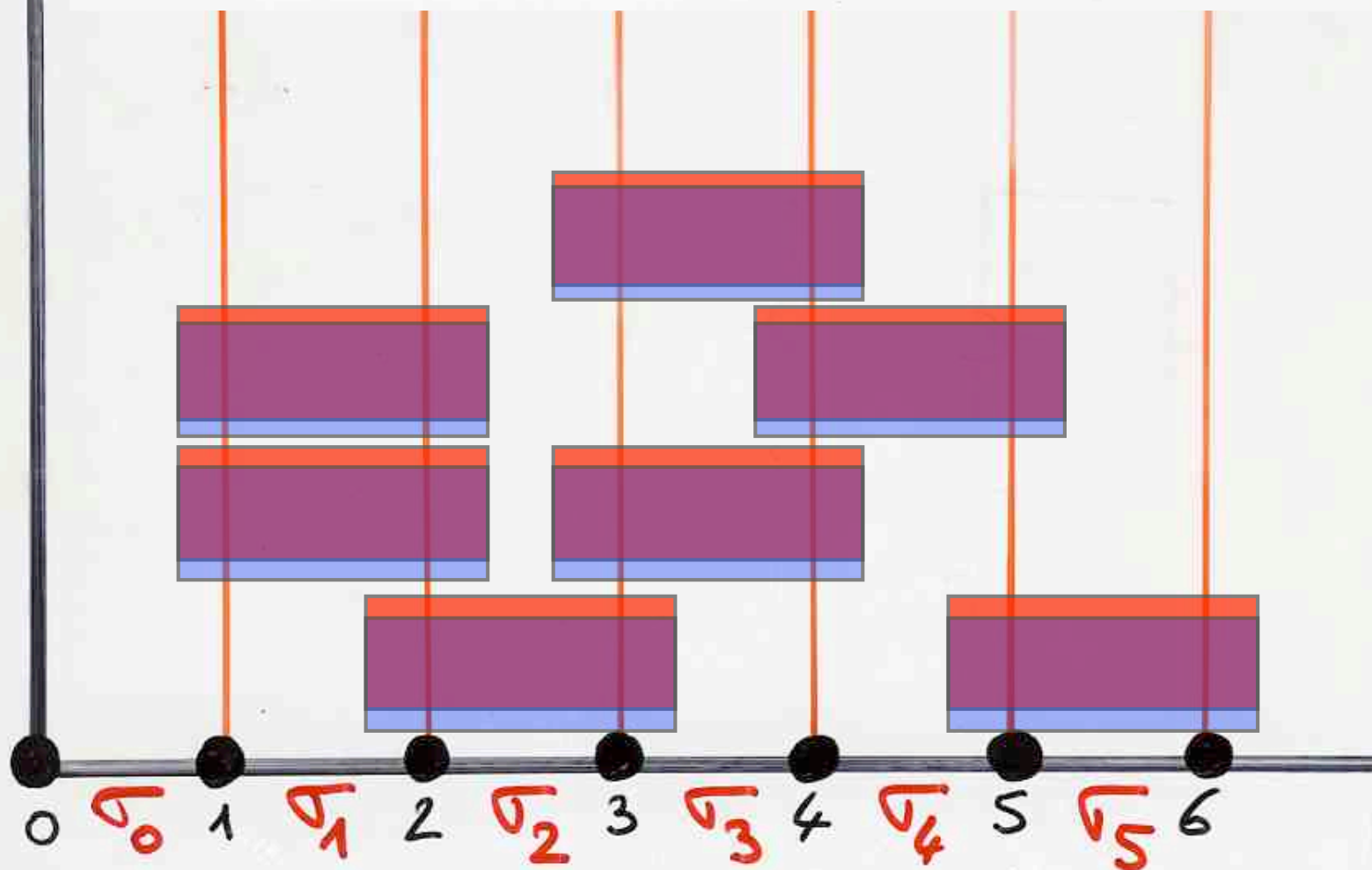
$$\pi = Id$$

$$W = \sigma_2 \sigma_3 \sigma_5 \sigma_1 \sigma_4 \sigma_1 \sigma_3$$



$$W = \sigma_2 \sigma_3 \sigma_5 \sigma_1 \sigma_4 \sigma_1 \sigma_3$$

$$W = \sigma_5 \sigma_2 \sigma_1 \sigma_1 \sigma_3 \sigma_4 \sigma_3$$



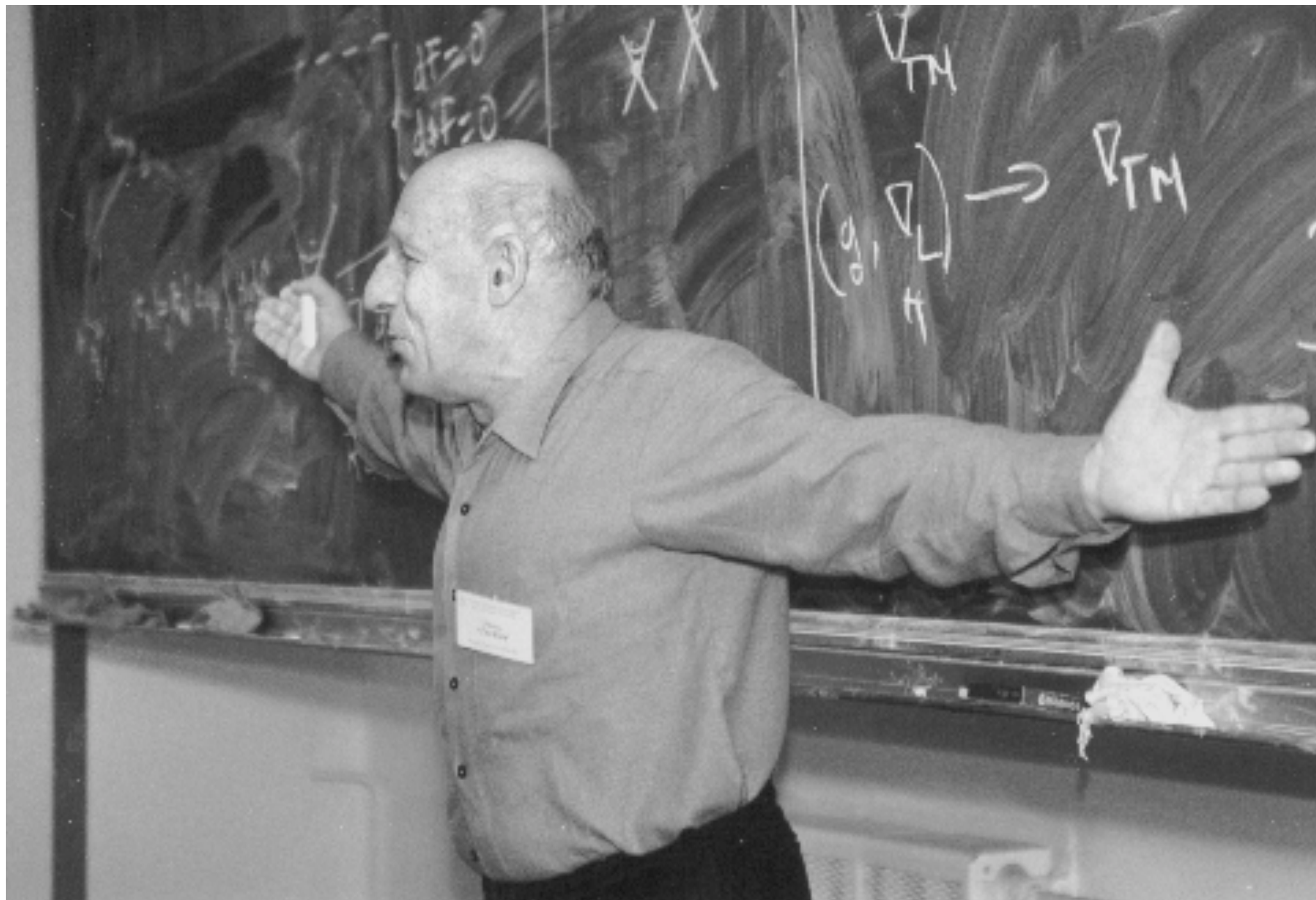
heaps of dimers
($i, i+1$)

on $\{0, 1, \dots, n-1\}$

generators $\{\sigma_0, \sigma_1, \dots, \sigma_{n-1}\}$

$$\sigma_i \sigma_j = \sigma_j \sigma_i$$

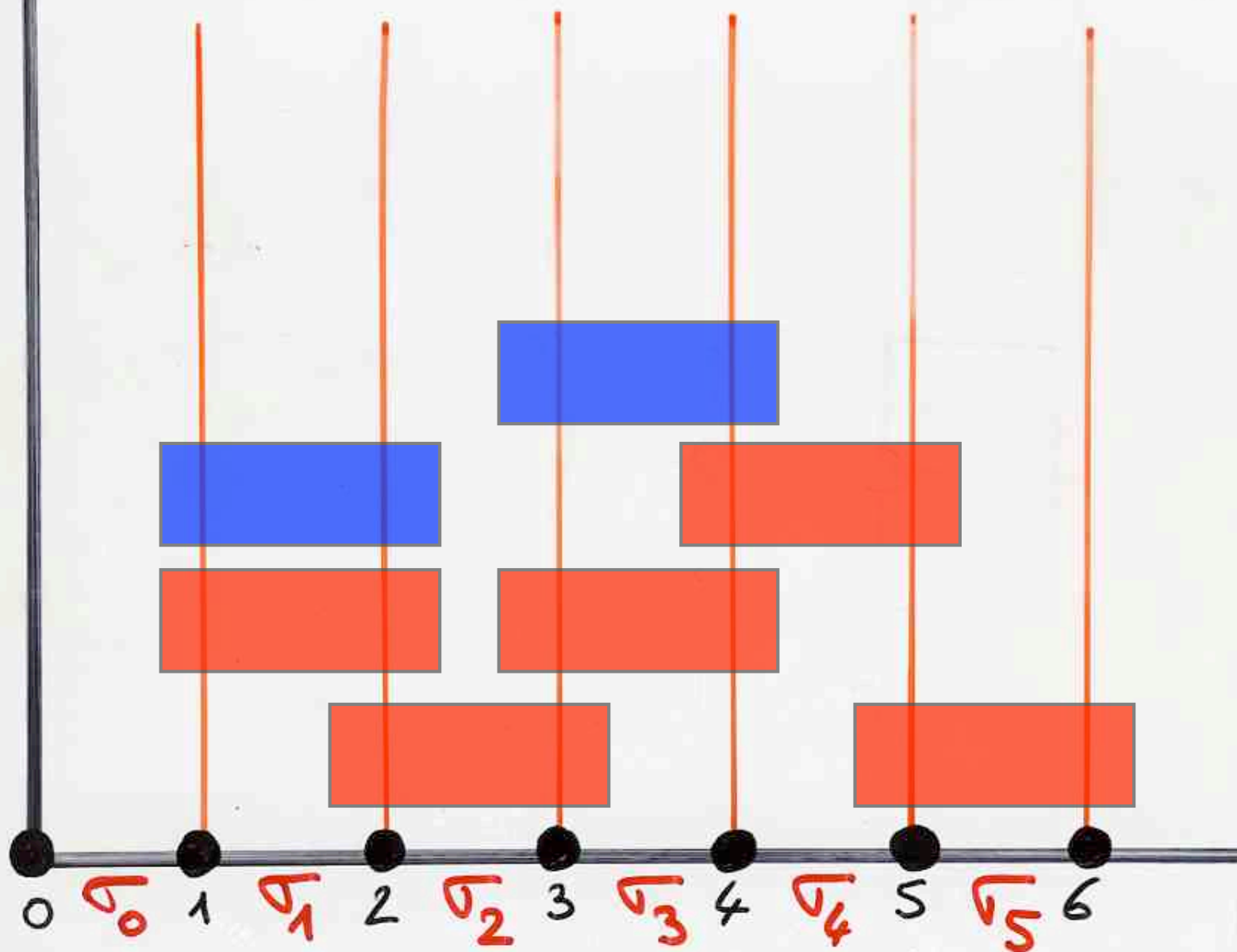
$$\text{iff } |i-j| \geq 2$$

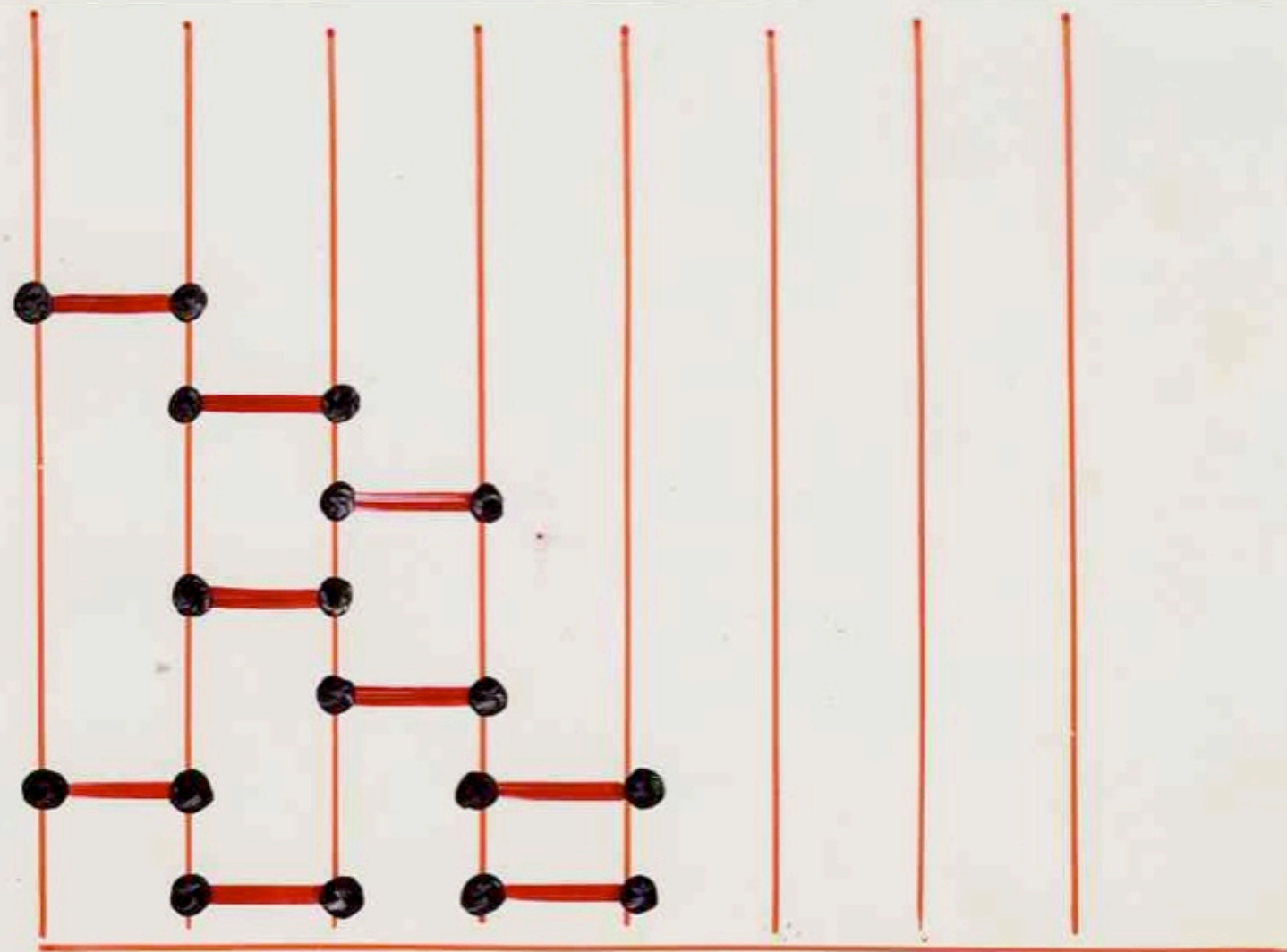




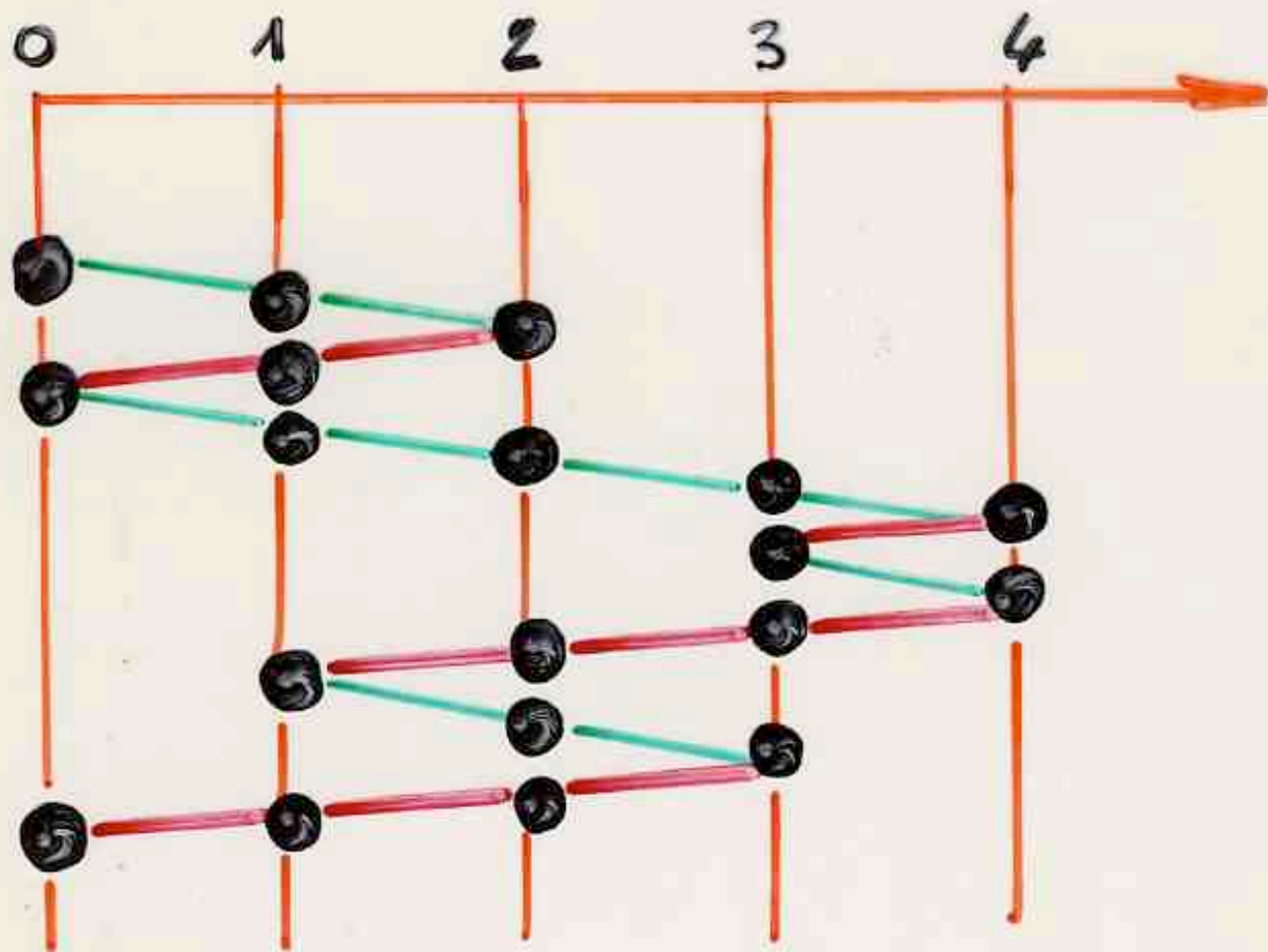
Les **Pyramides de dominos** sont des objets donnant une **interprétation** "géométrico-combinatoire" de la notion de "traces", ou classes de **mots** définis par certaines **commutations de variables**, notion introduite en informatique théorique pour rendre compte du **parallelisme** et de la **concurrence d'accès** à une **structure de données**.

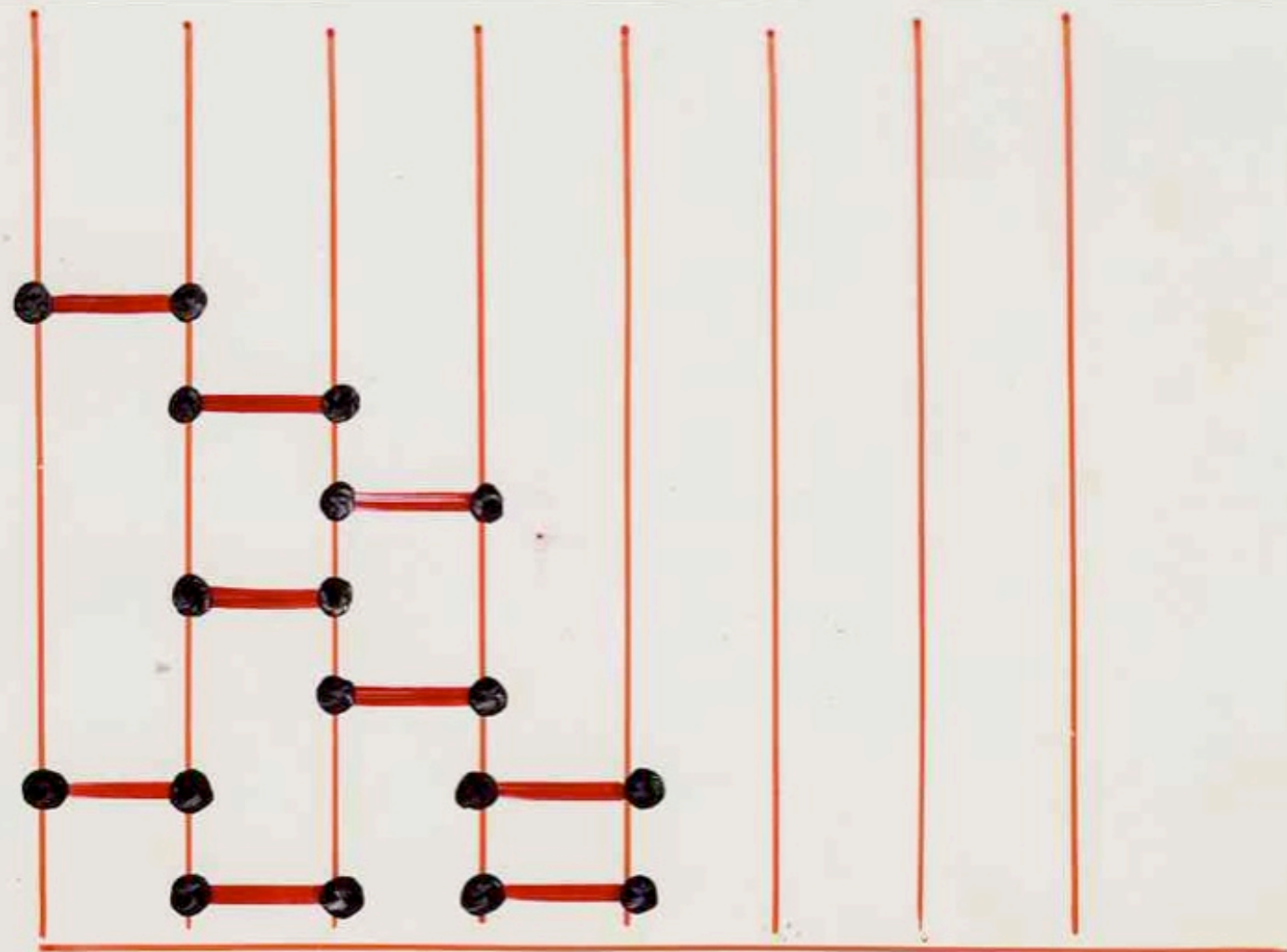
$$W = \sigma_2 \sigma_3 \sigma_5 \sigma_1 \sigma_4 \sigma_1 \sigma_3$$

