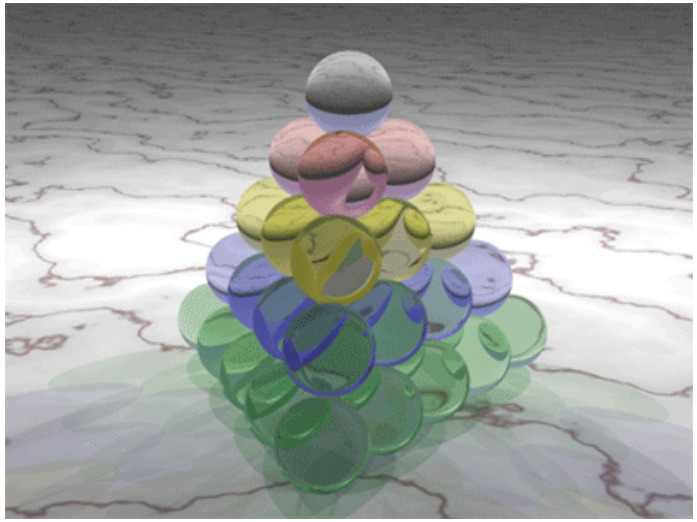
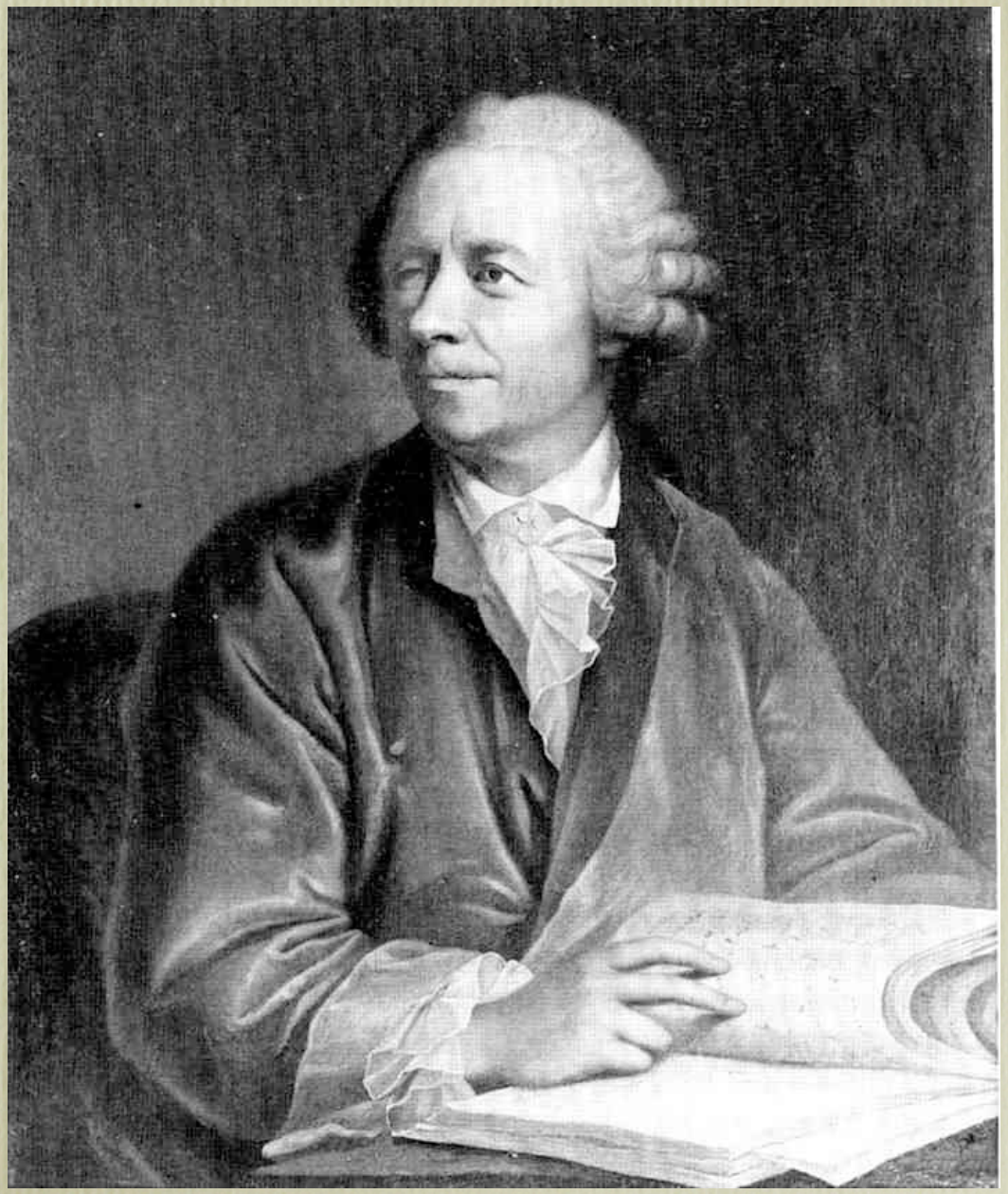
Arbres, triangulations, molécules d'ARN: quelles mathématiques, quelle informatique ?