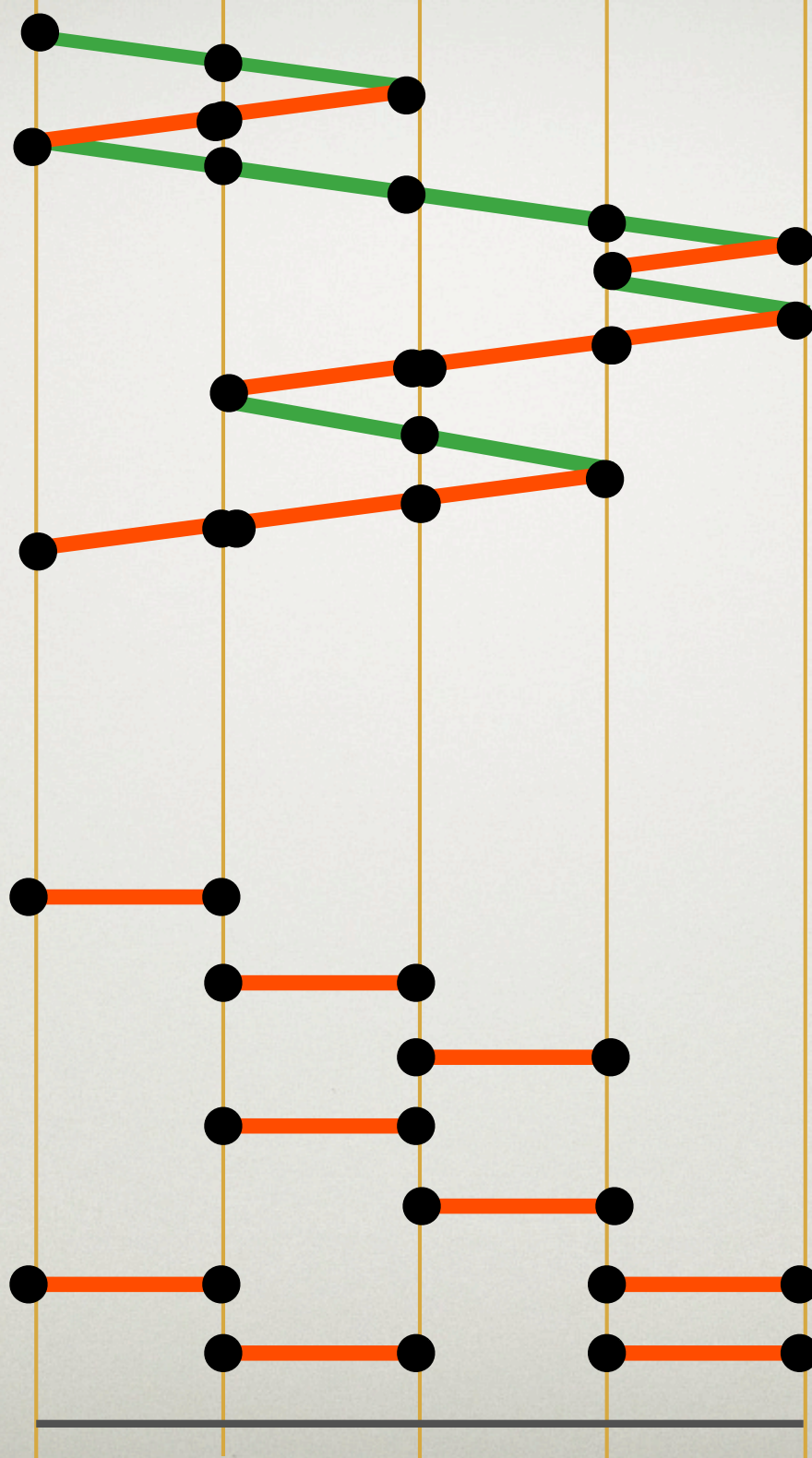




chemins de Dyck
et
pyramides de dominos

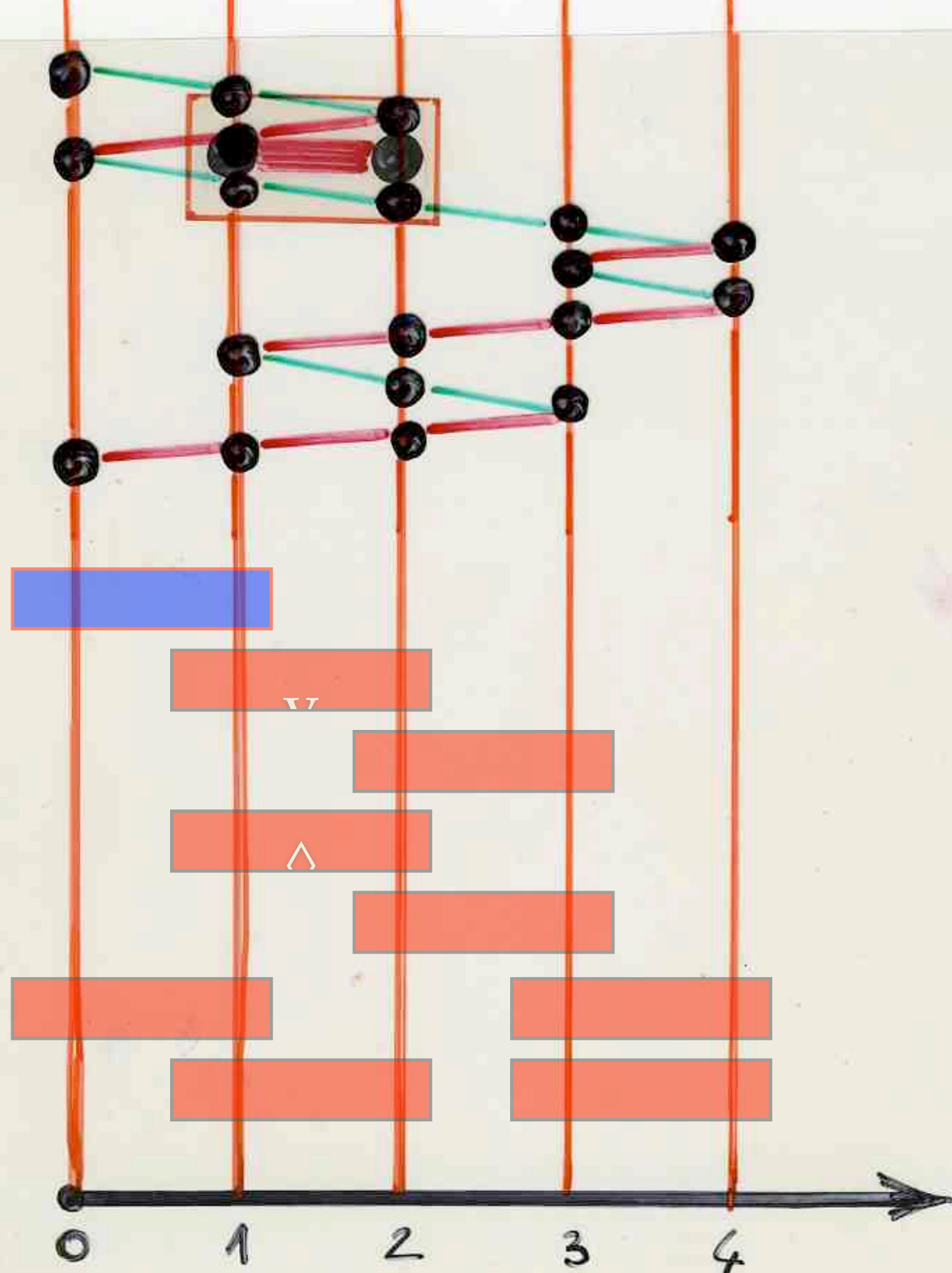


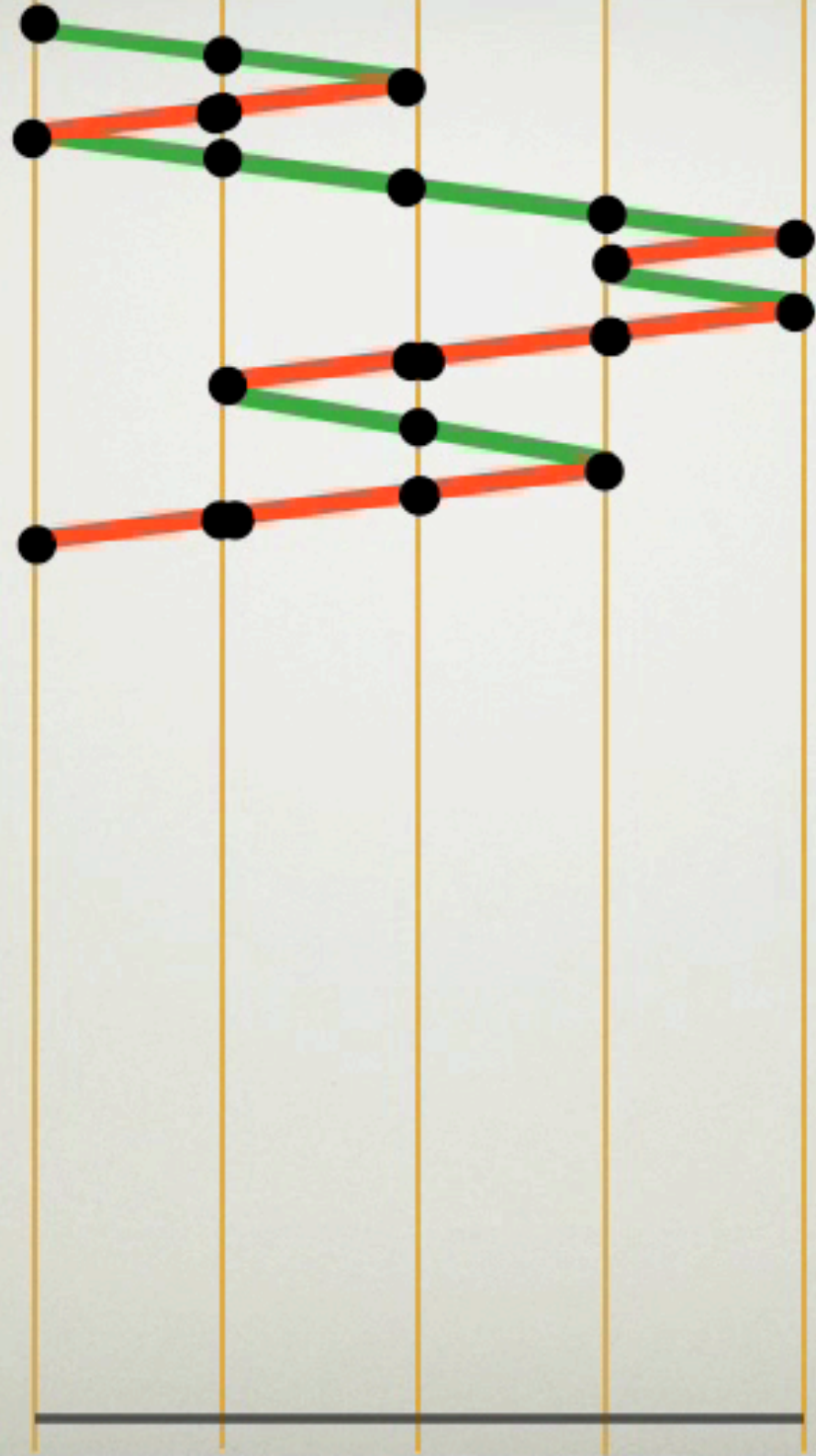




C'est vraiment une bijection !

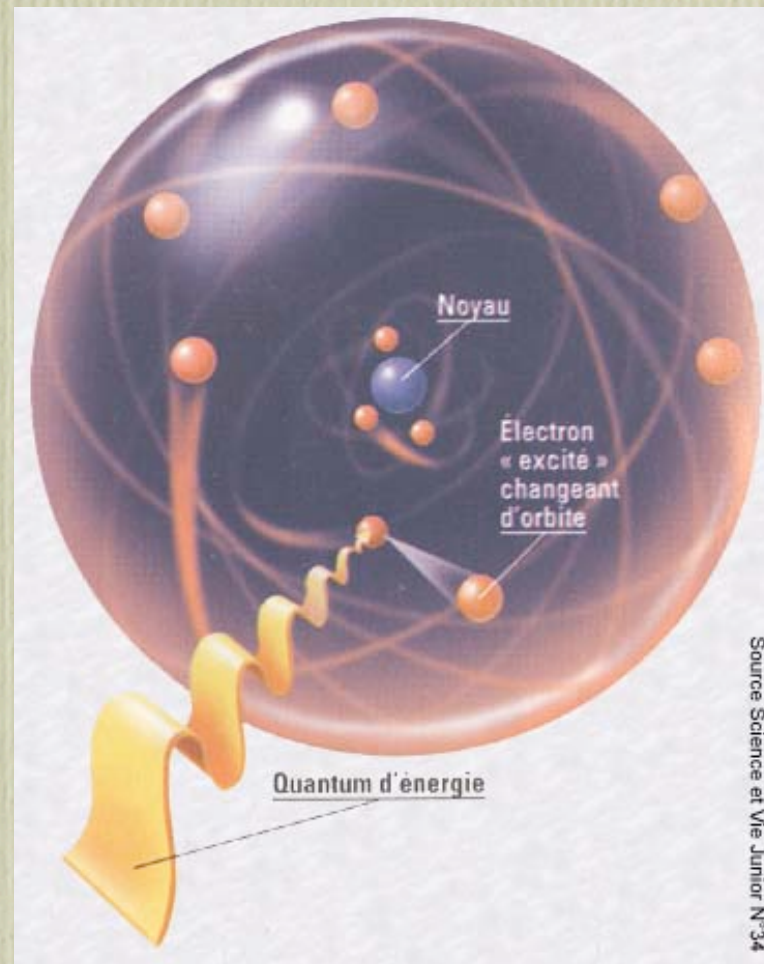
Vous pouvez retrouver l'ordre
dans lequel sont tombés les dominos
et ensuite reconstruire le chemin.

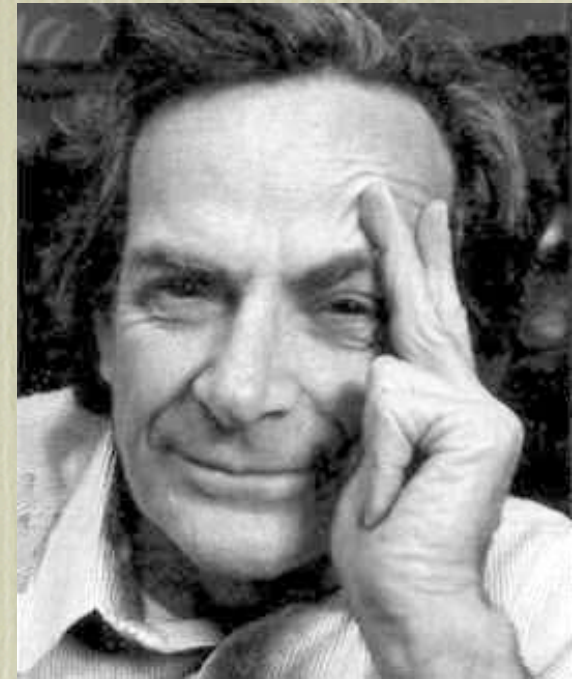
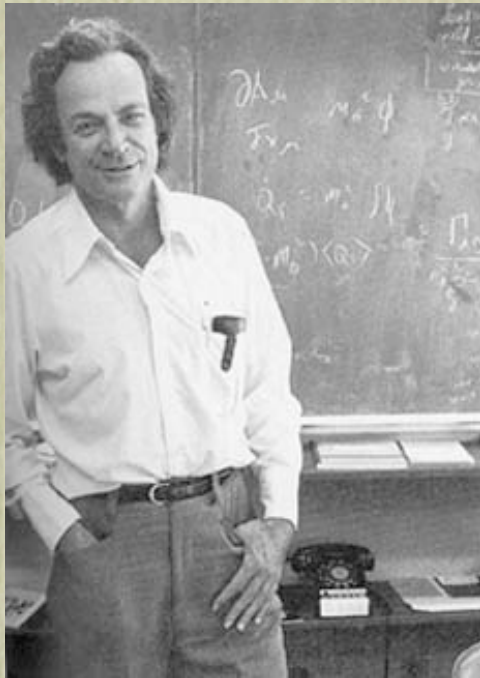




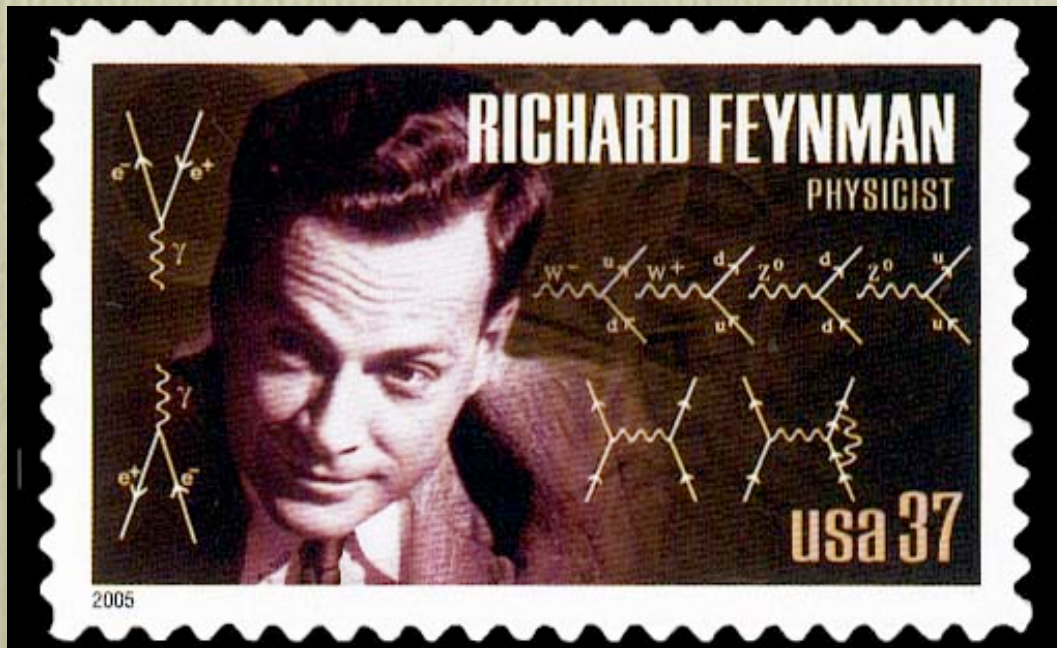
Des arbres dans les grains de lumière ...

le monde quantique





Richard Feynman
1918-1988



Diagrammes de Feynman

$$\sigma^{\gamma}(\Upsilon) = \text{---}\bigcirc\text{---}\bigcirc\text{---}\bigcirc\text{---}$$

$$\sigma^{\gamma}(\Upsilon) = \text{---}\bigcirc\text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

$$\sigma^{\gamma}(\Upsilon\Upsilon) = \text{---}\bigcirc\text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}\bigcirc\text{---}$$

$$\sigma^{\gamma}(\Upsilon\Upsilon) = \text{---}\bigcirc\text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}\bigcirc\text{---}$$

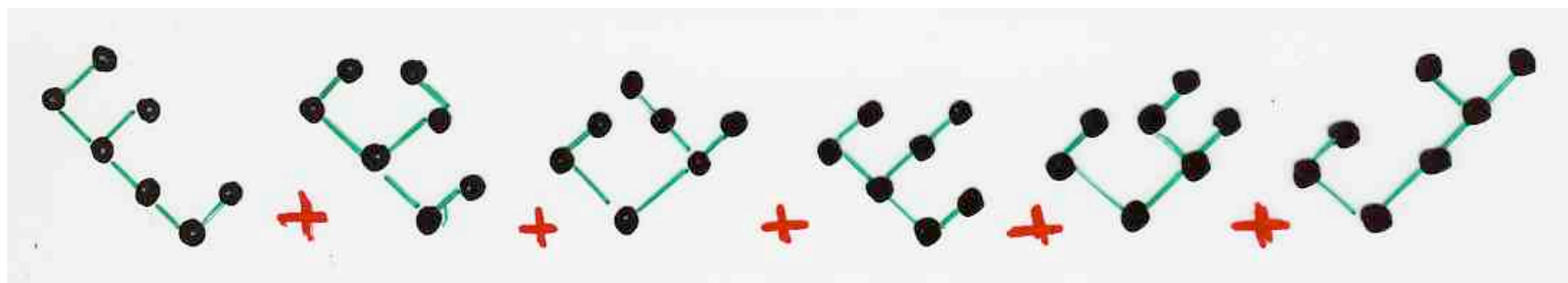
$$\sigma^{\gamma}(\Upsilon\Upsilon) = \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

$$+ \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

$$+ \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$



Renormalisation
quantique

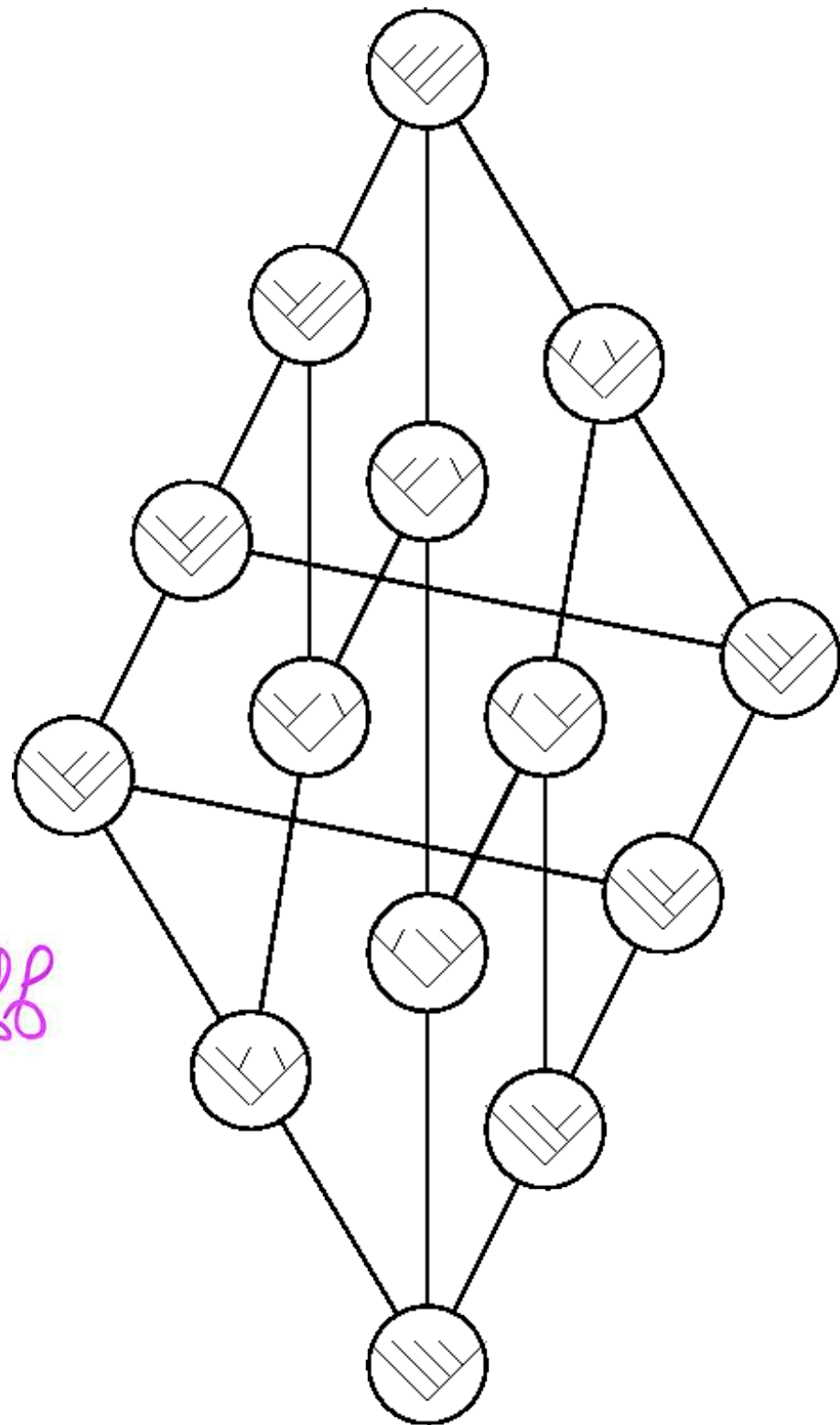


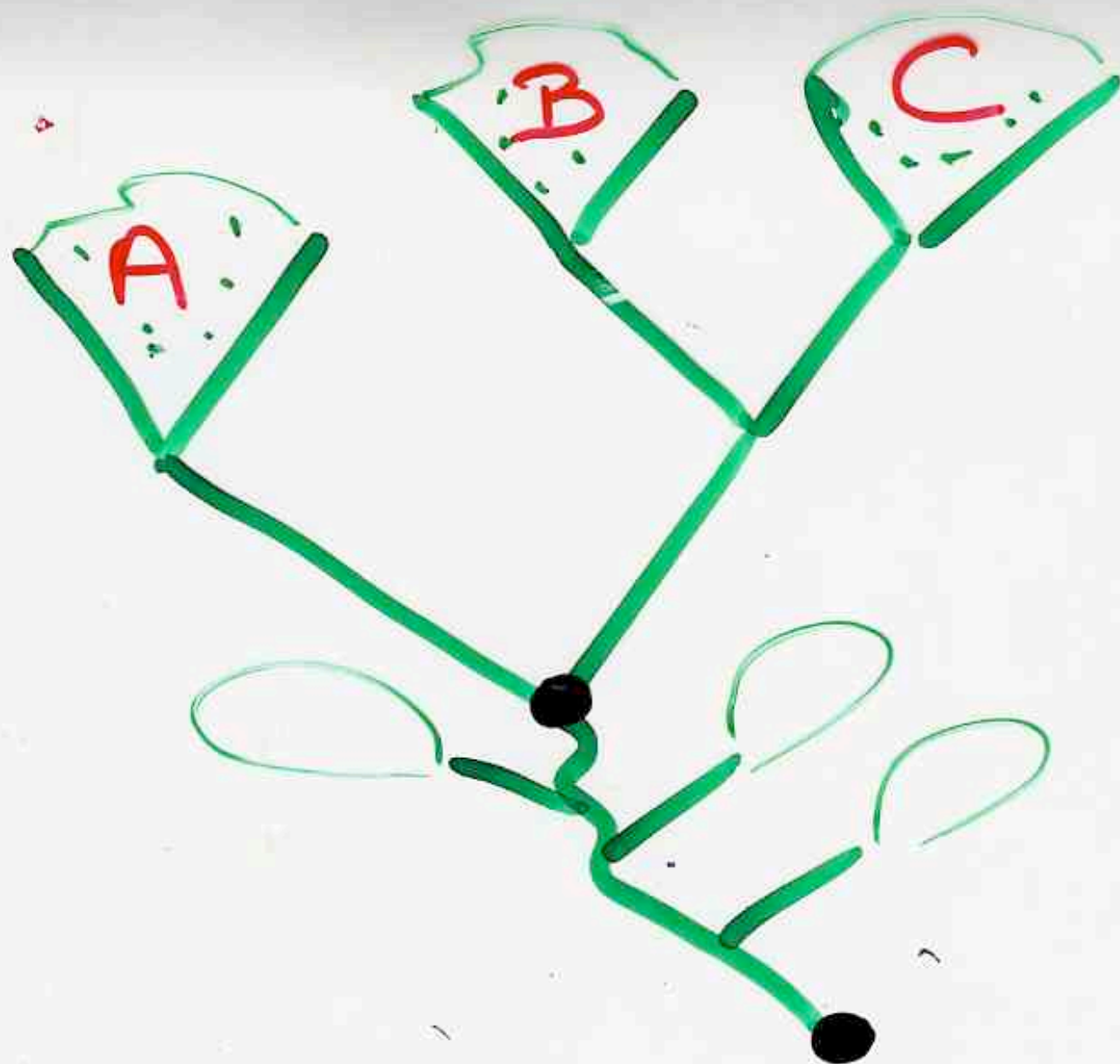
algèbre de Loday-Ronco

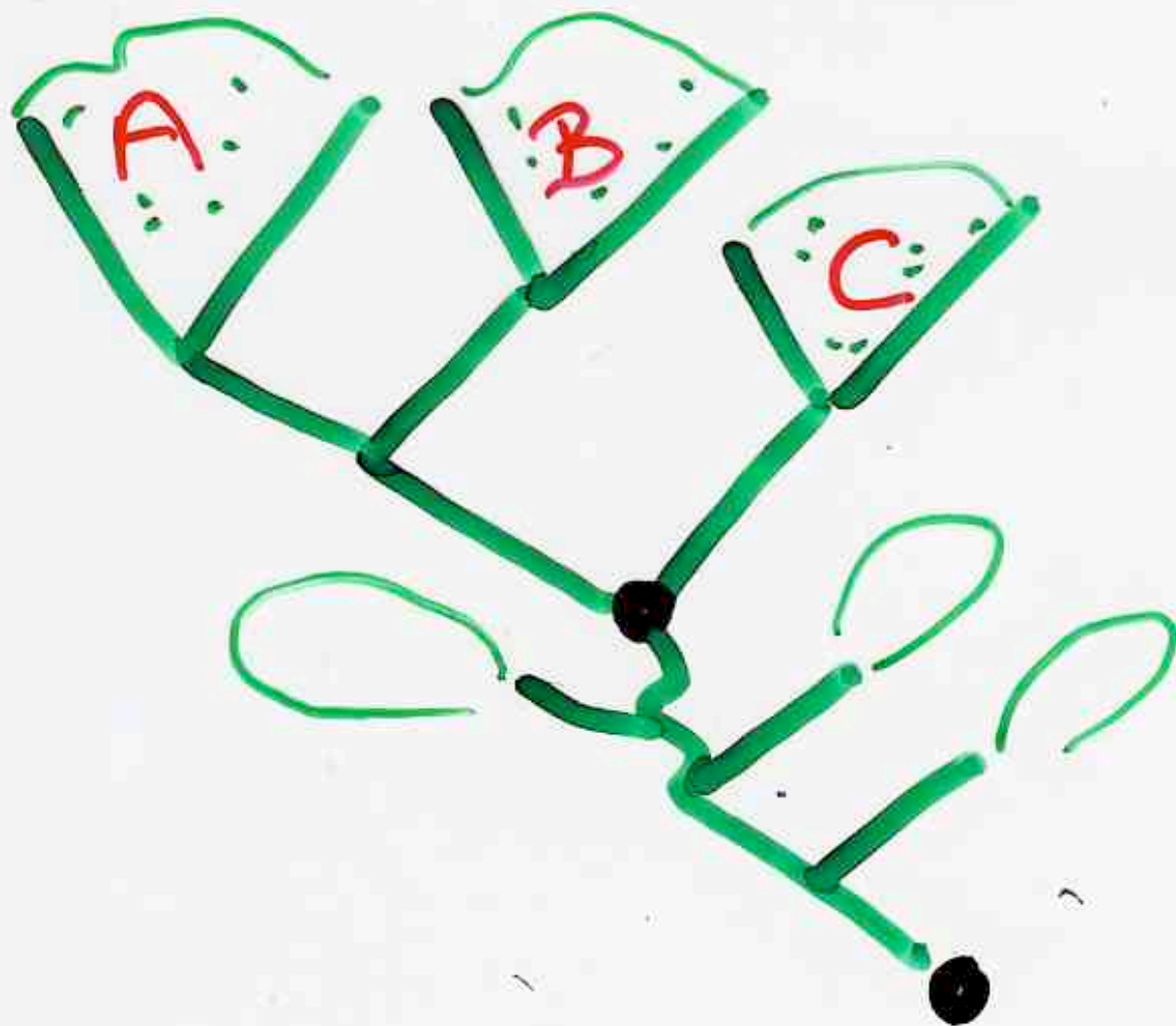
algèbre de Hopf
combinatoire

associaèdre

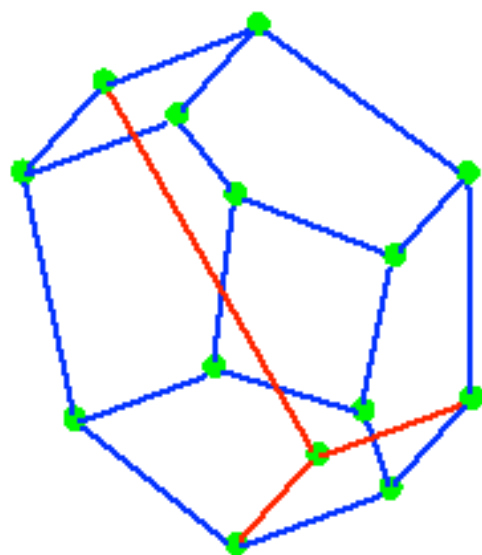
polytope de Stasheff





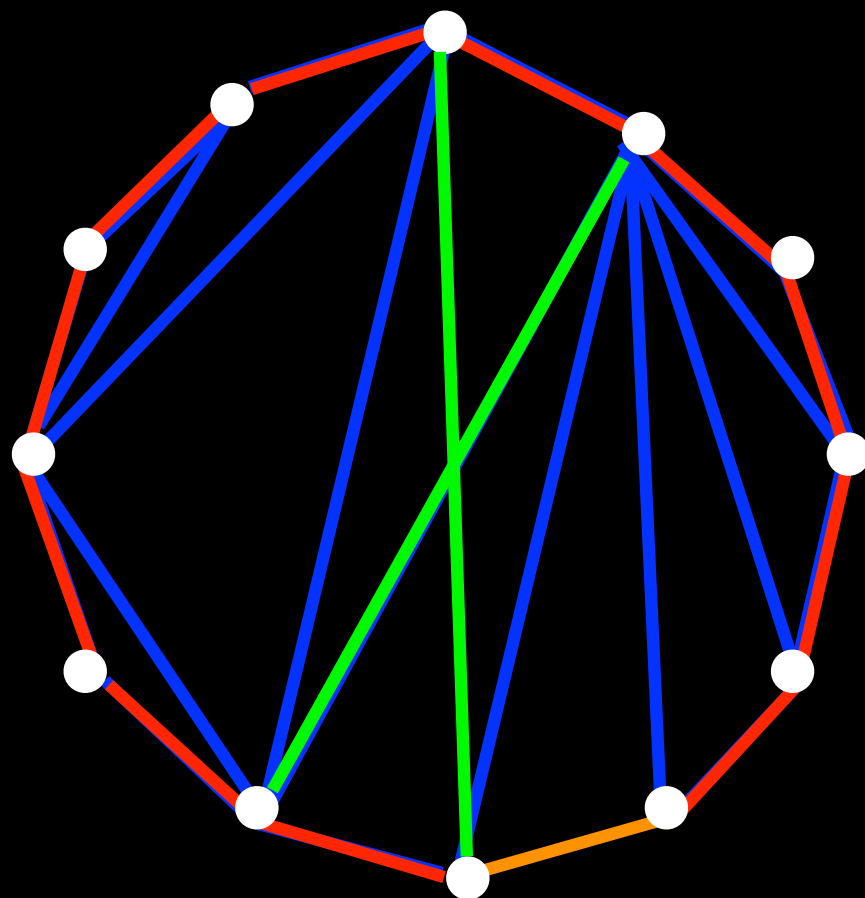


Physique



combinatoire





Des arbres dans les étoiles ...

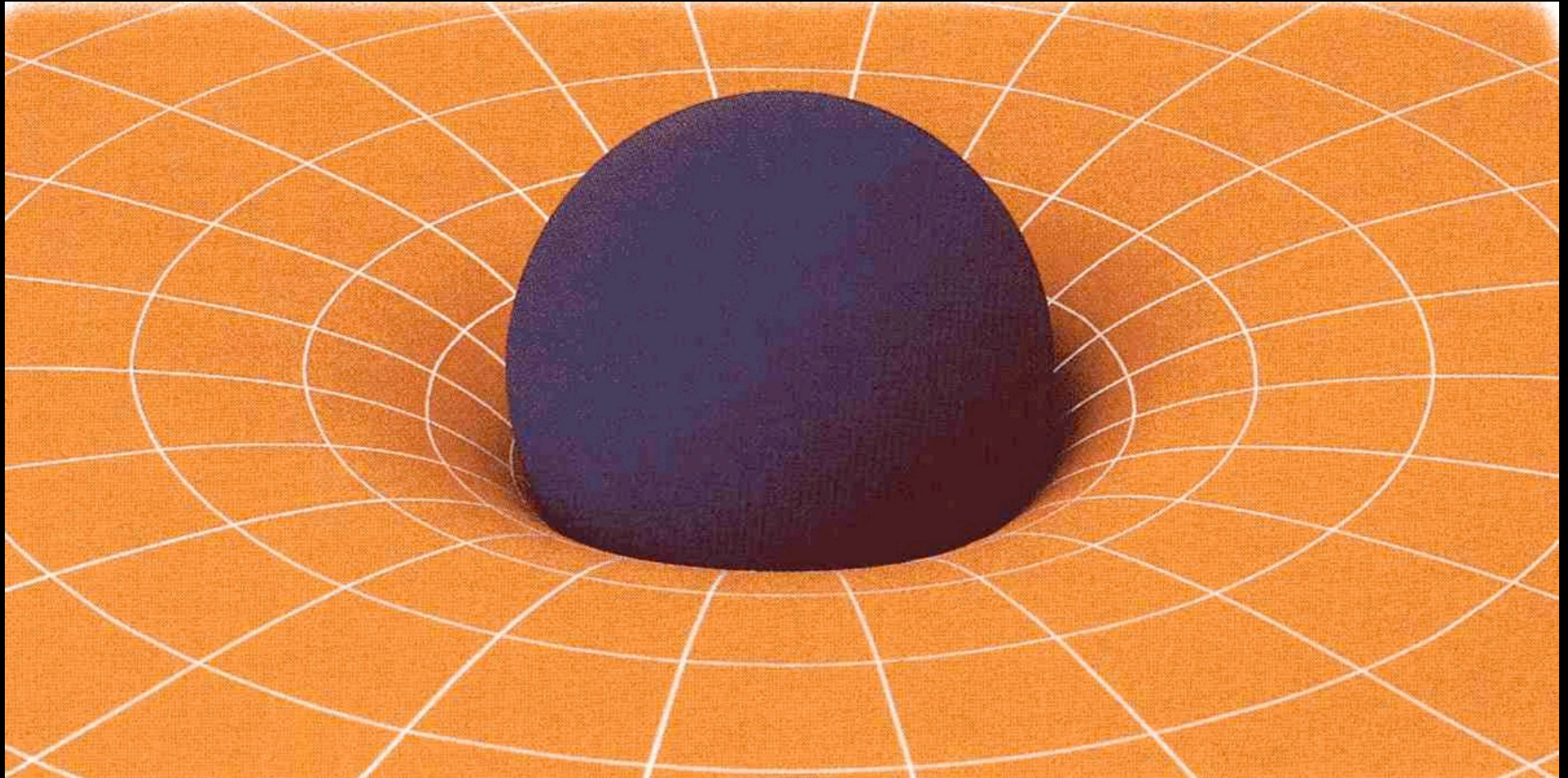


Théorie de la relativité restreinte générale

gravitation

Albert Einstein
1879-1955

Relativité générale
Mécanique quantique





Relativité générale
Mécanique quantique

gravitation
quantique

théorie des cordes

une particule comme une corde de violon ... ?
chaque fréquence correspond à une particule

nombres de Catalan ?



Alain Connes

$$\int e^{i \frac{S(A)}{\hbar}}$$

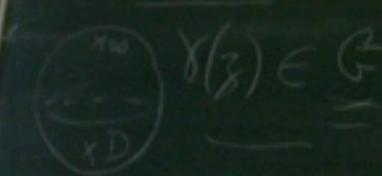
$$\frac{1}{k^2} = \int_0^\infty e^{-\lambda k^2} d\lambda$$

$$\int e^{-\lambda k^2} d^D k = \left(\frac{\pi}{\lambda}\right)^{D/2}$$

DIM-REG

D. KREIMER

Brk Dec



$$\chi(z) \in G$$

$$\chi(z) = \chi(z)$$

(D)

$$D_{||}(\bar{z}) =$$

M. MARCOLLI

$$\chi_N$$

Universal Singular Frac

$$\gamma_W(z, v) = T e^{-\frac{1}{z} \int_0^v u^Y(e) \frac{du}{u}}$$

$$\gamma_W(-z, v) = \sum_{n \geq 0} \sum_{k_j > 0}$$

$$\frac{e(-k_1)e(-k_2) \cdots e(-k_n)}{k_1(k_1+k_2) \cdots (k_1+k_2+\cdots +$$

Same coefficients as

Local Index Formula in NCC

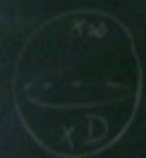
géométrie non-commutative

$$\int e^{-\frac{\lambda k^2}{2}} \quad \int e^{-\lambda k^2} d^D k = \left(\frac{\pi}{\lambda}\right)^{D/2}$$

$\mathcal{D}H\text{-REG}$

D. KREIMER

Brkhl // Dec



$$\gamma(z) \in \mathbb{G}$$

$$\gamma(z) = \gamma_-(z)$$

$$\rightarrow \gamma_+(D)$$

$$D: // (\gamma)$$

M. MARCOLLI

$$\gamma_N$$

Universal Singular Fra

$$\gamma_U(z, v) = T e^{-\frac{1}{2} \int_0^v u^Y(u) \frac{du}{u}}$$

$$\gamma_U(-z, v) = \sum_{n \geq 0} \sum_{k_j > 0}$$

$$\frac{e(-k_1) e(-k_2) \cdots e(-k_n)}{k_1 (k_1 + k_2) \cdots (k_1 + k_2 + \cdots + k_n)}$$

Same coefficients as

Local Index Formula in NC

géométrie non-commutative

plus de points ...

$$e^{\frac{i\hbar}{2m} \Delta}$$

$$\int e^{-\lambda k^2} d^D k = \left(\frac{\pi}{\lambda}\right)^{D/2}$$

$D, KREIMER$ $Birkhoff$ Dec

$$\gamma(z) \in G \quad \gamma(z) = \gamma_-(z)^{-1} \gamma_+(z)$$

$$\xrightarrow{\quad} \gamma_+(D)$$

Universal Singular Fra

$$\gamma_U(z, v) = T e^{-\frac{1}{2} \int_0^v u^Y(e) dt}$$

$$\gamma_U(-z, v) = \sum_{\eta \geq 0} \sum_{k_j > 0}$$

$$\frac{e(-k_1)e(-k_2) \cdots e(-k_n)}{k_1(k_1+k_2) \cdots (k_1+k_2+\cdots + k_n)}$$

Same coefficients as

Local Index Formula in NC

$$D_{i//}(\bar{z}) \rightarrow$$

M. MARCOLLI

$$\gamma_n$$

géométrie non-commutative

plus de droites ...

Universal Singular Frame

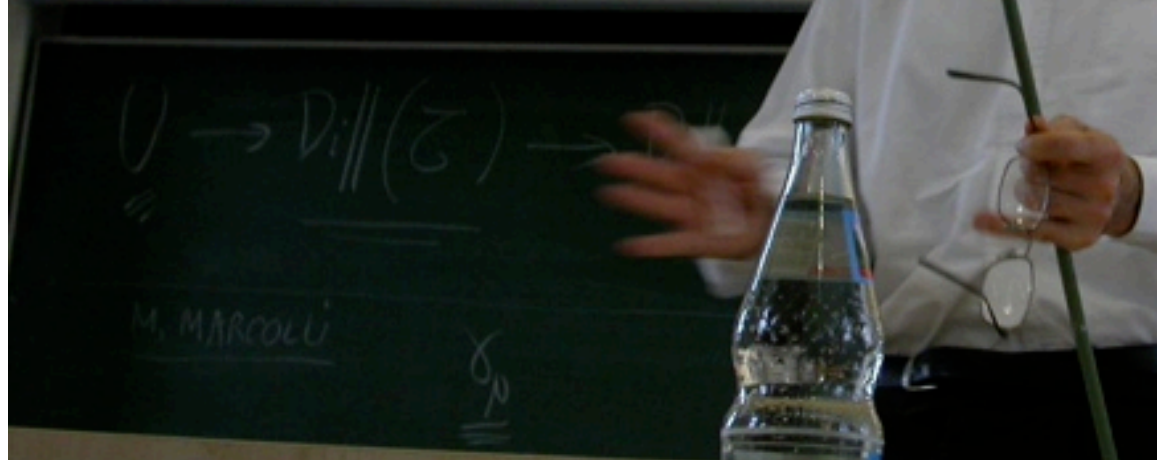
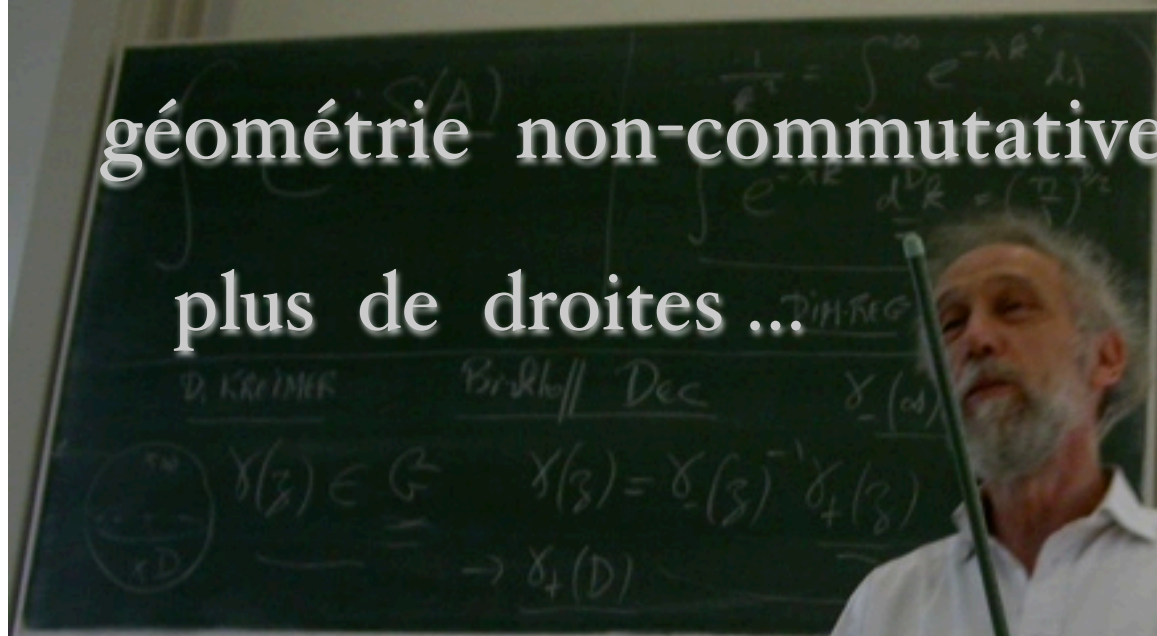
$$\gamma_U(z, v) = T e^{-\frac{1}{z} \int_0^v u^Y(e) \frac{du}{u}} \in U$$

$$\gamma_U(-z, v) = \sum_{n \geq 0} \sum_{k_j > 0} \dots$$

$$\frac{e(-k_1)e(-k_2)\dots e(-k_n)}{k_1(k_1+k_2)\dots(k_1+k_2+\dots+k_n)} v^{\sum k_j}$$

Same coefficients as in

Local Index Formula in NCG (ac + i



géométrie non-commutative ni d'espace ...

Universal Singular Frame

$$\gamma_U(z, v) = \text{Te}^{-\frac{1}{z} \int_0^v u^Y(e) \frac{du}{u}} \in U$$

$$\gamma_U(-z, v) = \sum_{n \geq 0} \sum_{k_j > 0} \frac{e(-k_1)e(-k_2) \cdots e(-k_n)}{k_1(k_1 + k_2) \cdots (k_1 + k_2 + \cdots + k_n)} v^{\sum k_j}$$

$$\frac{e(-k_1)e(-k_2) \cdots e(-k_n)}{k_1(k_1 + k_2) \cdots (k_1 + k_2 + \cdots + k_n)} v^{\sum k_j}$$

Same coefficients as in

Local Index Formula in NCG (ac + 1)

$$\frac{1}{x} = \int_0^\infty e^{-xk} dk$$

D. KREIMER Birkhoff Dec χ

$\chi(\mathbb{Z}) \in G$ $\chi(\mathbb{Z}) = \chi_+$

$\rightarrow \chi_+(D)$

$U \rightarrow \text{Diff}(\mathbb{C})$

M. MARCOLLI χ_n

géométrie non-commutative

AB BA

Universal Singular Frame

$$\gamma_U(z, v) = \text{Tr} e^{-\frac{1}{z} \int_0^v u^Y(e) \frac{du}{u}} \in U$$

$$\gamma_U(-z, v) = \sum_{n \geq 0} \sum_{k_j > 0} \frac{e(-k_1) e(-k_2) \cdots e(-k_n)}{k_1 (k_1 + k_2) \cdots (k_1 + k_2 + \cdots + k_n)} v^{\sum k_j}$$

$$\frac{e(-k_1) e(-k_2) \cdots e(-k_n)}{k_1 (k_1 + k_2) \cdots (k_1 + k_2 + \cdots + k_n)} v^{\sum k_j}$$

Same coefficients as in

Local Index Formula in NCG (ac

D. KREIMER

Brakhoff Dec

γ



$\gamma(\gamma) \in G$

$\gamma(\gamma) = \gamma(\gamma^{-1})$

$\rightarrow \gamma_+(D)$

$U \rightarrow \text{Diff}(\mathbb{C})$

M. MARCOLLI

γ_n

geometría no-conmutativa

AB BA

et le temps?