xavier viennot

LaBRI, CNRS, Université Bordeaux 1


4000 BASEL
e-k+f=2
Leonhard Euler
1707-1783
AUSGABETAG 6. MÄRZ 2007
LEONHARD EULER
1707-1783
130
Leonhard Euler 300
300. Geburtstag
300ème anniversaire
300° anniversario
300th anniversary



Leonhard
Euler

Opera Omnia, 80 volumes

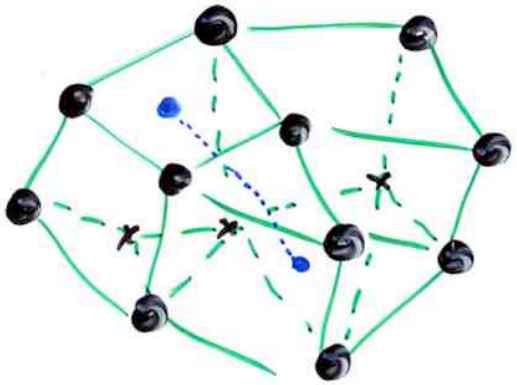
Eneström E001, E800
40 livres

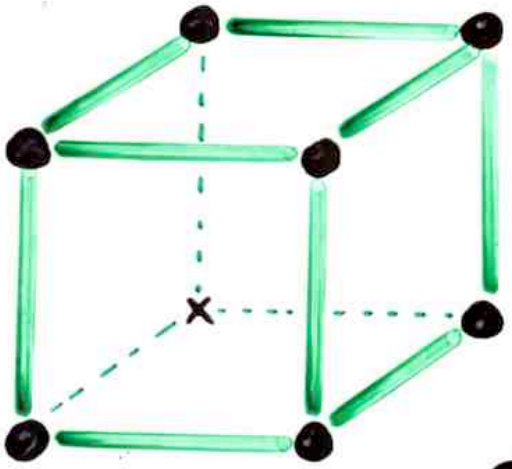
correspondance 3000 lettres

mathématiques, physique,
optique, mécanique, astronomie,
sciences et techniques, “ingéniérie”,
navigation, géographie, musique