Universal Singular Frame

$$\gamma_U(z, v) = \text{Tr} e^{-\frac{1}{z} \int_0^v u^Y(e) \frac{du}{u}} \in U$$

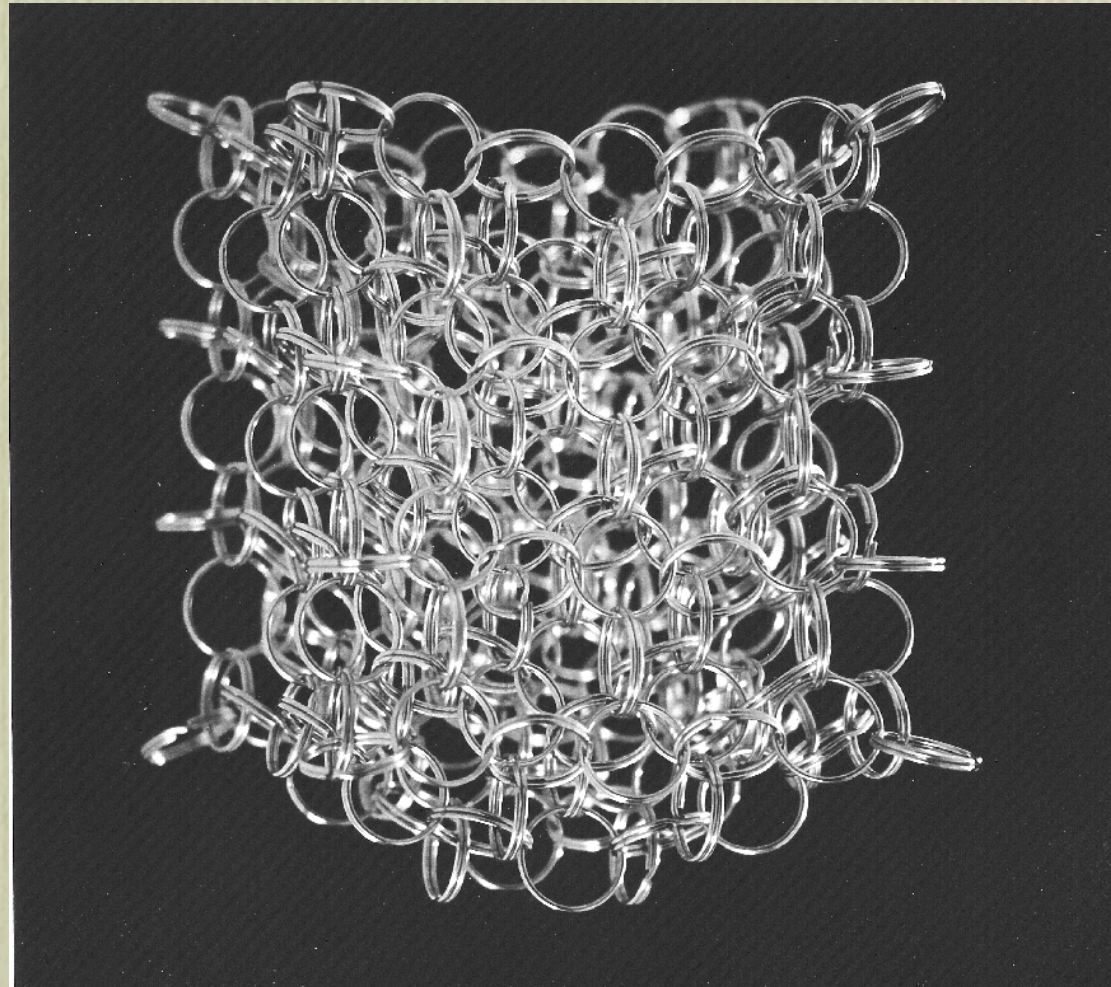
$$\gamma_U(-z, v) = \sum_{\eta \geq 0} \sum_{k_j > 0} \dots$$

$$\frac{e(-k_1)e(-k_2)\cdots e(-k_n)}{k_1(k_1+k_2)\cdots(k_1+k_2+\cdots+k_n)} v^\Sigma$$

Same coefficients as in

Local Index Formula in NCG (ac +

gravitation quantique à boucles



gravitation quantique à boucles



Carlo Rovelli

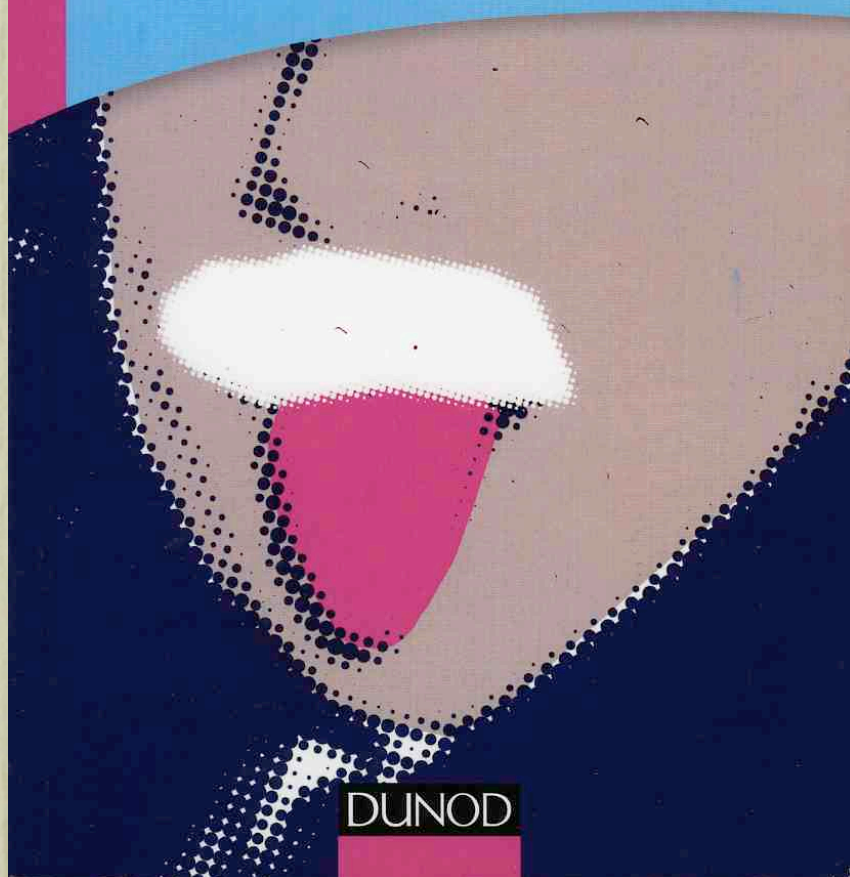
QUAI DES SCIENCES

CARLO ROVELLI

ET SI LE TEMPS N'EXISTAIT PAS ?

UN PEU DE SCIENCE
SUBVERSIVE

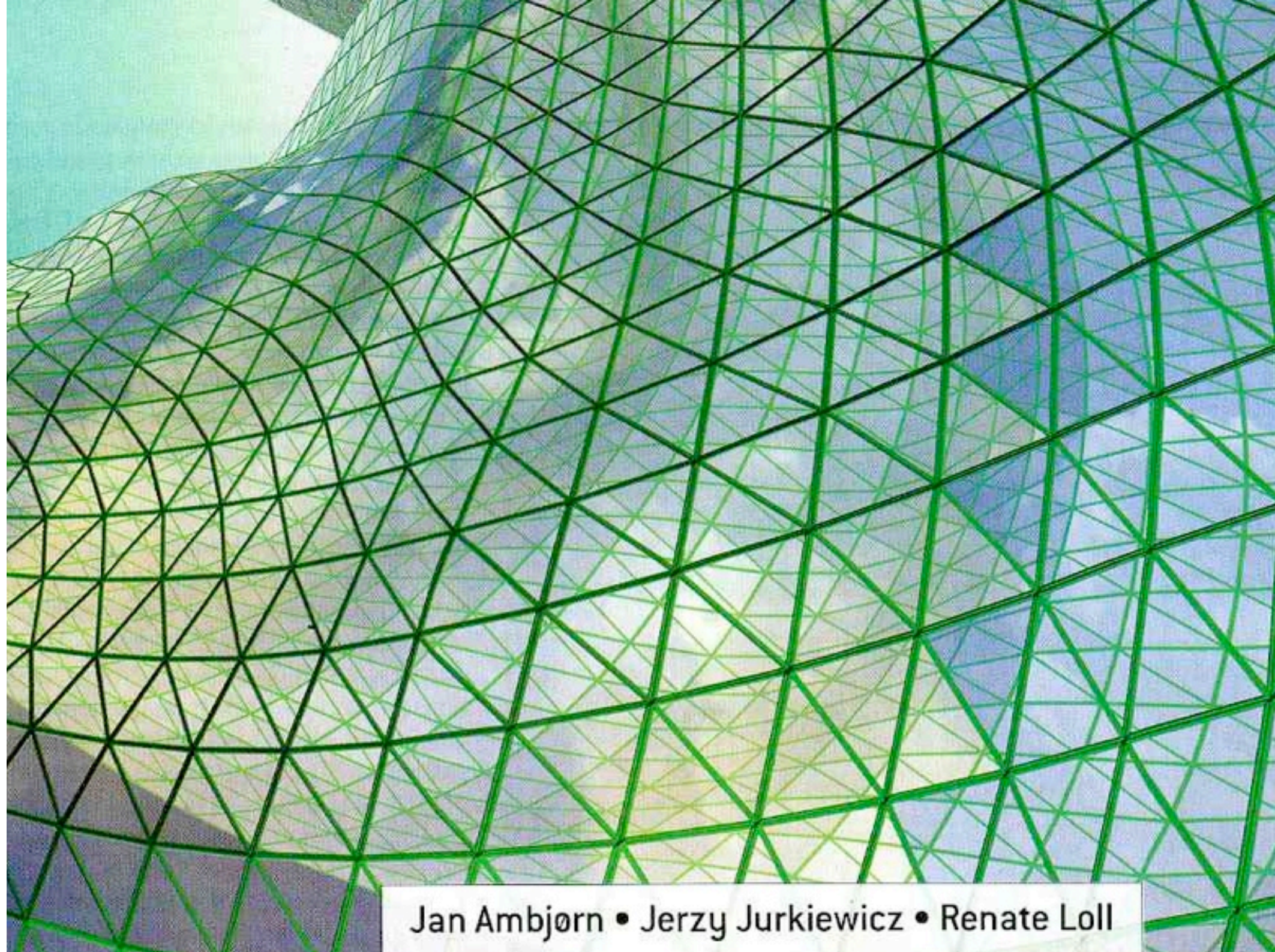
DUNOD





gravitation quantique

triangulations causales dynamiques



Jan Ambjørn • Jerzy Jurkiewicz • Renate Loll

pourlascience.com

POUR LA SCIENCE

Septembre 2008

Édition française de Scientific American

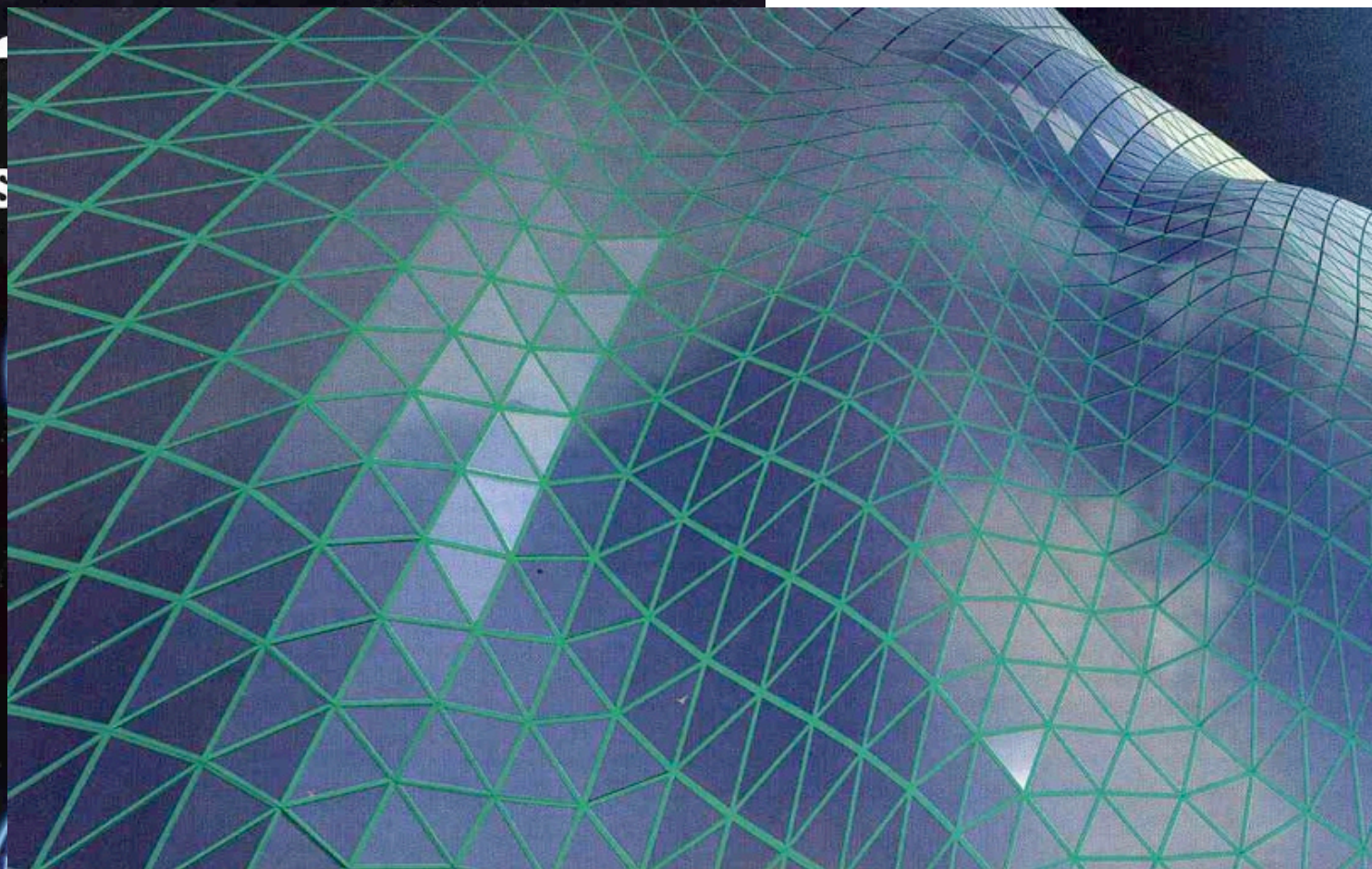
Le ver Des algues

- L'Univers quantique auto-organisé
- Que s'est-il passé à Tougouska il y a 100 ans ?
- Comment détecter les images truquées
- D'où viennent les larves ?

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L'univers quantique auto-organisé

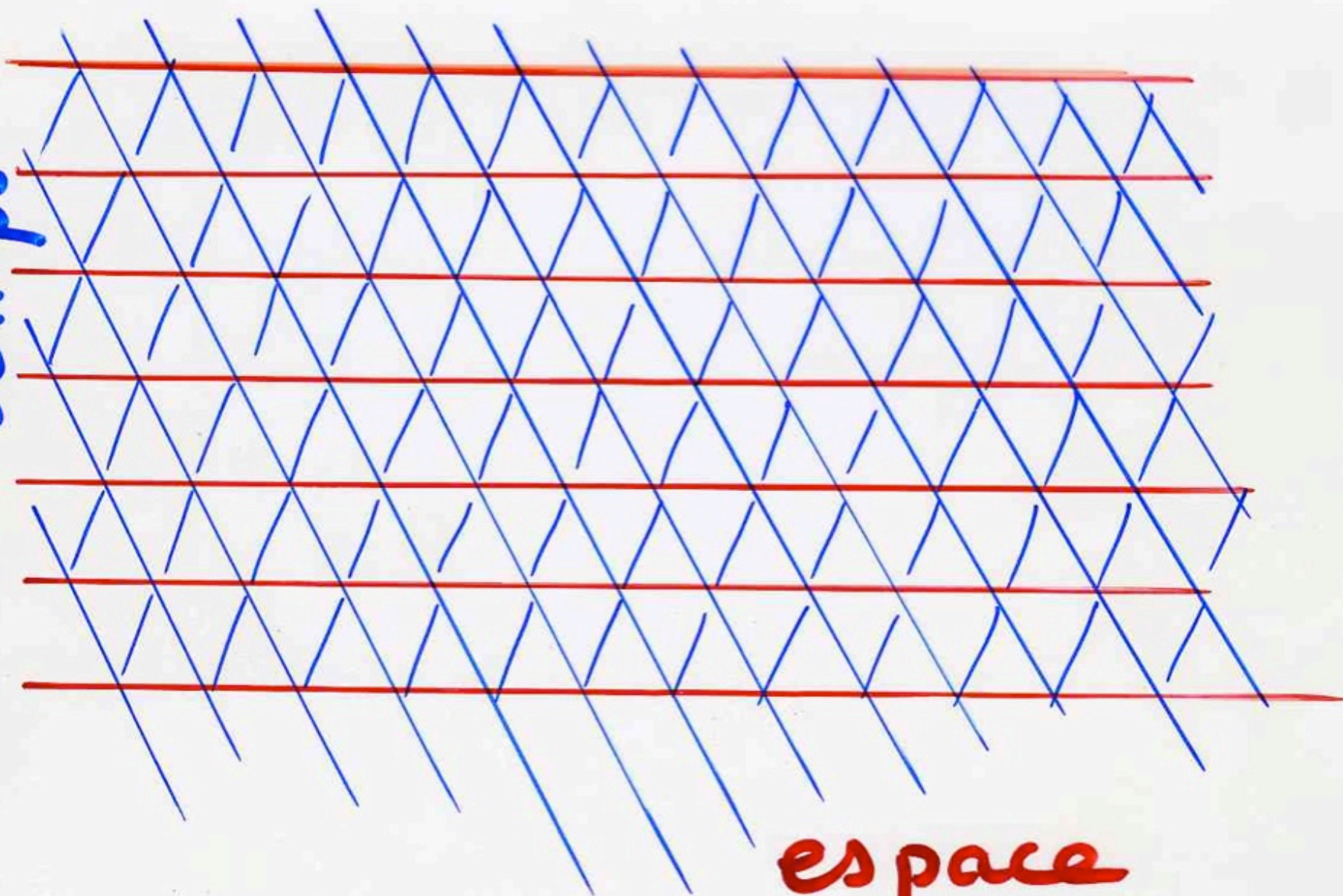
Lorentzian
quantum
gravity

+++
space
-
time

2D

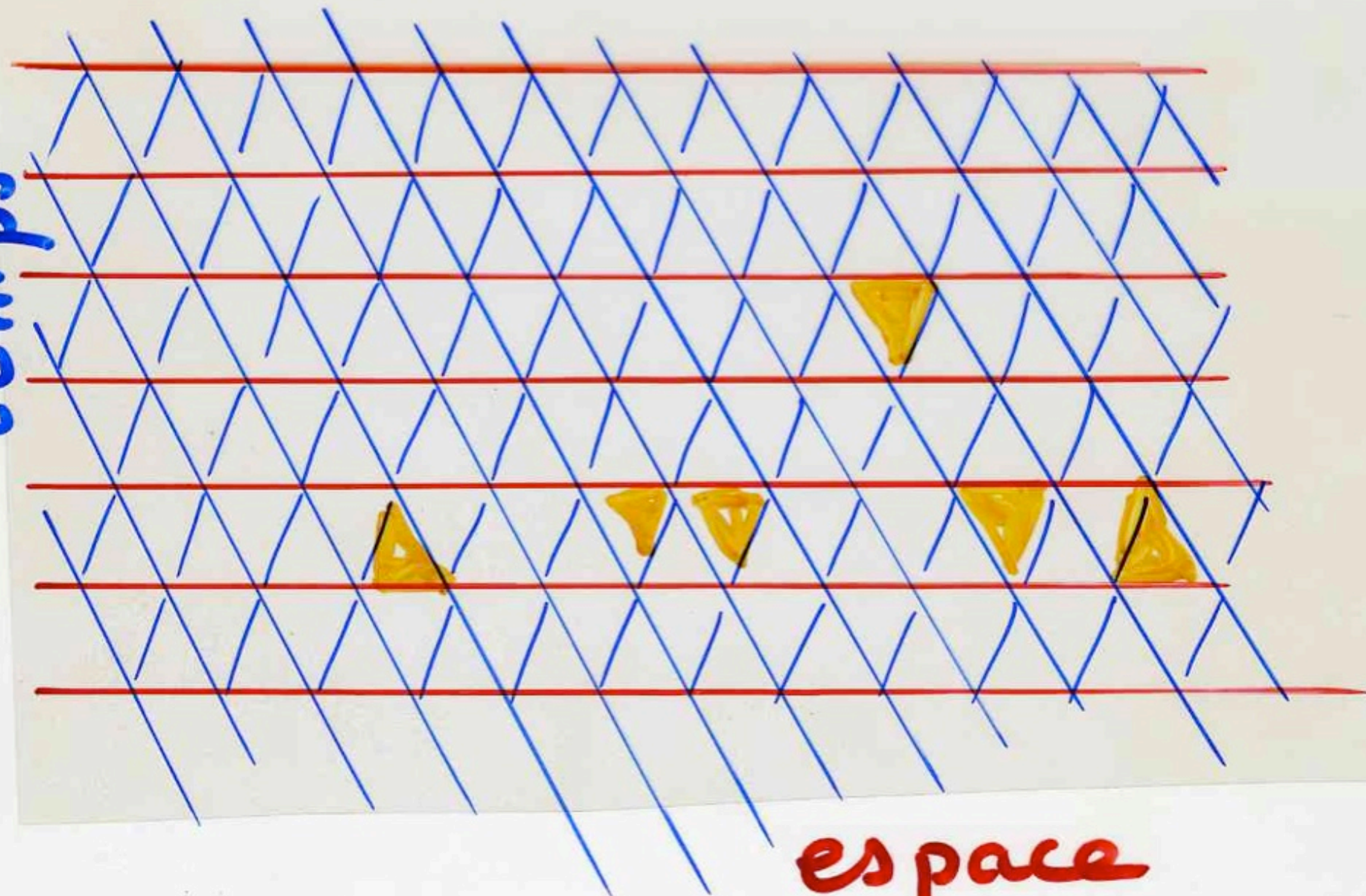
1 + 1
space time

temps



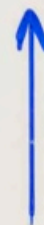
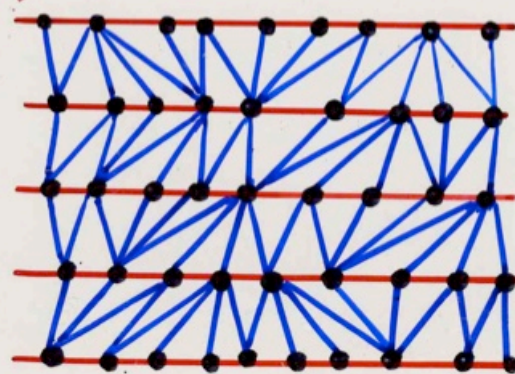
espace

temps

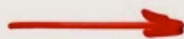


espace

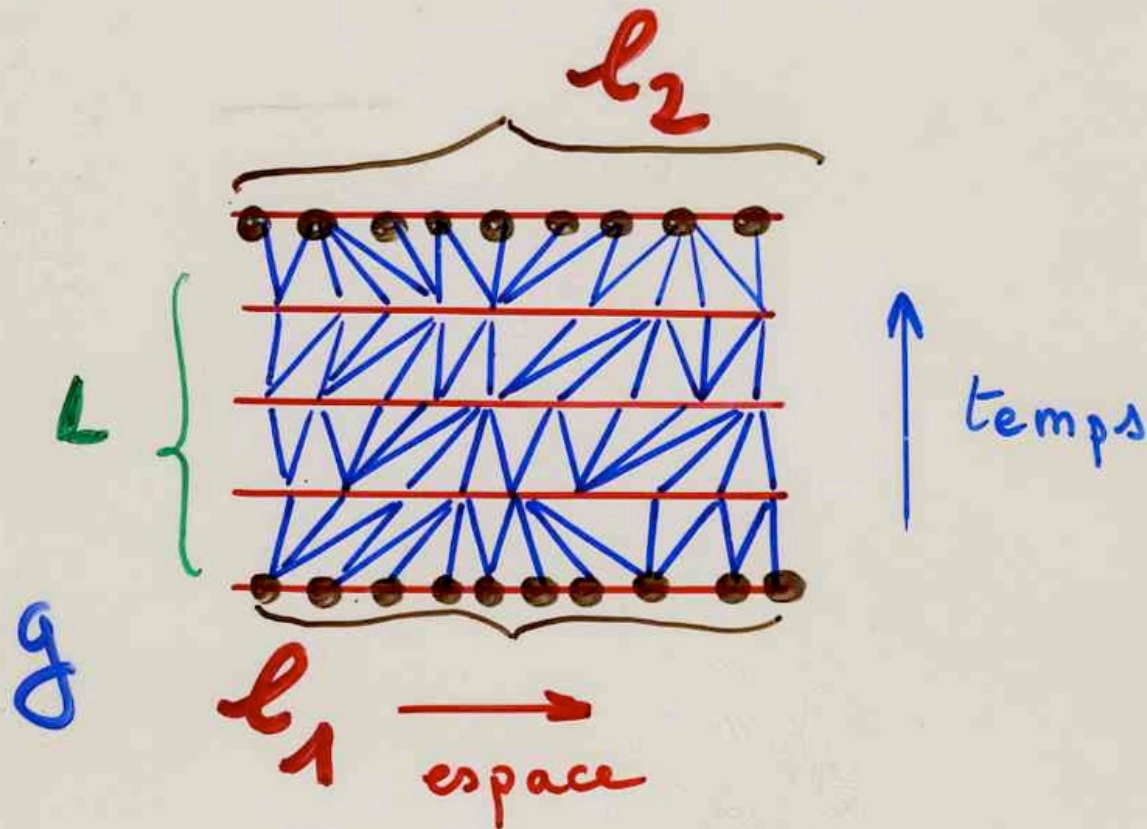
Lorentzian triangulation



temps



espace



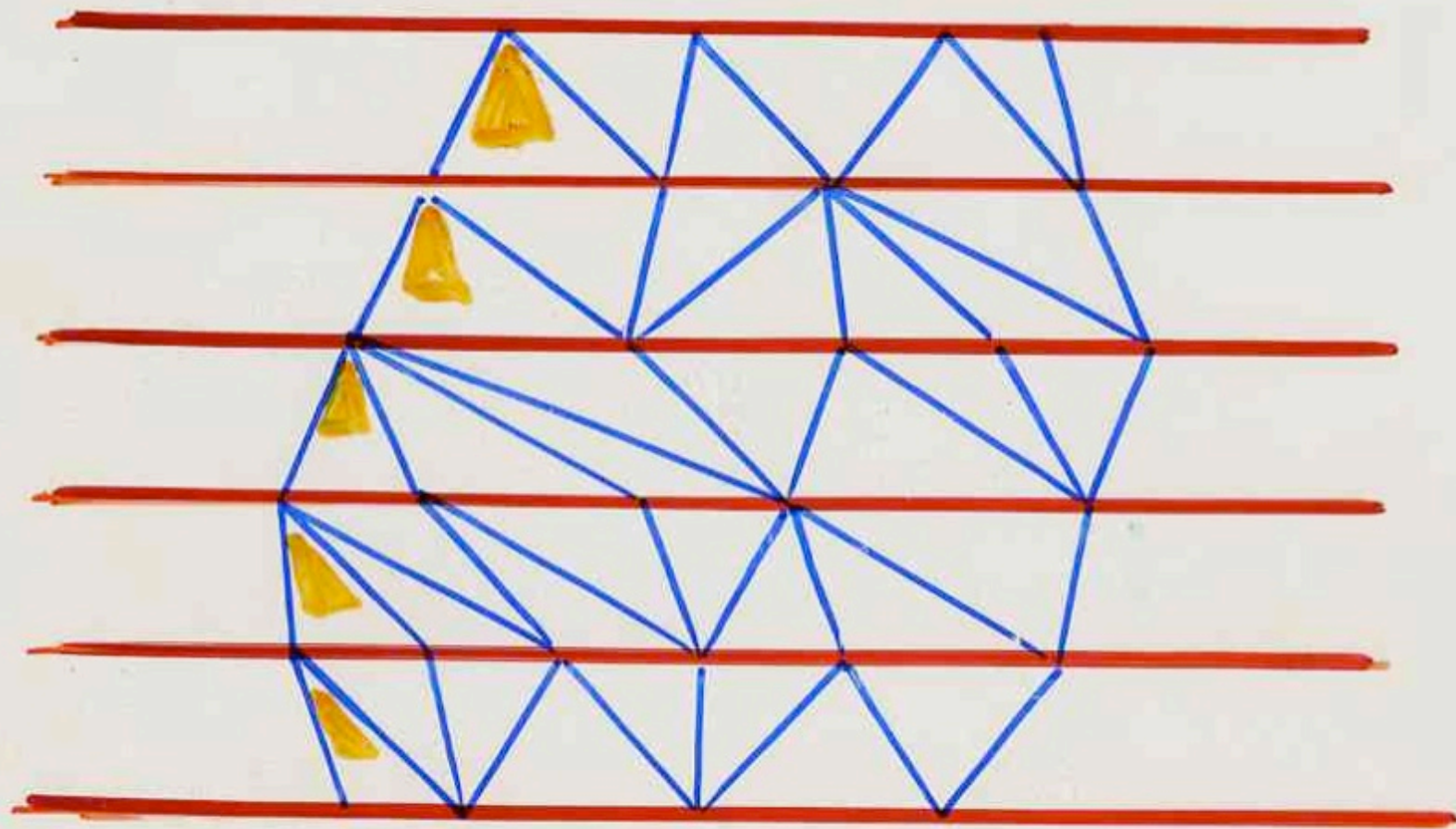
Path integral amplitude
for the propagation from
geometry l_1 to l_2