grand pédagogue

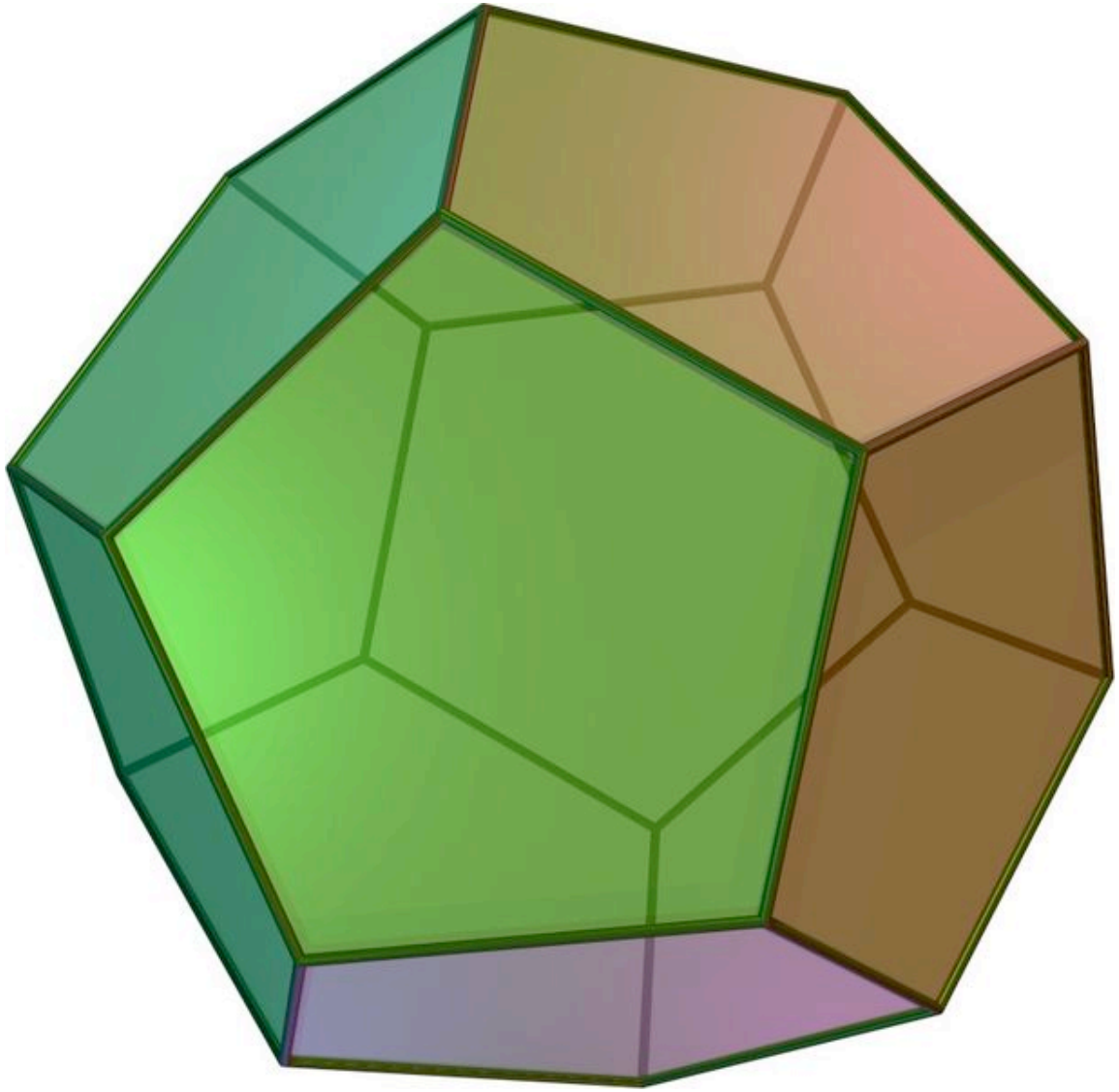
Polyèdre convexe





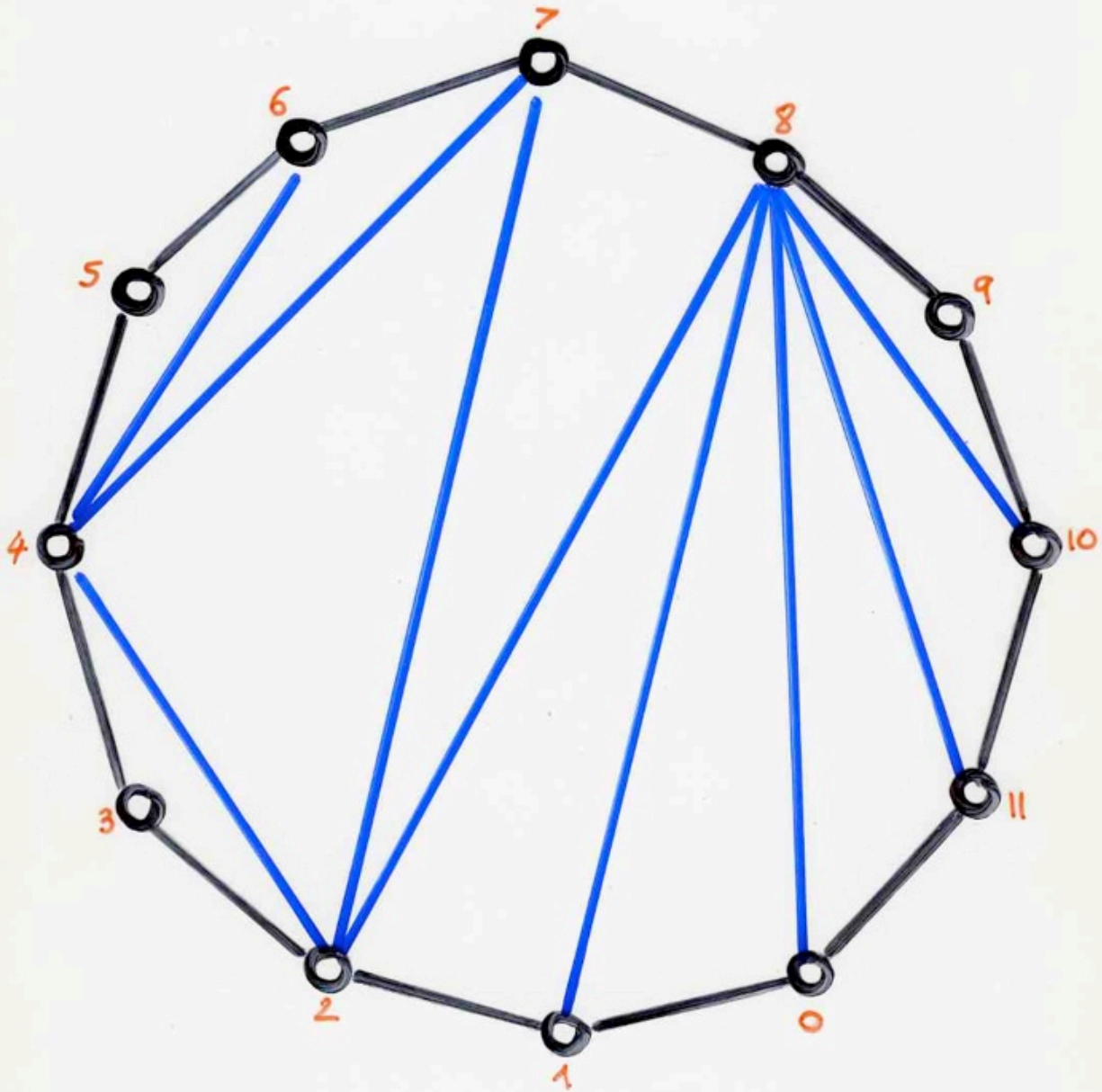
$$S - A + F =$$

$$8 - 12 + 6 = 2$$



Une lettre oubliée d'Euler à Christian Goldbach ....

Berlin, 4 Septembre 1751


7
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2
0
1

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

$$= \frac{(2n)!}{(n+1)!\ n!}$$

$$n! = 1 \times 2 \times \dots \times n$$

$$C_4 = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8}{1 \times 2 \times 3 \times 4 \quad 1 \times 2 \times 3 \times 4 \times 5}$$

$$= 14$$

1 1 2 5 14 42

nombres de Catalan

NOVI COMMENTARII ACADEMIAE SCIENTIARVM IMPERIALIS PETROPOLITANAE

TOM. VII.

pro Annis MDCCLVIII. et MDCCLIX.

PETROPOLI
TYPIS ACADEMIAE SCIENTIARVM
MDCCLXI.