Catalan

number



$$C_n = \frac{1}{(n+1)} \binom{2n}{n}$$



métamorphose:

triangulation

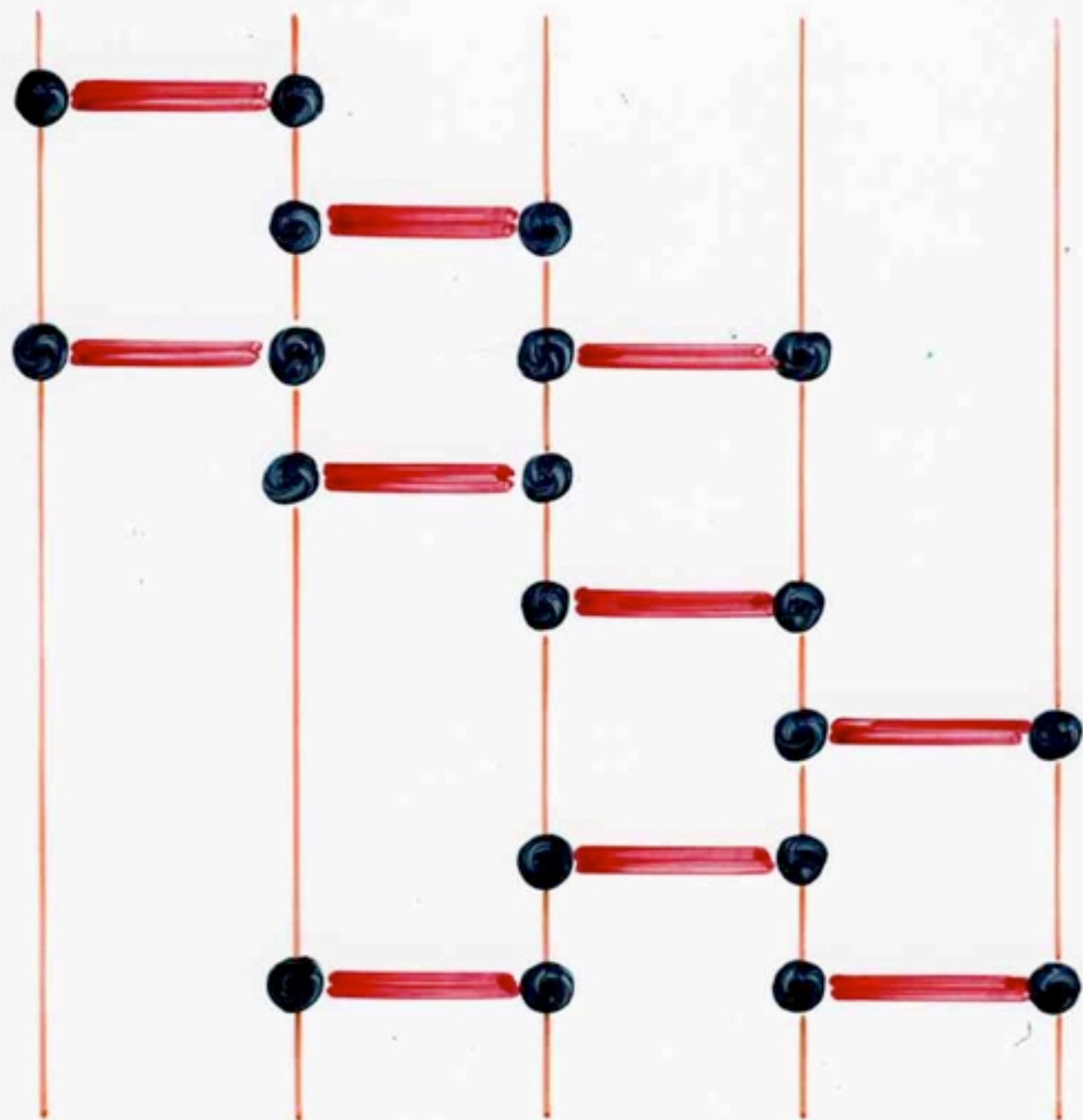
arbres binaire

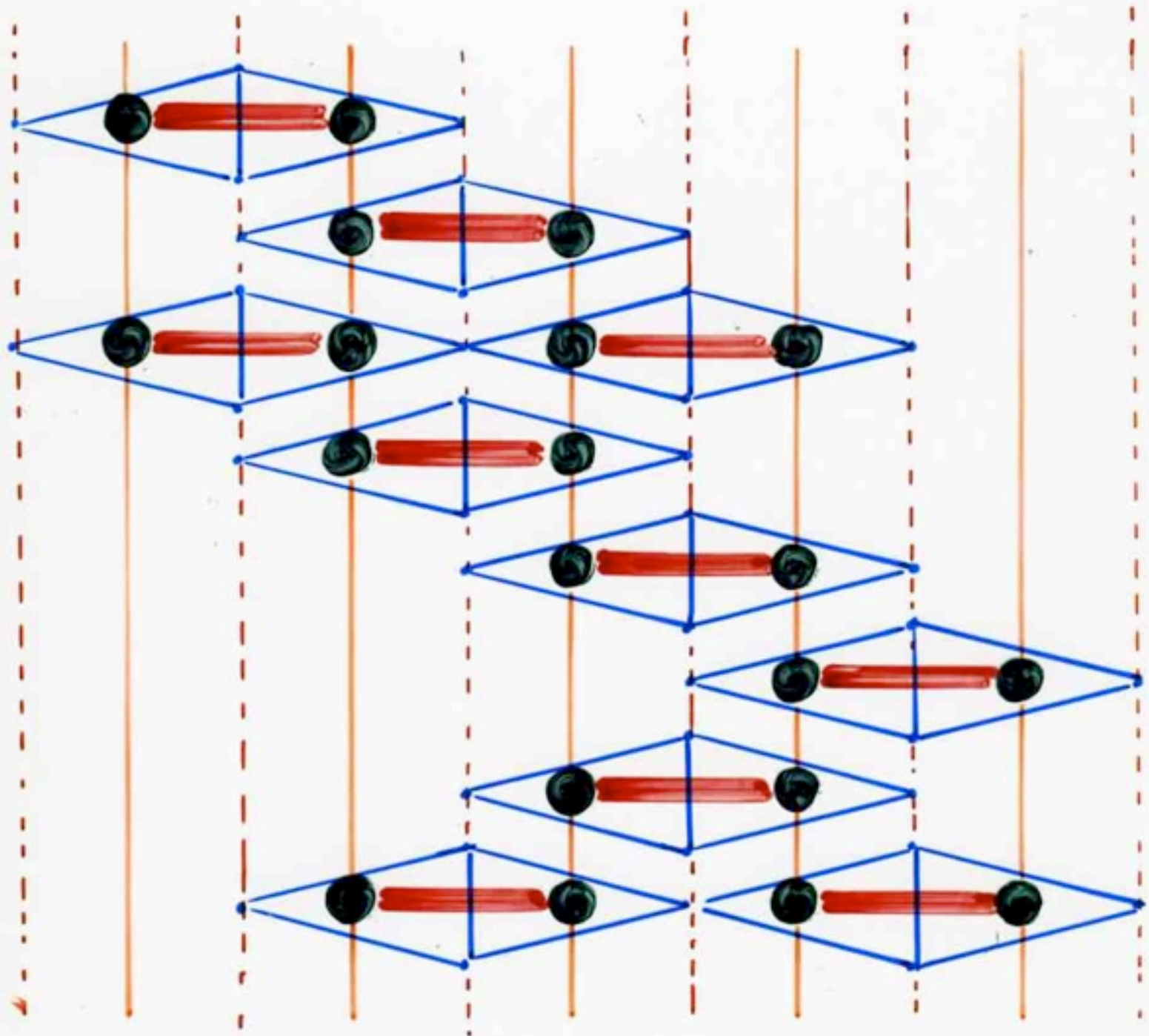
chemin de Dyck

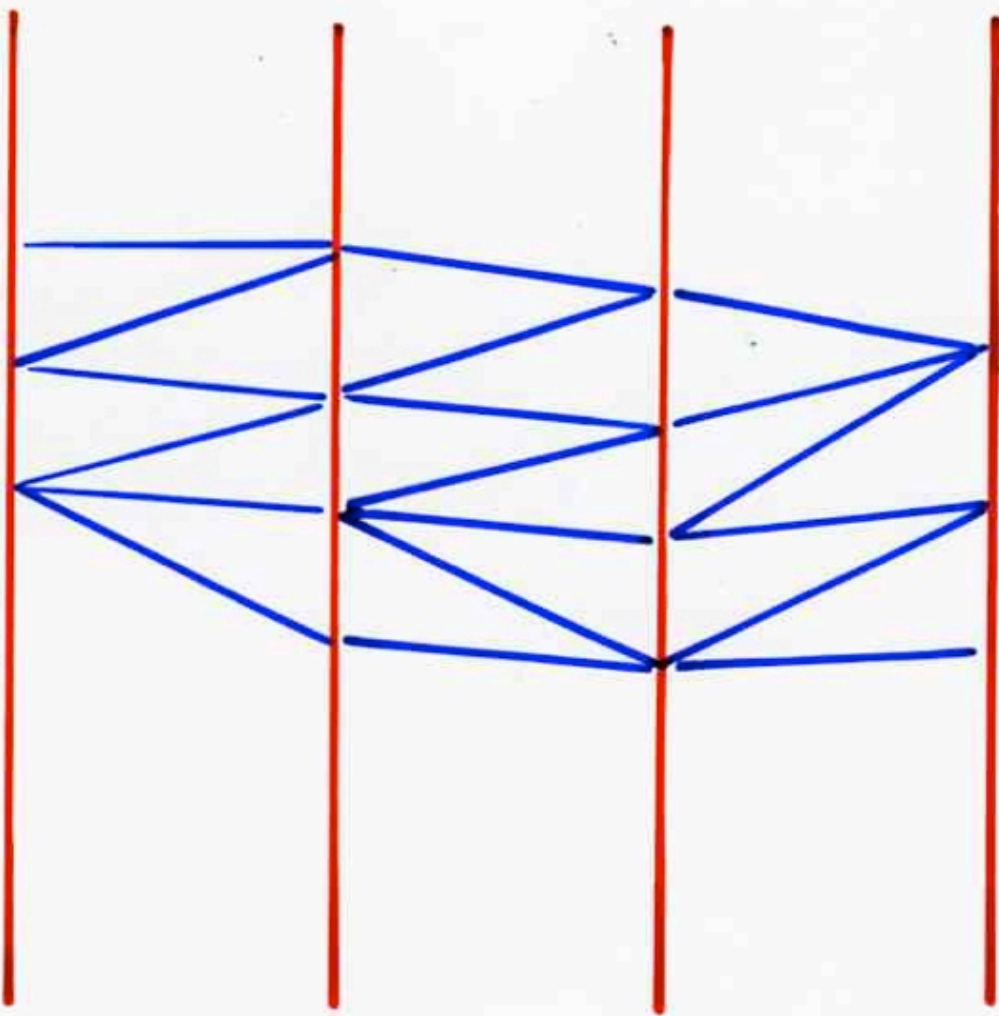
pyramide de dominos

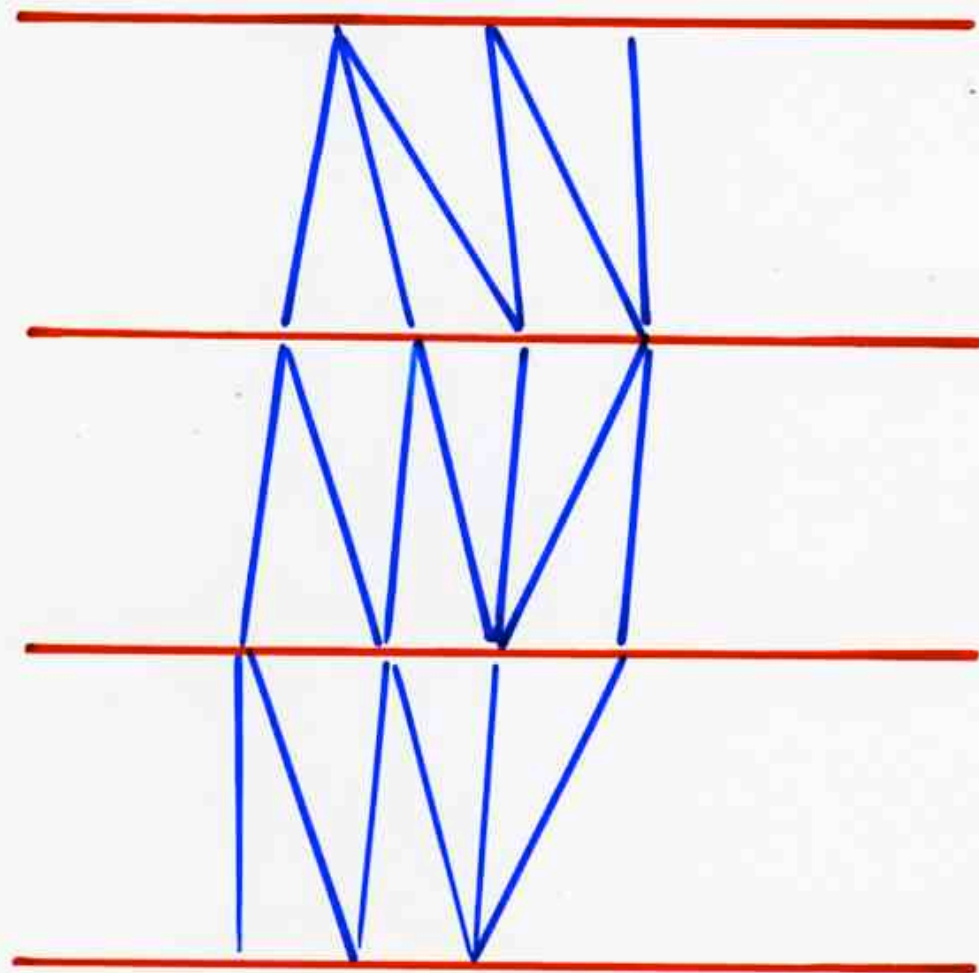
triangulation Lorentzienne

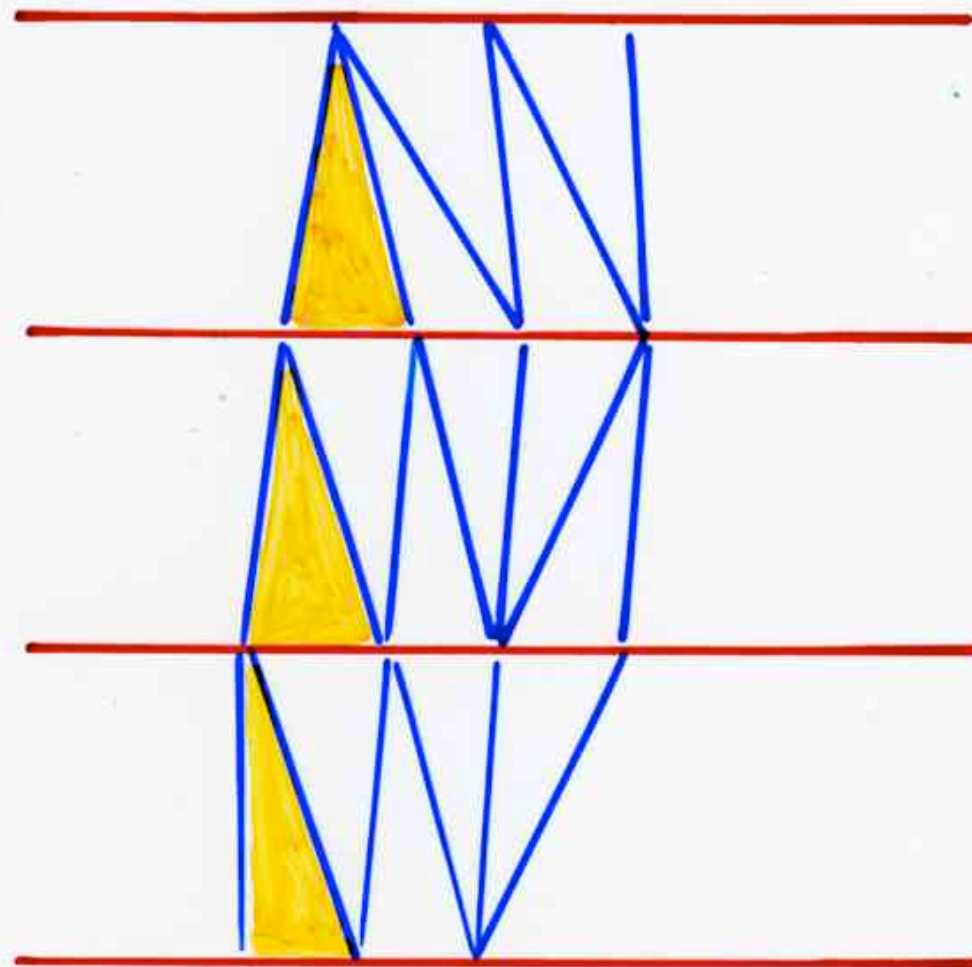
Des empilements aux
triangulations Lorentziennes