VI.

Enumeratio modorum, quibus figurae planae rectilineae per diagonales diuiduntur in triangula.

Auct. I. A. de Segner pag. 203.

Quando in Geometria area figurarum pluribus lateribus inclusarum definiri debet, eae per diagonales in triangula resolui solent, quia tum cuiusque trianguli areae ex cognitis lateribus facile determinantur. Quo pluribus autem lateribus figura est praedita, eo pluribus modis eam hoc modo in triangula resolui posse, vel leuiter attendenti statim est manifestum. Ita cum in quadrilaterum duas diagonales ducere liceat, quadrilaterum duplici modo in bina triangula diuiditur. Pentagonum autem quintuplici modo, diagonalibus ducendis, in triangula resolui posse reperitur, hexagonum vero 14 modis, et heptagonum 42 modis, octogonum 132 modis, enneagonum 429 modis etc. quae omnium modorum possibilium enumeratio, quo magis cum laterum numero eorum multitudo crescit, eo fit difficilior et taediosior. Quare quaestio omnino curiosa, et Geometrarum attentione digna videtur, qua lege isti resolutionum numeri pro laterum multitudine progrediantur, vt inde pro quouis polygono resolutionum numerus rite definiri queat? Hinc Ill. Auctor modo prorsus singulari et ingenioso legem progressionis horum nu-

merorum exponit, ac rigorose demonstrat, dum docet, quomodo pro quouis polygono resolutionum numerus, ex cognita resolutione polygonorum simpliciorum, quae paucioribus constant lateribus, colligi debeat. Hac ratione, si a simplicissimis incipiamus, hanc inuestigationem continuo ad polygona plurium laterum extendere licet; sicque Ill. Auctor sub finem tabellam adiecit, in qua istiusmodi resolutiones ad polygona 20 laterum vsque exhibet. Liceat autem nobis, a summo quodam Geometra, qui eandem hanc tabulam calculo subiecit, admonitis, obseruare, hanc tabulam, ob quendam calculi errorem, tantum vsque ad polygona 15 laterum esse iustam, quippe pro hoc polygono resolutionum numerus non est 751900, vt tabella habet, sed 742900, sequentes quoque numeri, dum forte nouus error irrepsit, primo ad 17 vsque latera nimis sunt magni, deinde vero nimis parui, dum pro 20 lateribus resolutionum numerus est 477638700. Facilius haec apparent, si ex lege primum obseruata, qua quilibet numerus ex omnibus praecedentibus colligitur, alia ad computum facilior eliciatur, cuius ope quilibet numerus ex solo praecedente definiatur. Ita si pro polygono n laterum numerus resolutionum sit P, pro polygono sequente $n+1$ laterum numerus resolutionum erit $\frac{4n-6}{n}$ P. Quin etiam hinc, sine consideratione praecedentium, statim indefinite pro polygono n laterum numerus resolutionum ita per factores exprimitur, vt sit:

$$\frac{2}{2}\cdot\frac{6}{3}\cdot\frac{10}{4}\cdot\frac{14}{5}\cdot\frac{18}{6}\cdot\frac{22}{7}\cdot\ \ldots\ldots\ \frac{4n-10}{n-1}$$

vbi numeratores quaternario, denominatores vero vnitate crescunt. Hinc sequentem nouam tabulam, benevole

vole nobiscum communicatam, adiungere e re visum est, quod Ill. Auctori huius schediasmatis non displiciturum esse speramus.

Num. laterum	numerus resolutionum.	num. laterum	numerus resolutionum.
III	1	XV	742900
IV	2	XVI	2674440
V	5	XVII	9694845
VI	14	XVIII	35357670
VII	42	XIX	129644790
VIII	132	XX	477638700
IX	429	XXI	1767263190
X	1430	XXII	6564120420
XI	4862	XXIII	24466267020
XII	16796	XXIV	91482563640
XIII	58786	XXV	343059613650.
XIV	208012		

VII.

Methodus simplex et vniuersalis omnes omnium aequationum radices detegendi.