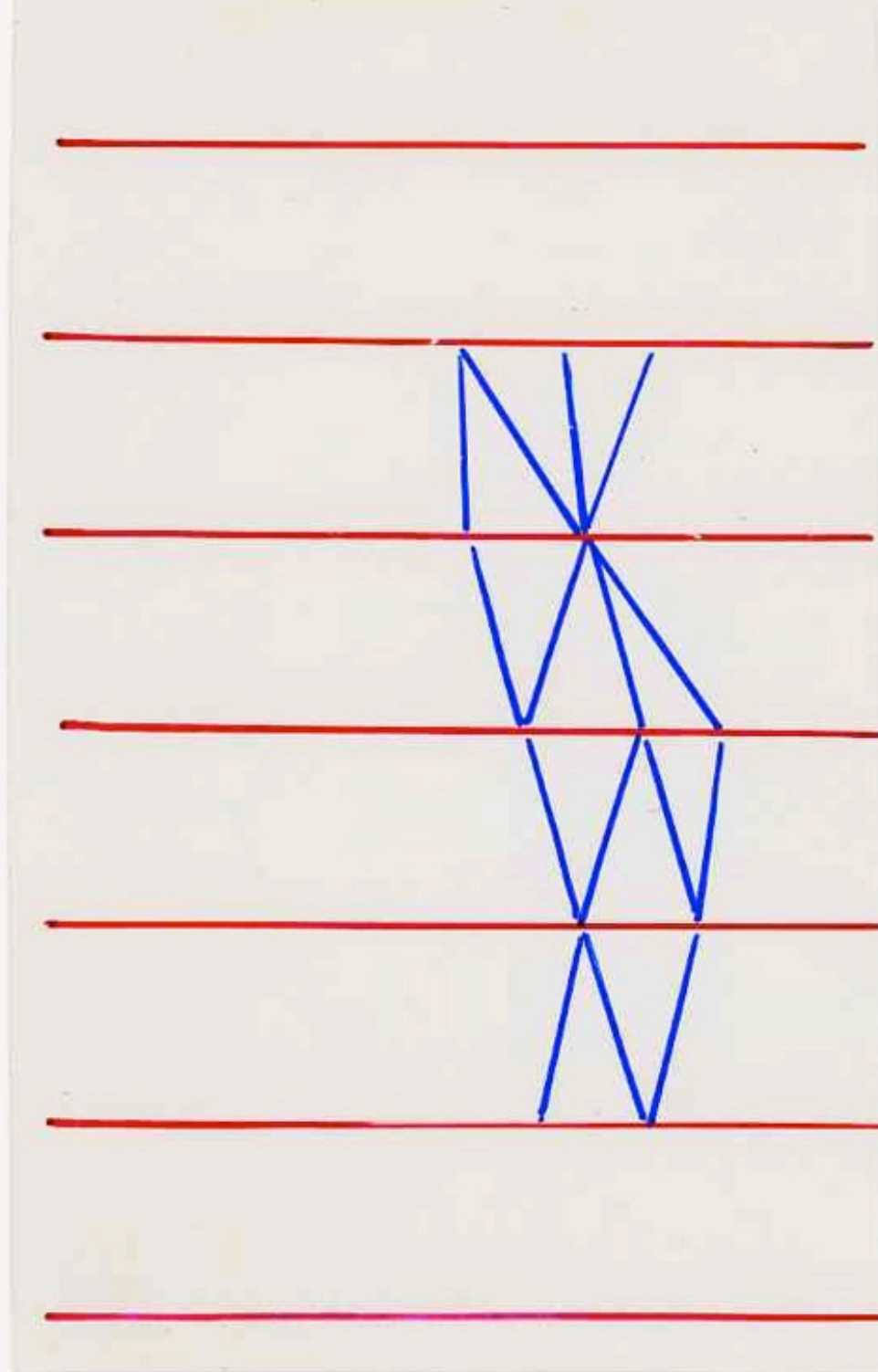


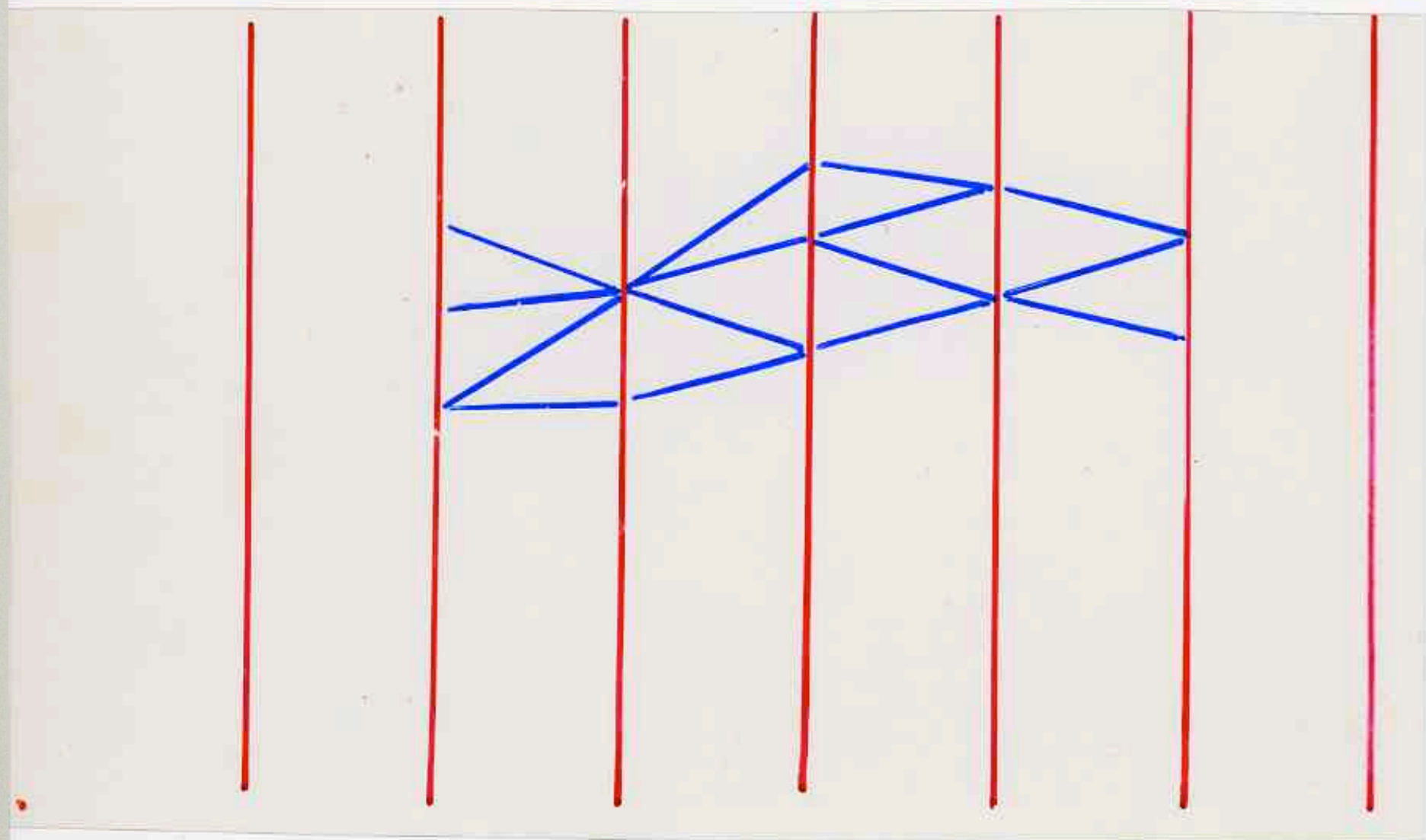


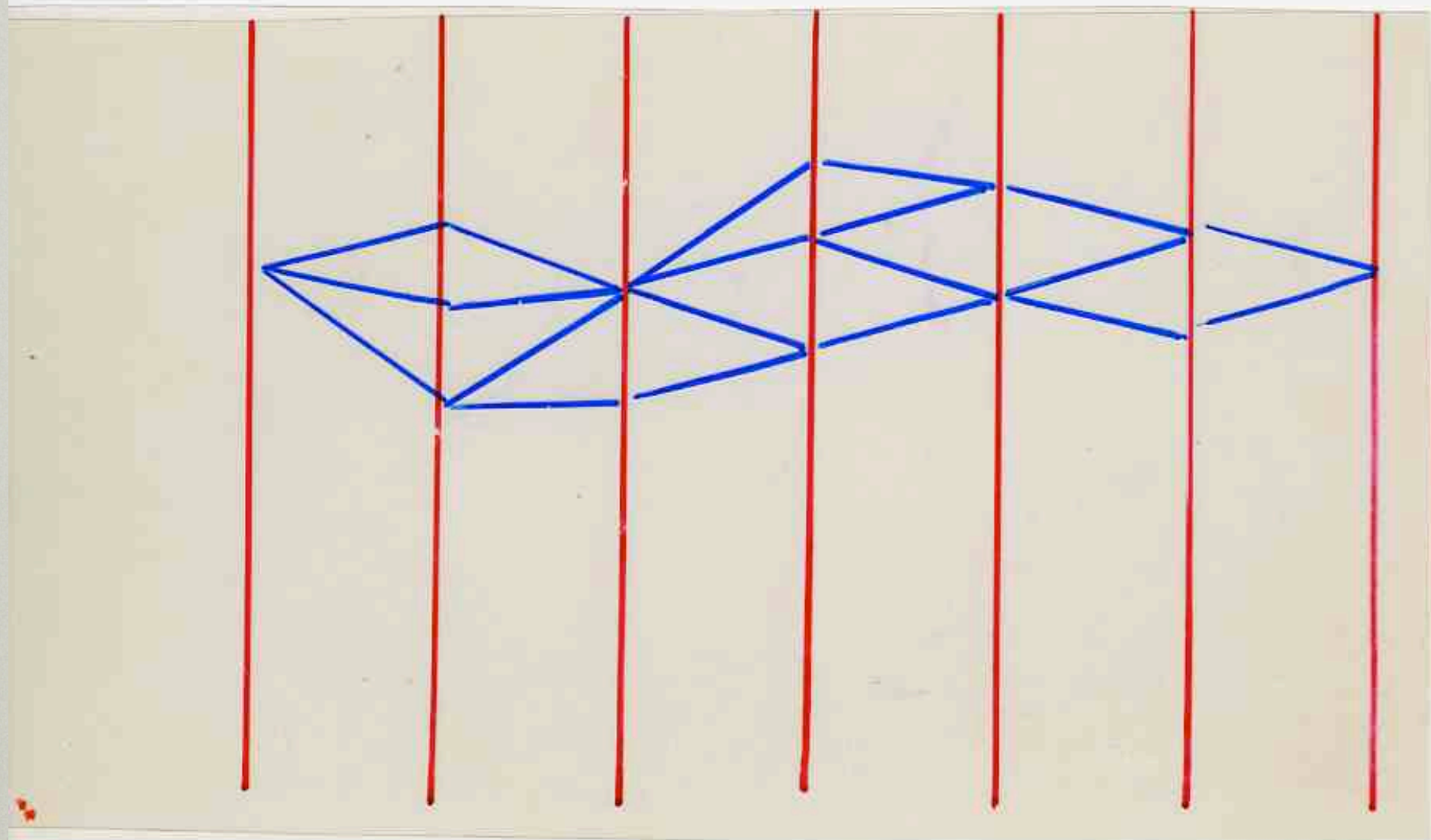


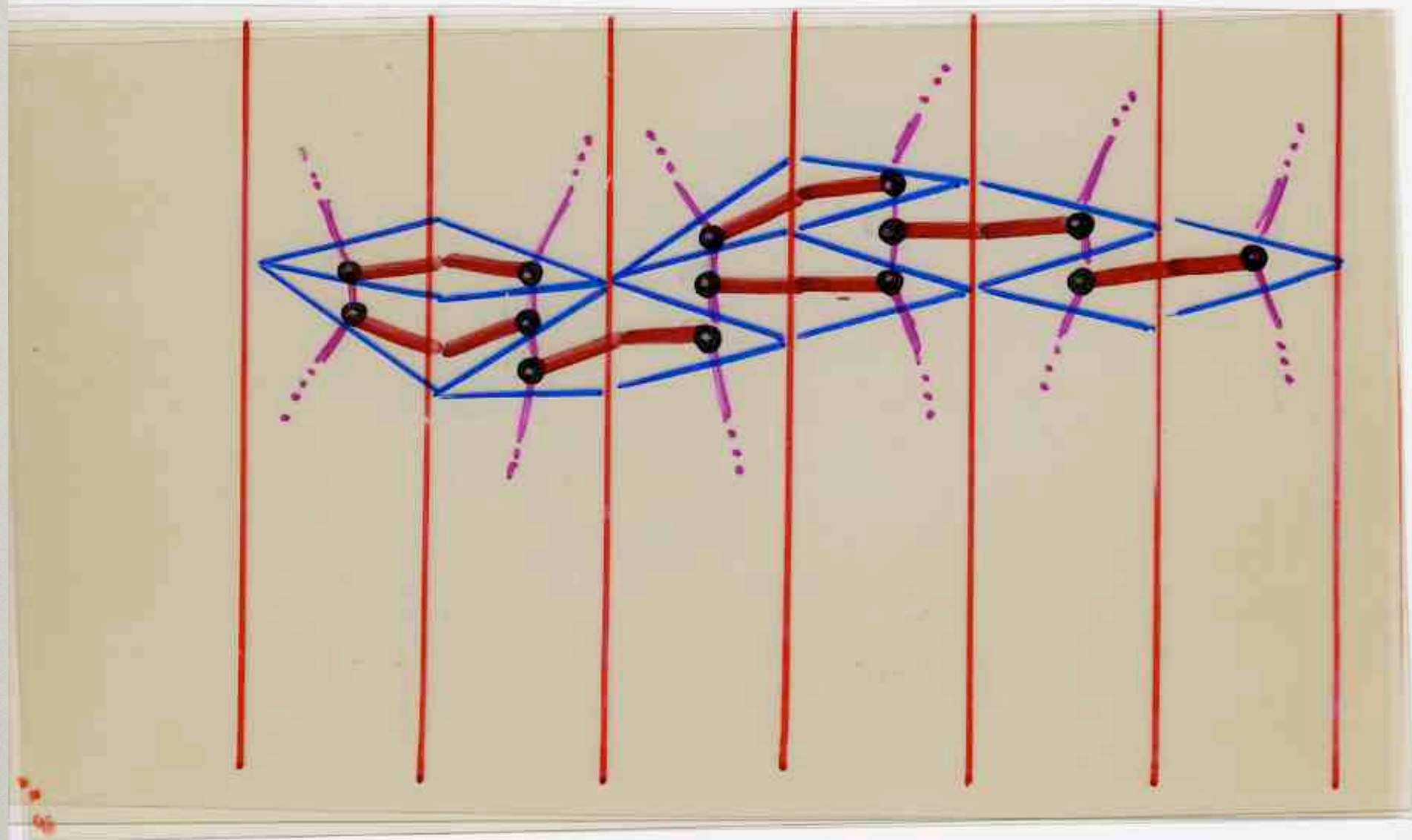
bijection réciproque:

des triangulations Lorentziennes
aux empilements

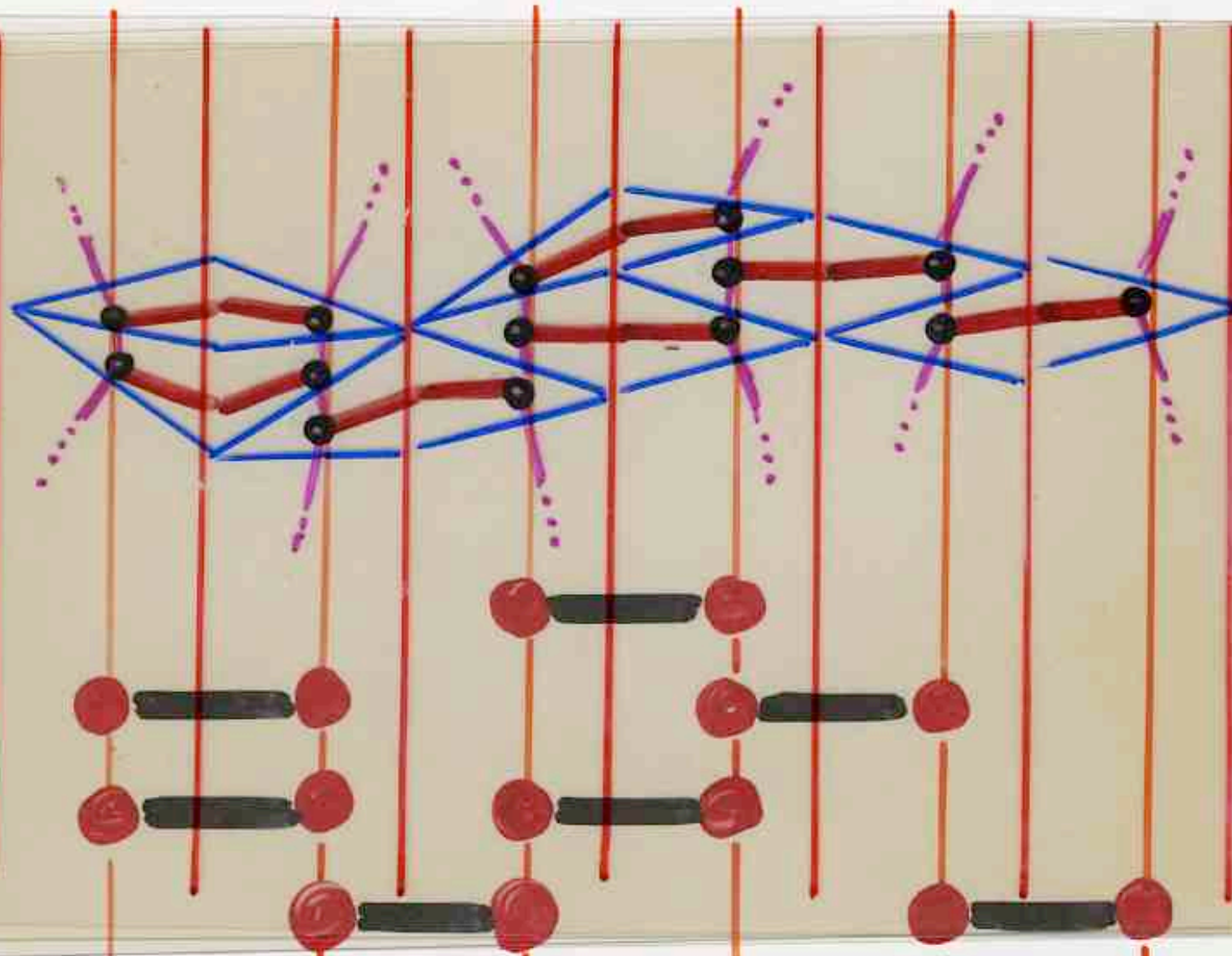


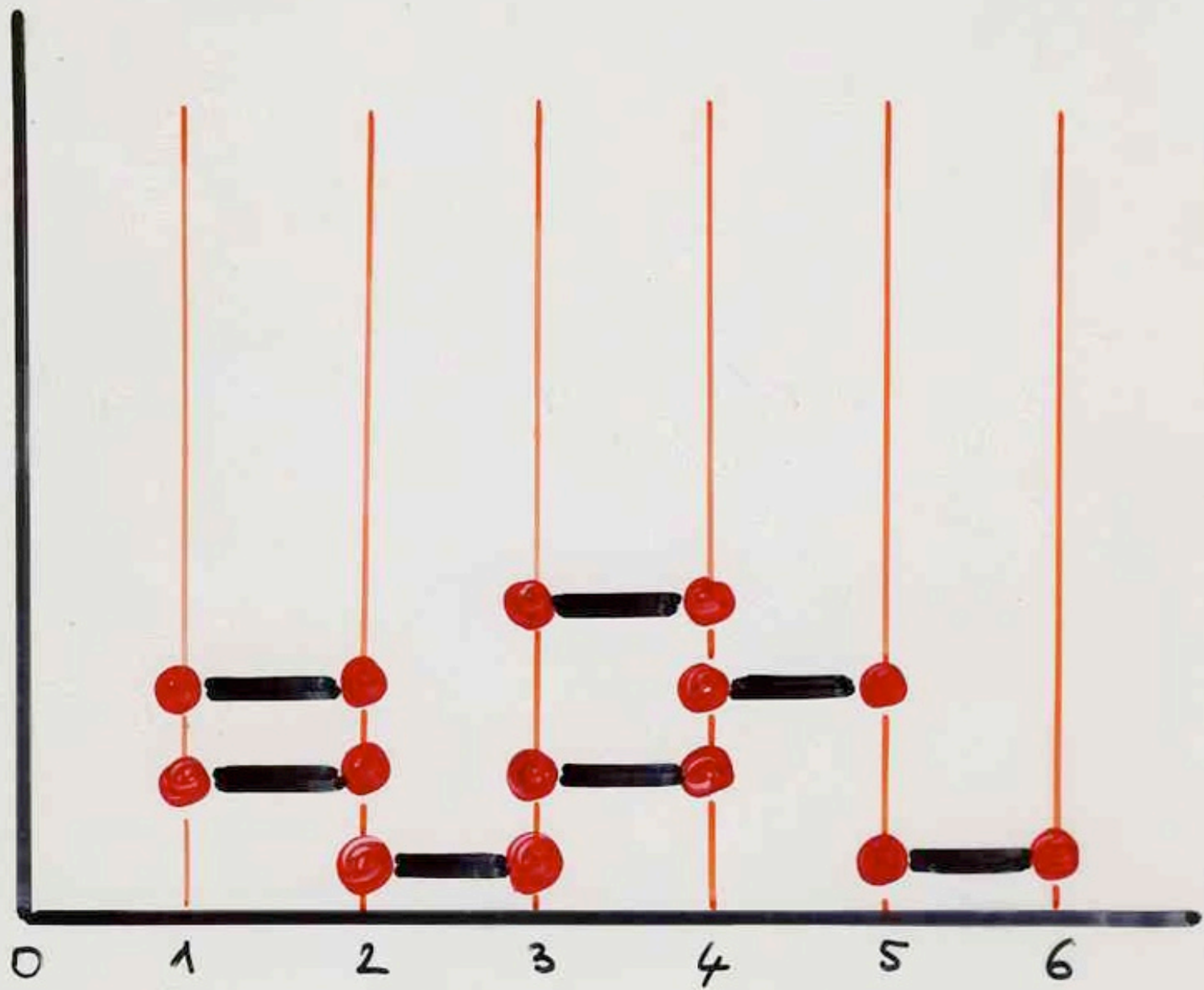


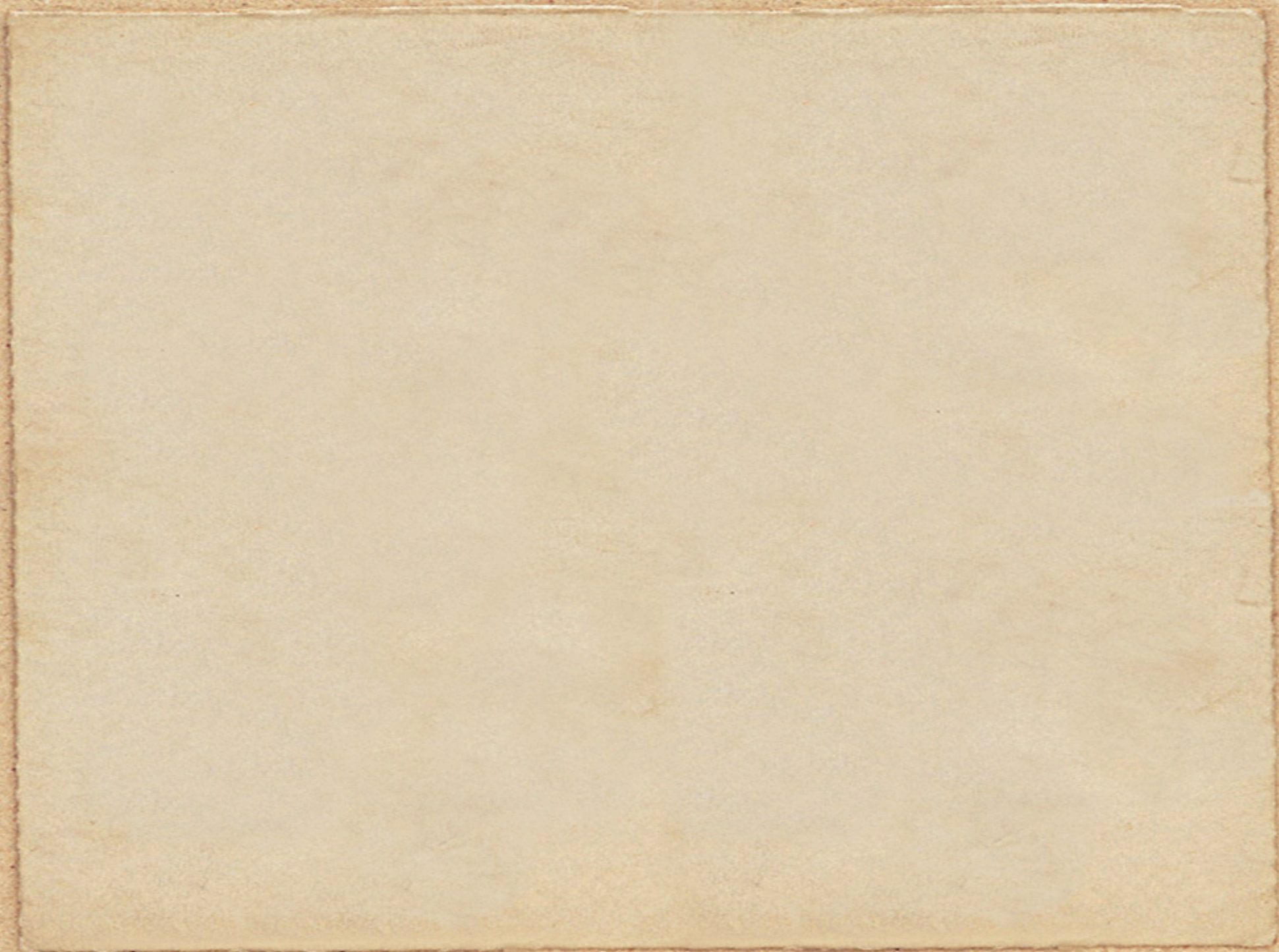


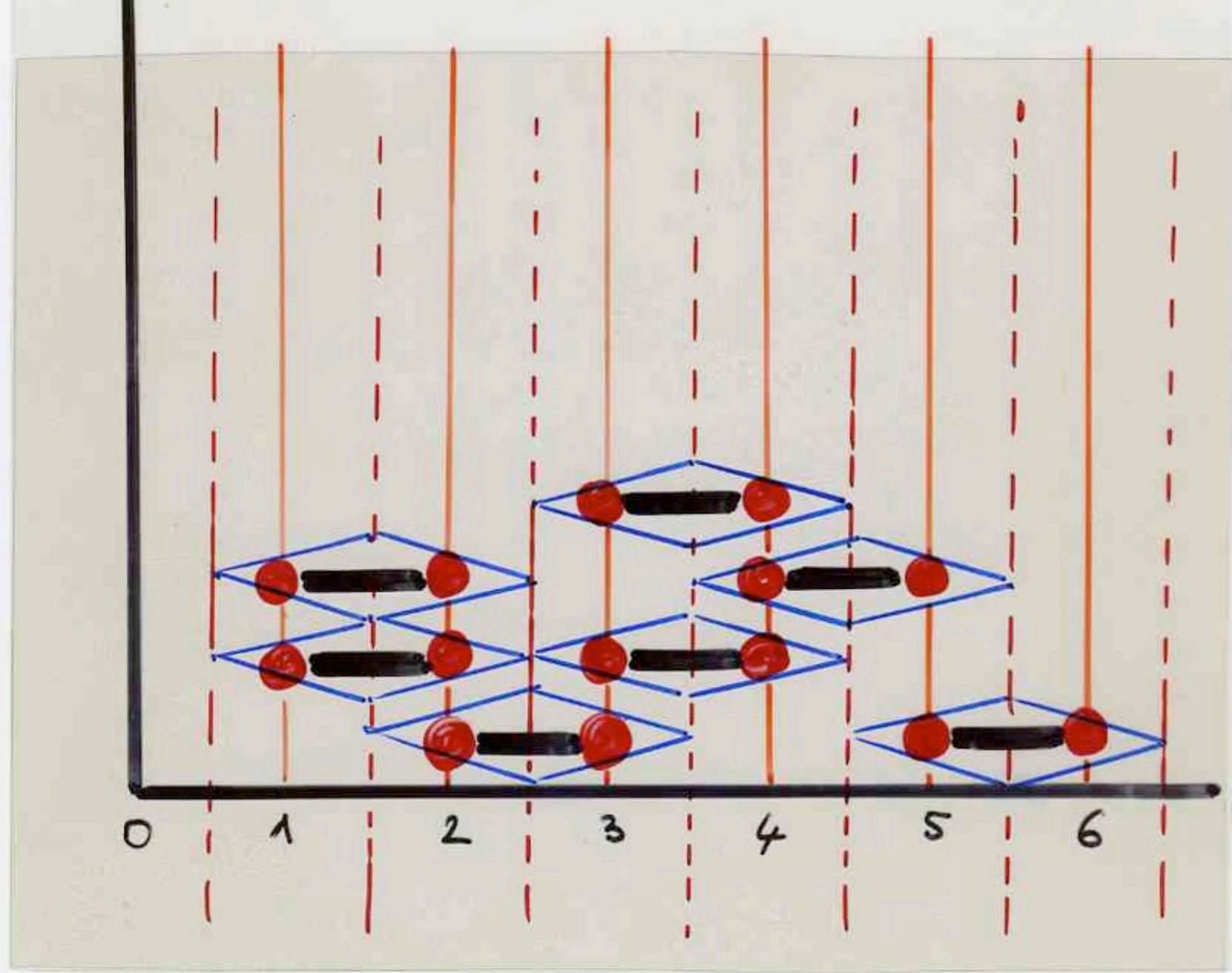


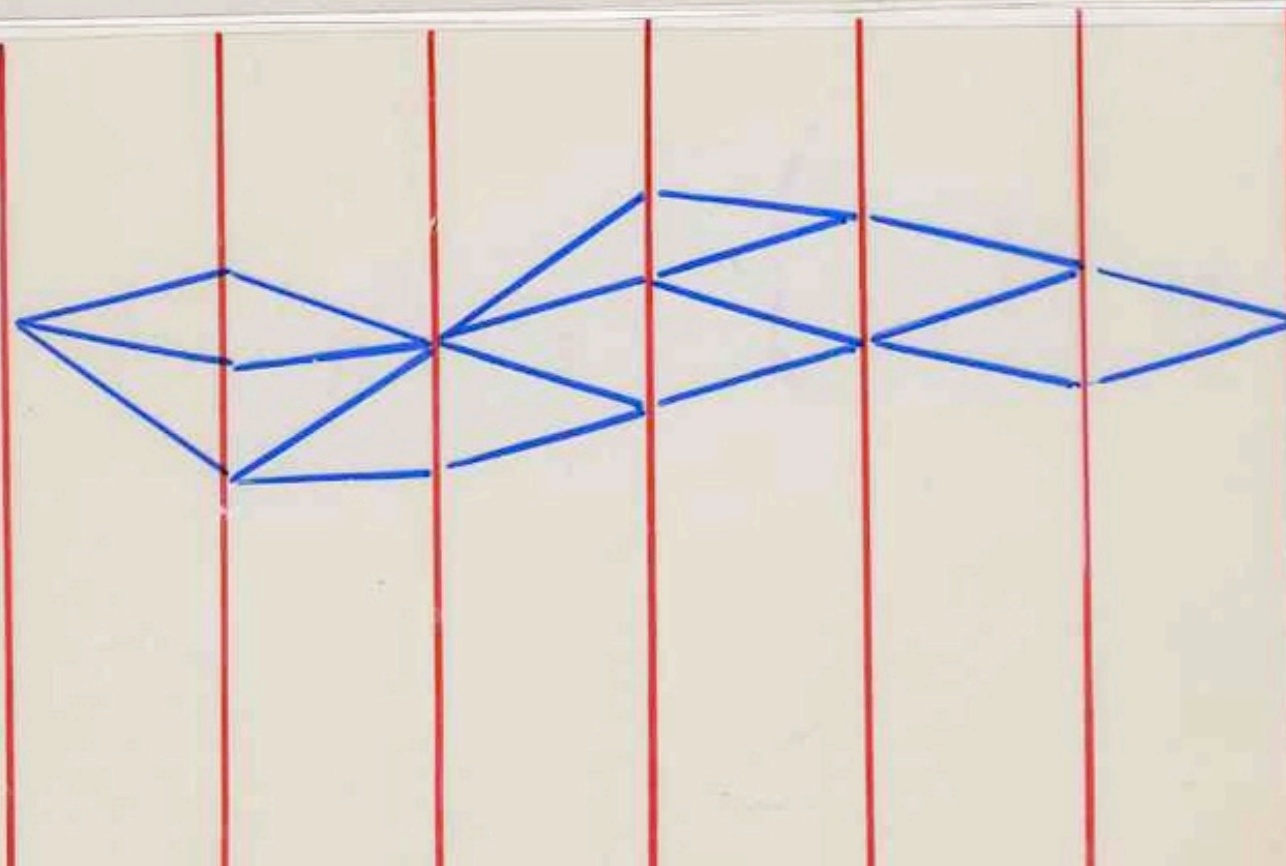
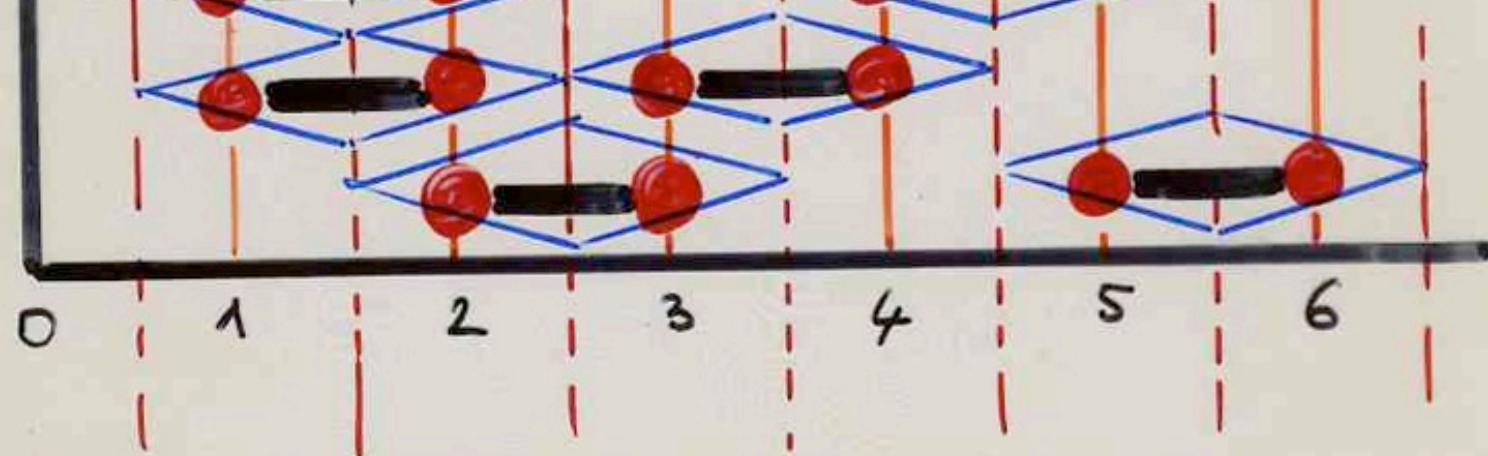
0 1 2 3 4 5 6

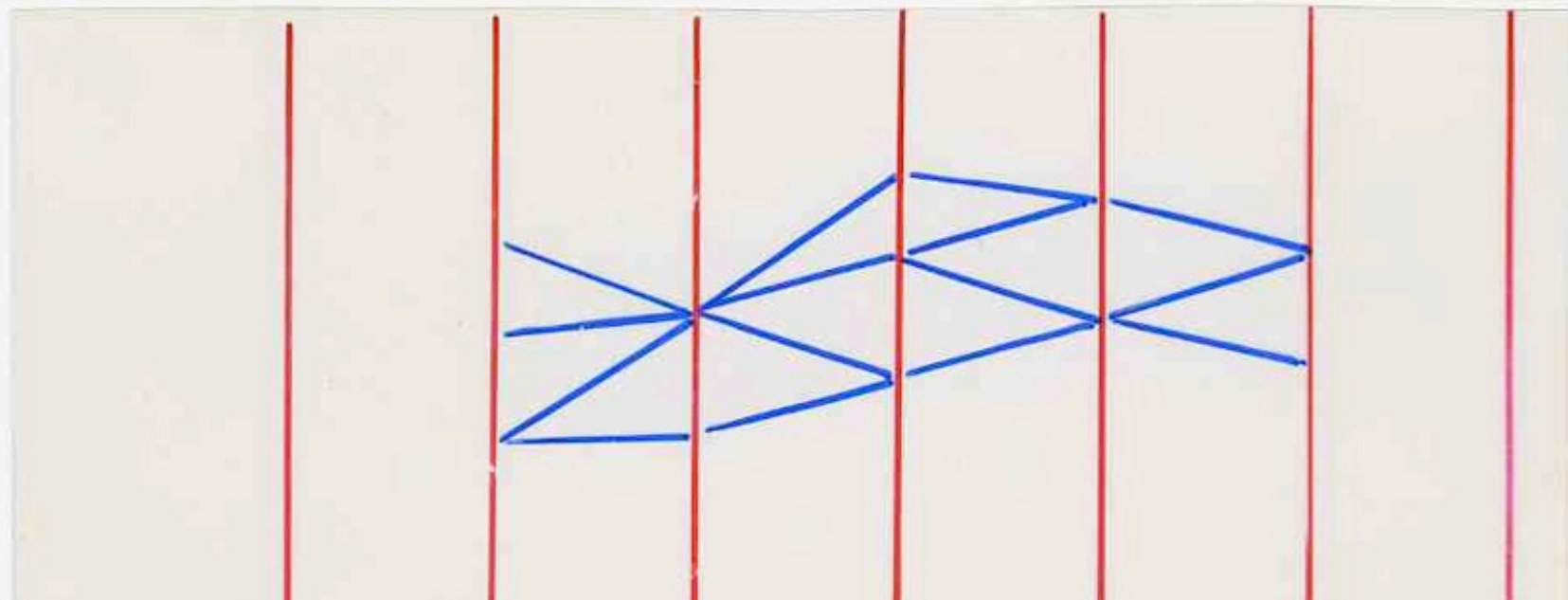
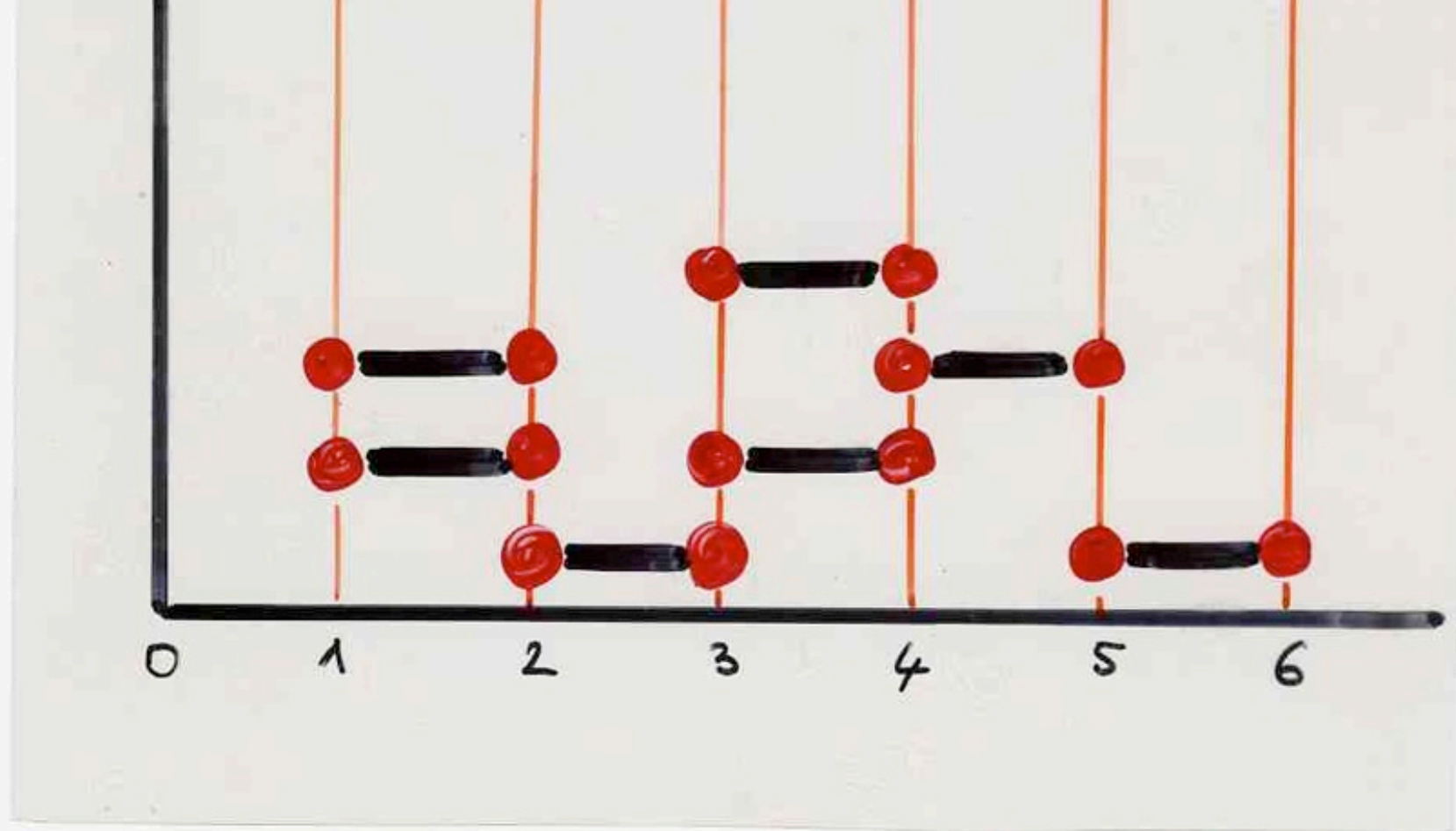


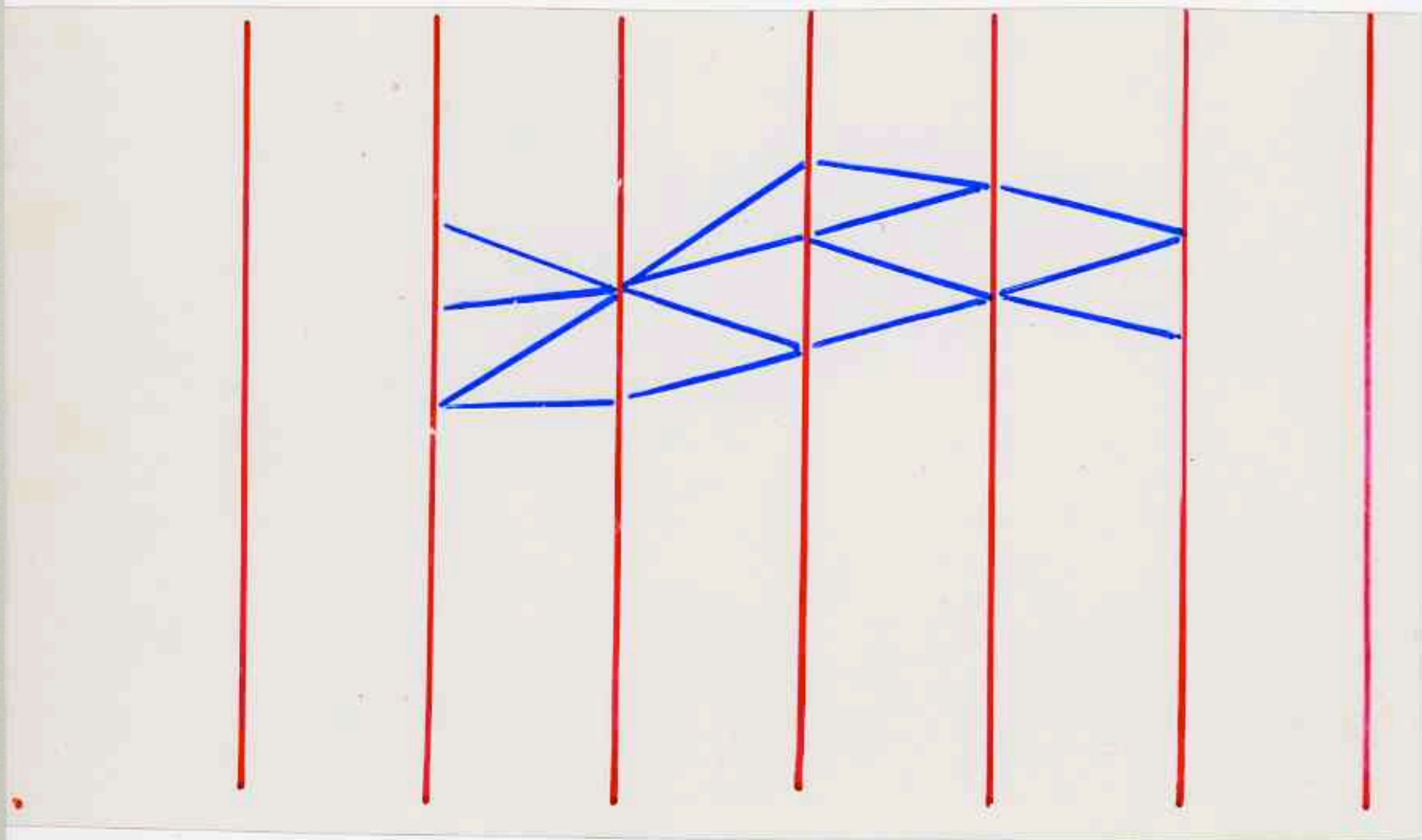


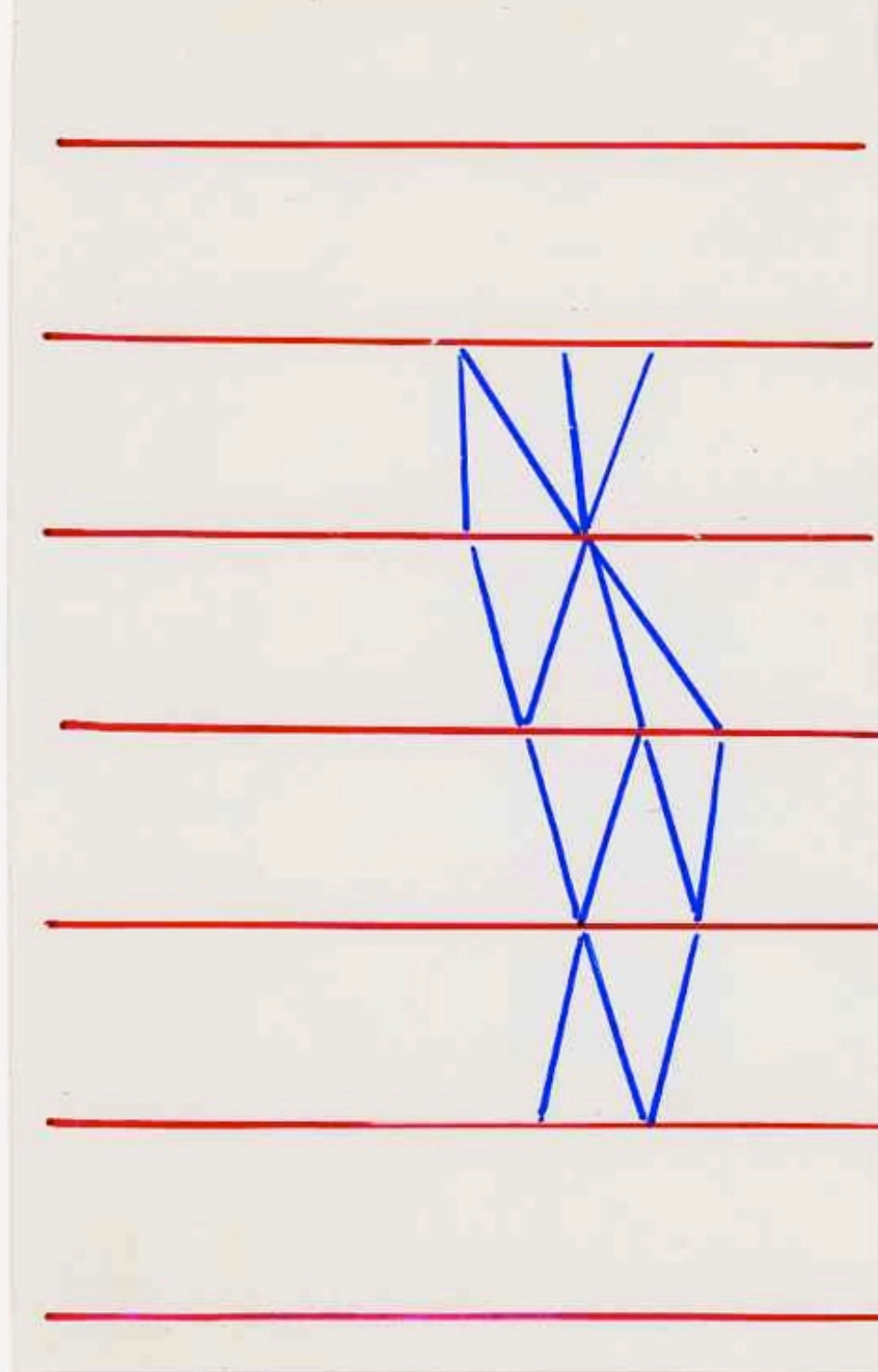




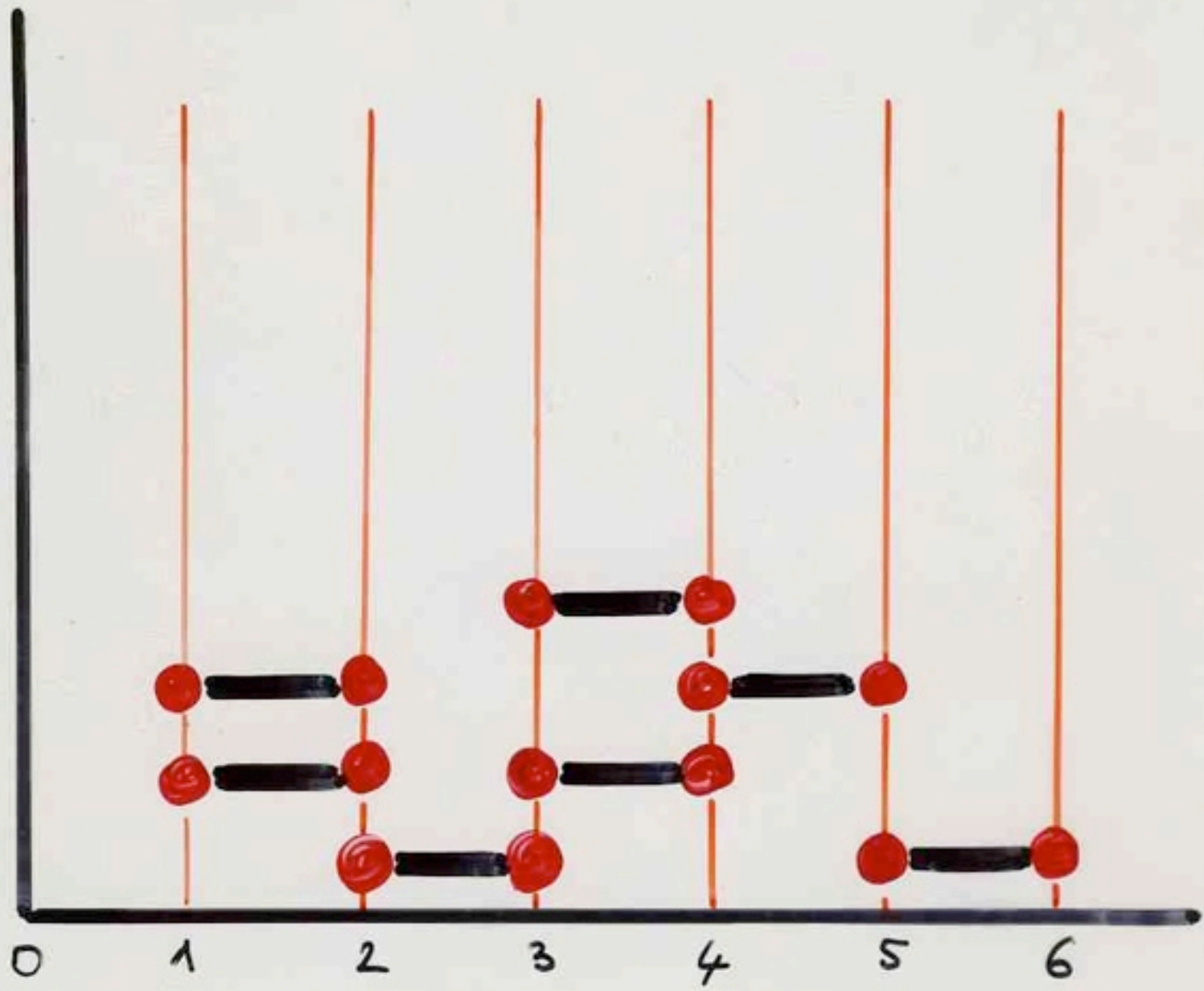


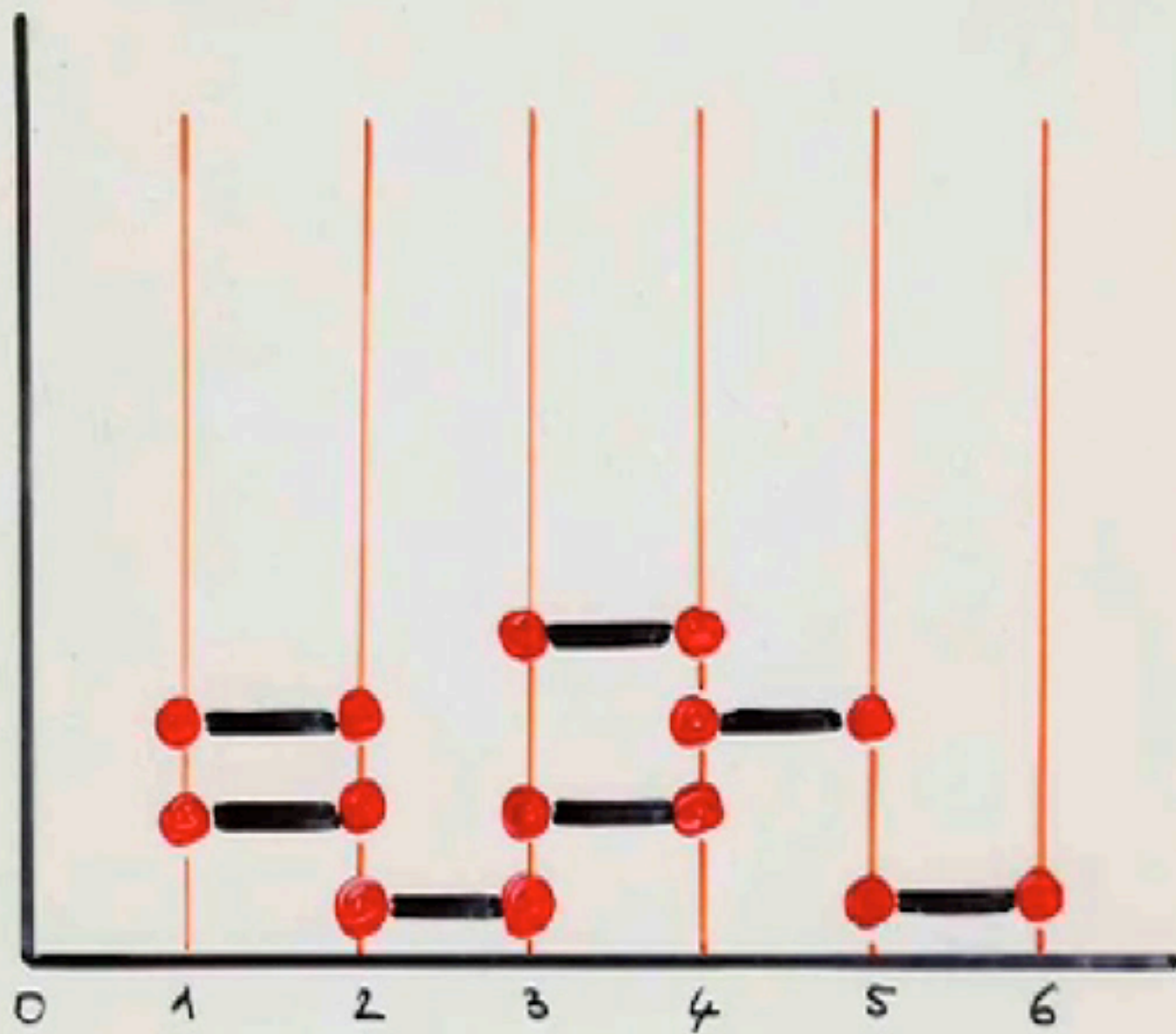


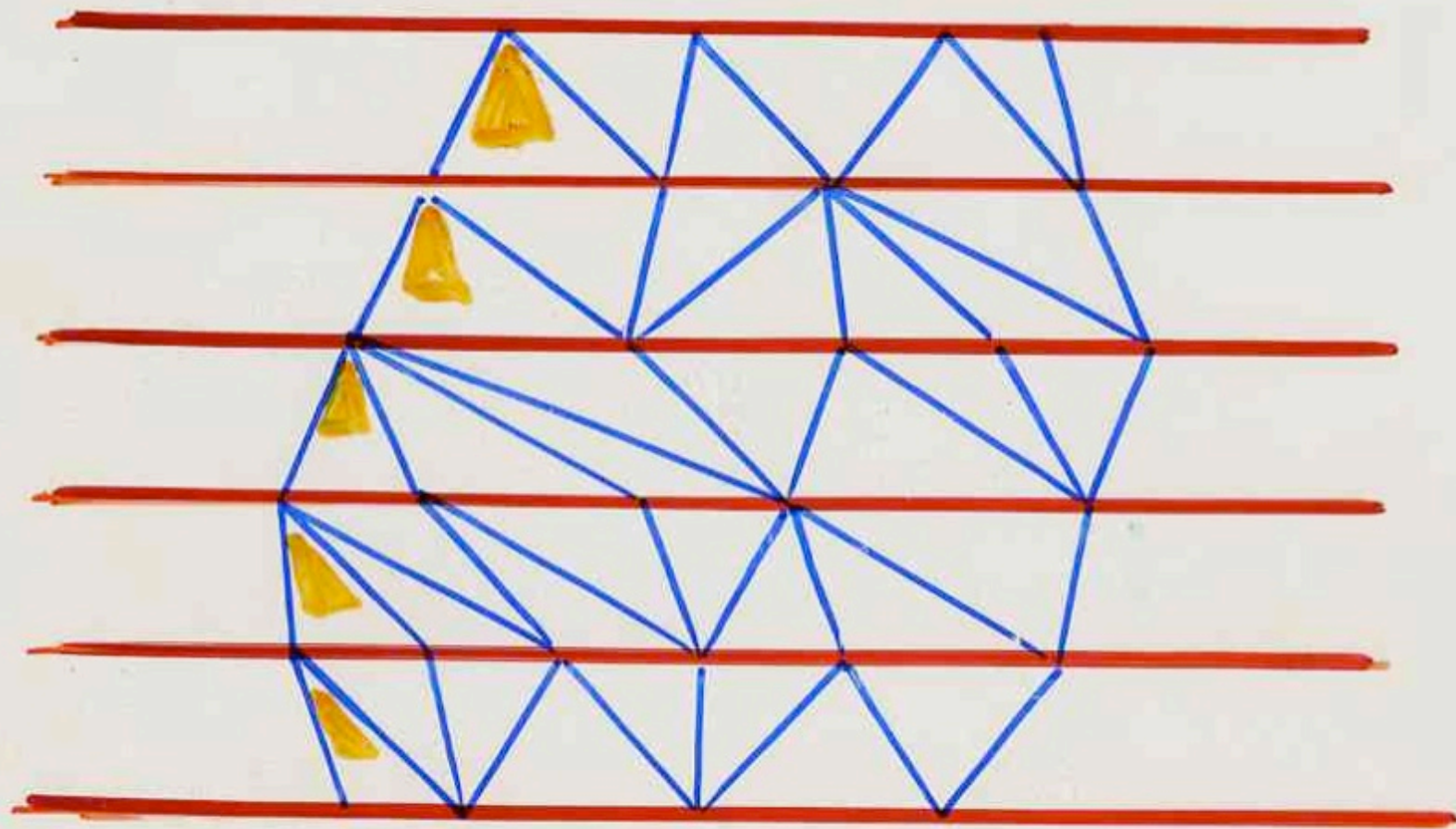




Des empilements aux
triangulations Lorentziennes







Les **bijections** **combinatoires**
permettent de mieux **comprendre**
la structure même des objets
combinatoires.

Mieux **comprendre** conduit à des
algorithmes plus efficaces sur ces
objets ...

et aussi à découvrir et démontrer
de nouvelles **formules d'énumération**
sur des objets plus compliqués,
comme par exemple les
triangulations lorentziennes générales
(= sans **conditions aux bords**) en **2D**,
résolvant ainsi des problèmes ouverts
en **physique théorique**.

(Bousquet-Mélou, Rechnitzer) 2002

$$C(t) = \frac{Q}{(1-Q) \left[1 - \sum_{k \geq 1} \frac{Q^{k+1}}{1 - Q^k (1+Q)} \right]}$$

$$Q(t) = \frac{1 - 2t - \sqrt{1 - 4t}}{2t}$$

generating function for
half-pyramid $(\neq \emptyset)$

$$= \sum_{n \geq 1} C_n t^n$$

Catalan

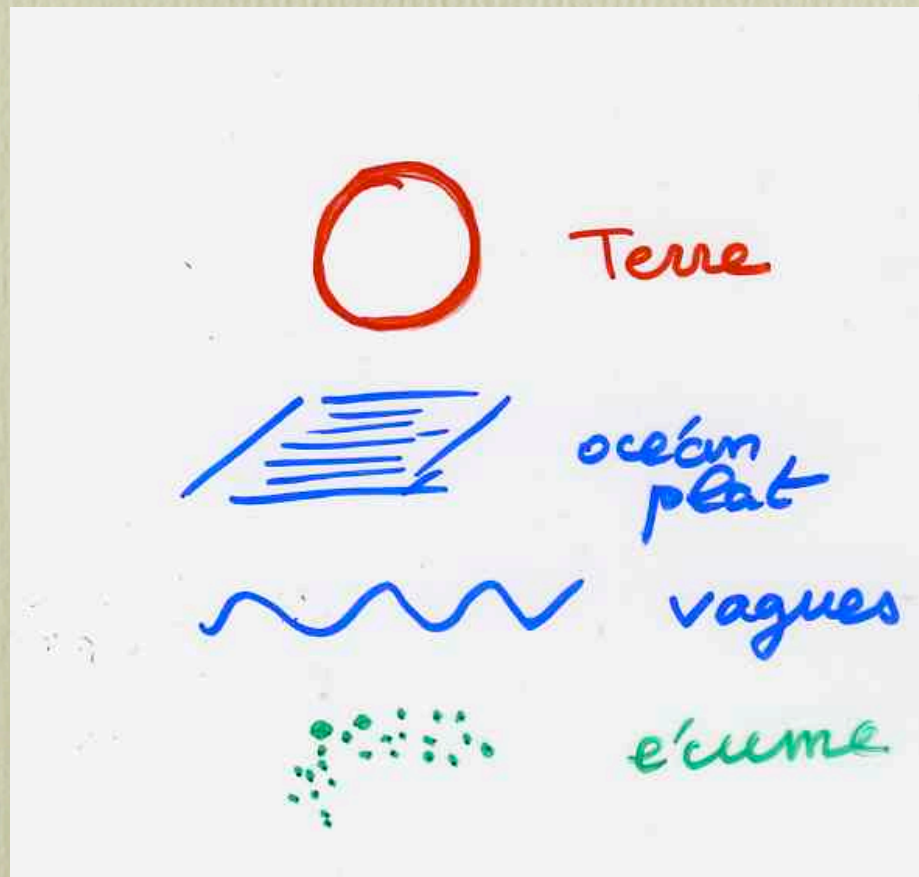
gravitation quantique

quatre échelles
l'univers
échelle humaine
particules élémentaires

comme
la terre vue de l'espace
l'océan vu d'avion
les vagues de l'océan

l'échelle de Planck

l'écume de la mer





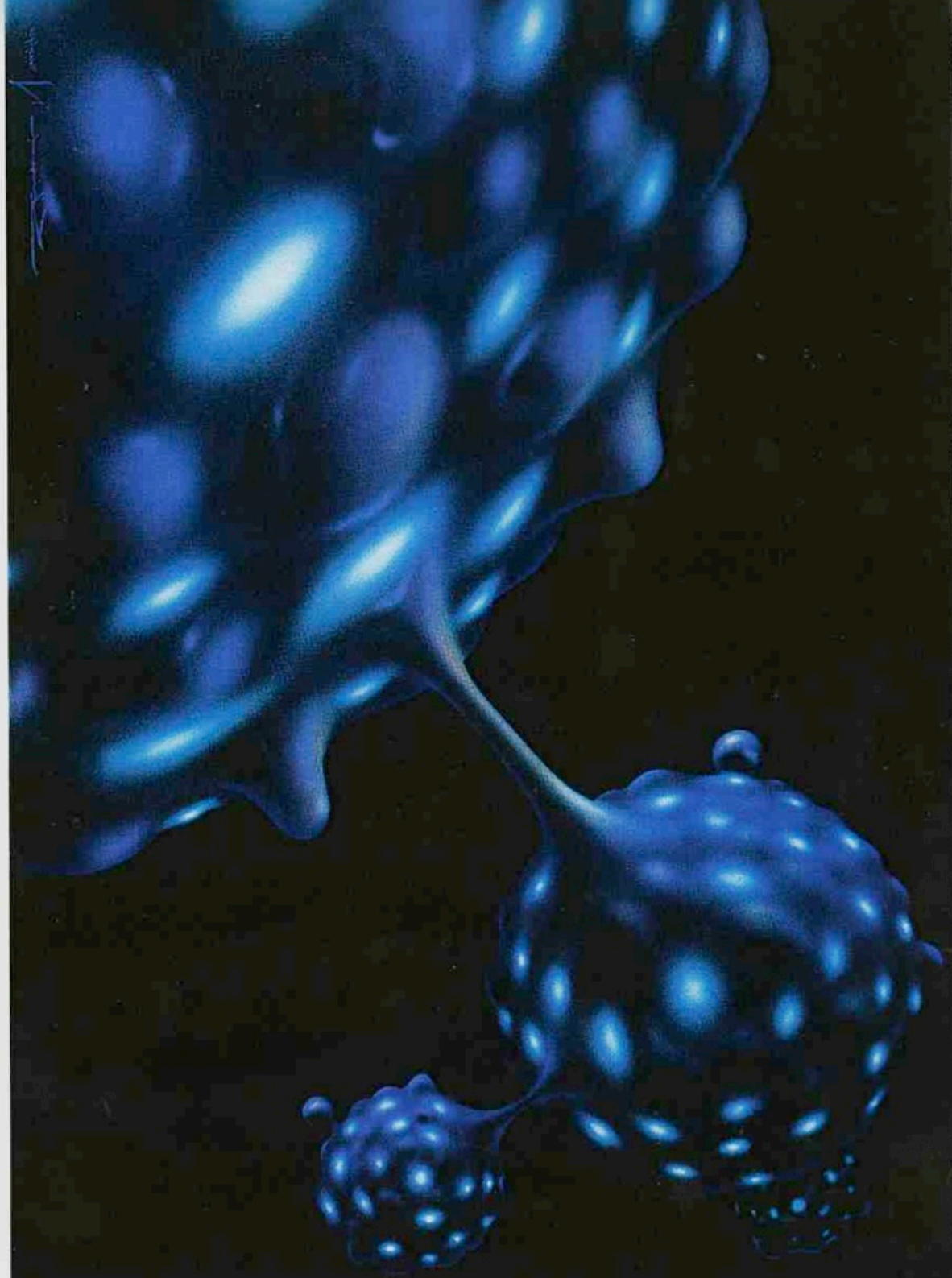






Dessin
de
S. Numazawa

Ciel & Espace





épilogue











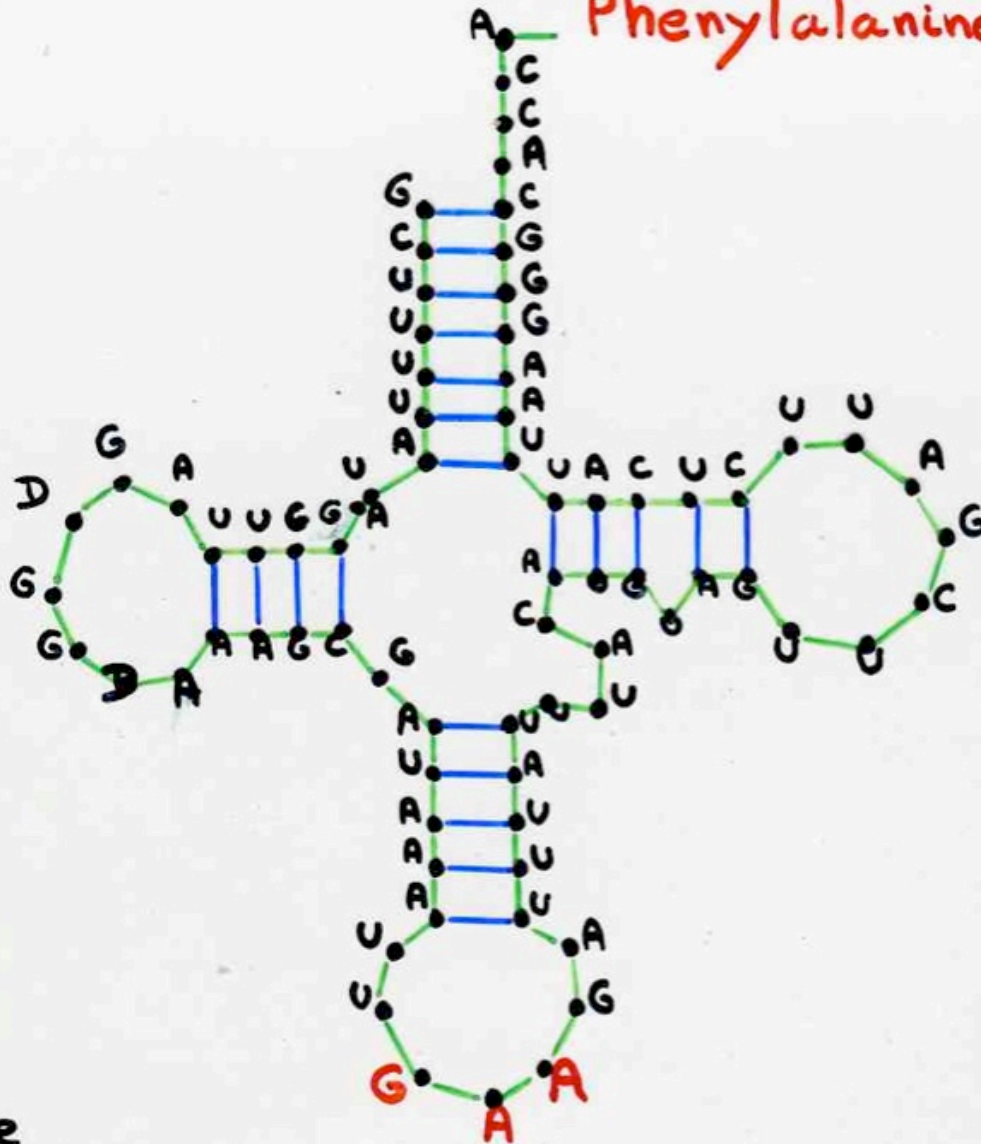
NATIONAL GEOGRAPHIC





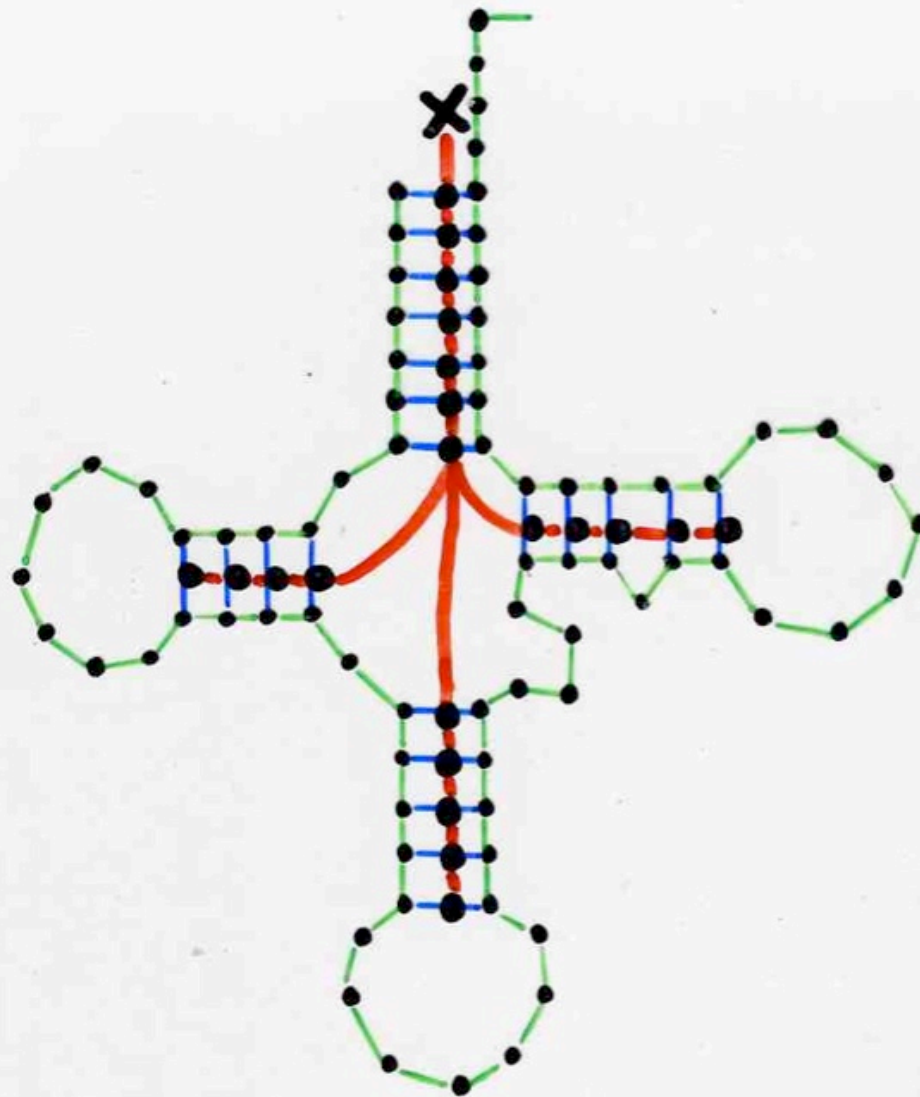
Trees everywhere

Phenylalanine

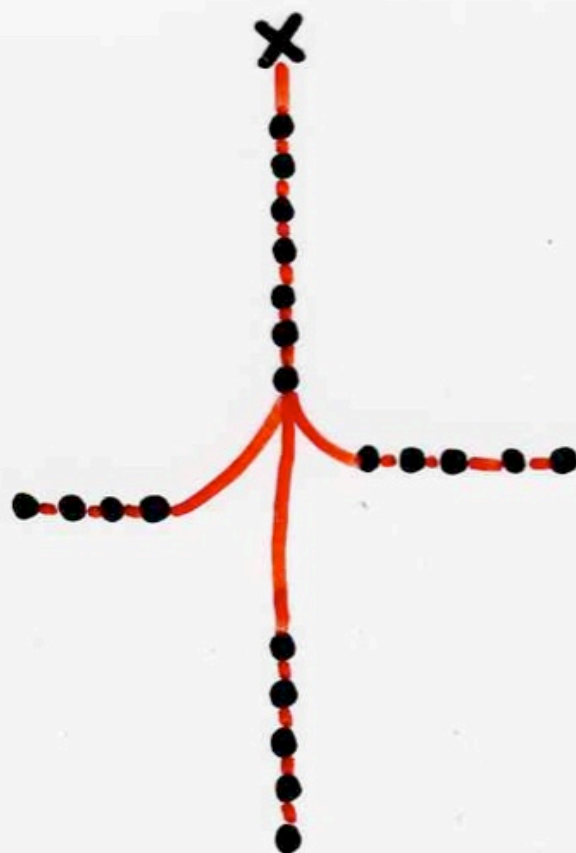


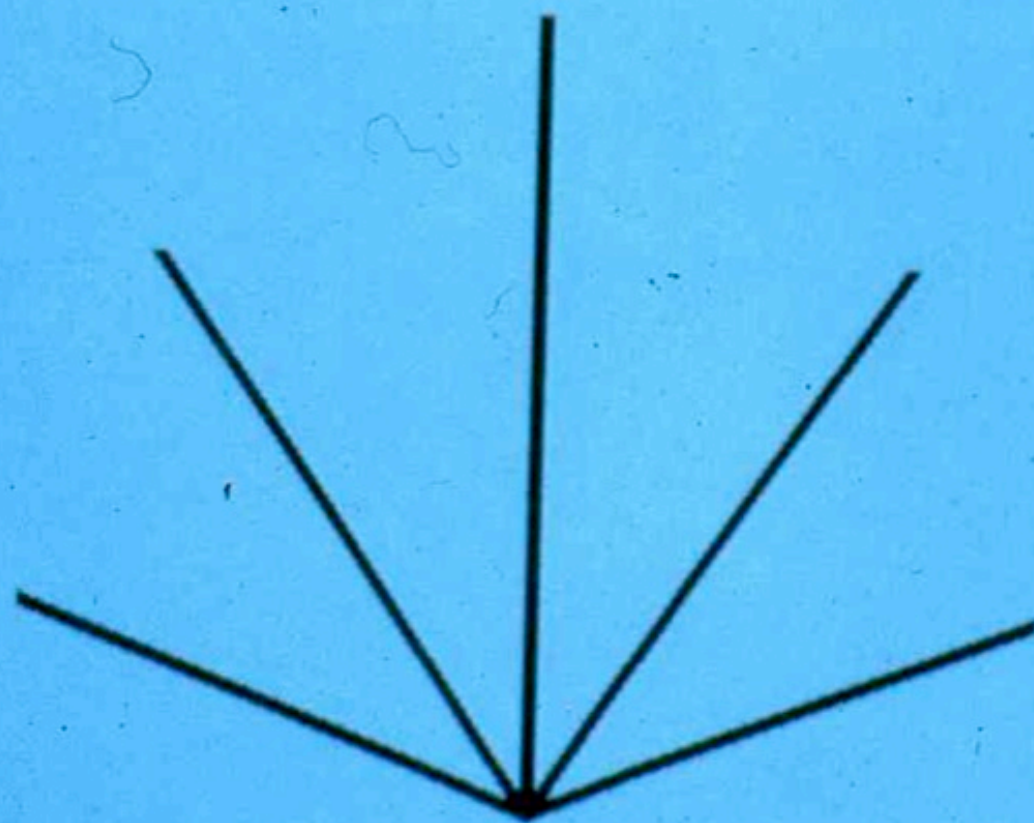
A denine
U racyle
G uanine
C ytosine

tARN^{Phe}



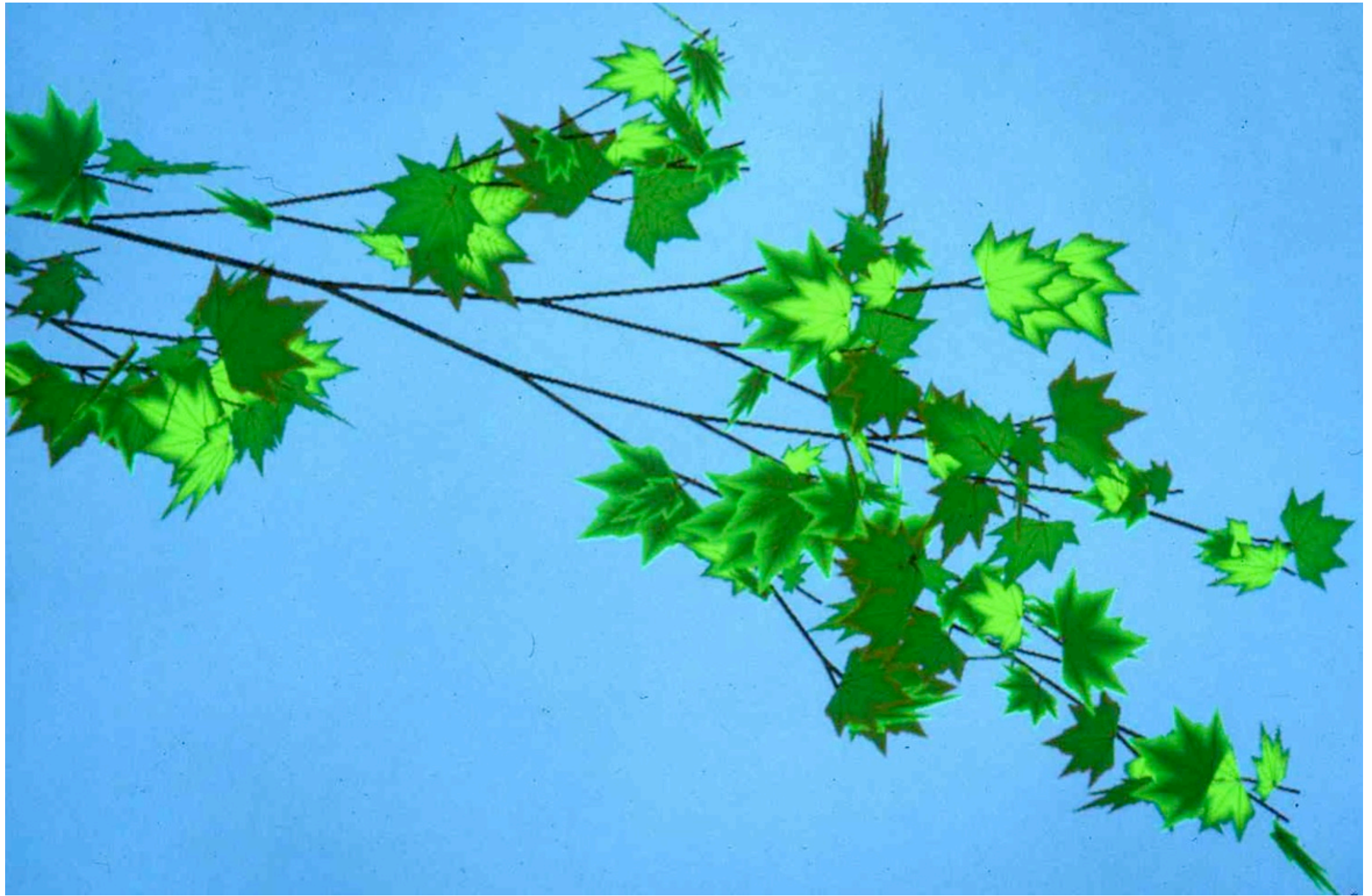
tARN^{Phe}















Il y a des arbres dans les étoiles,
des arbres dans les grains de lumière.

Les théories mathématiques s'interpellent,
s'entrecroisent, renaissent, se fondent entre elles.

Les grands Maîtres se parlent à travers les siècles
dans le jardin merveilleux des Mathématiques.



C'est la fin
merci à tous !

réalisation:
Xavier Viennot

au violon:
Gérard H.E. Duchamp
Mariette Freudentheil

texte cosmique:
Marcia Pig Lagos

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