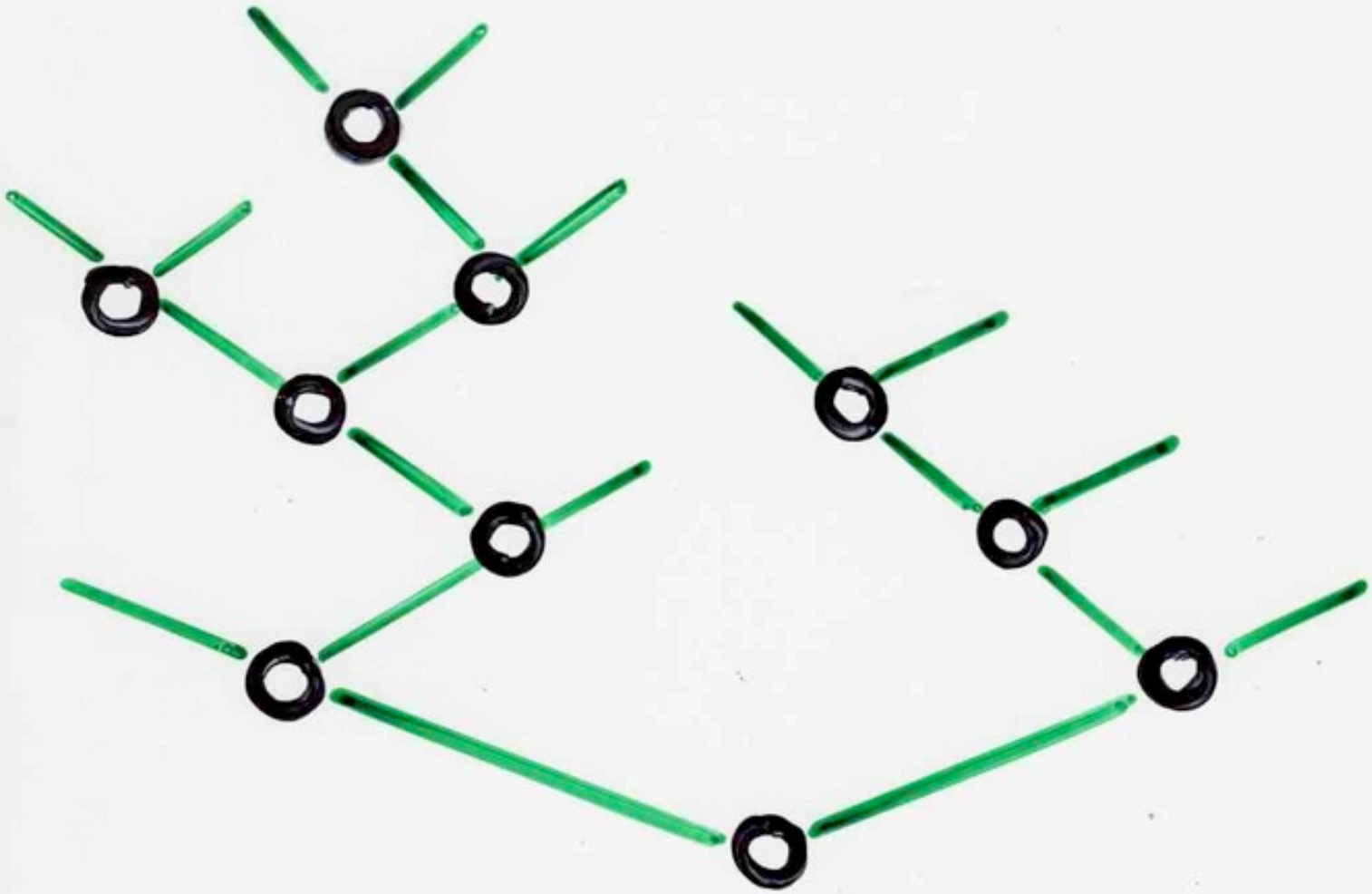
$$2(2n+1)C_n = (n+2)C_{n+1}$$

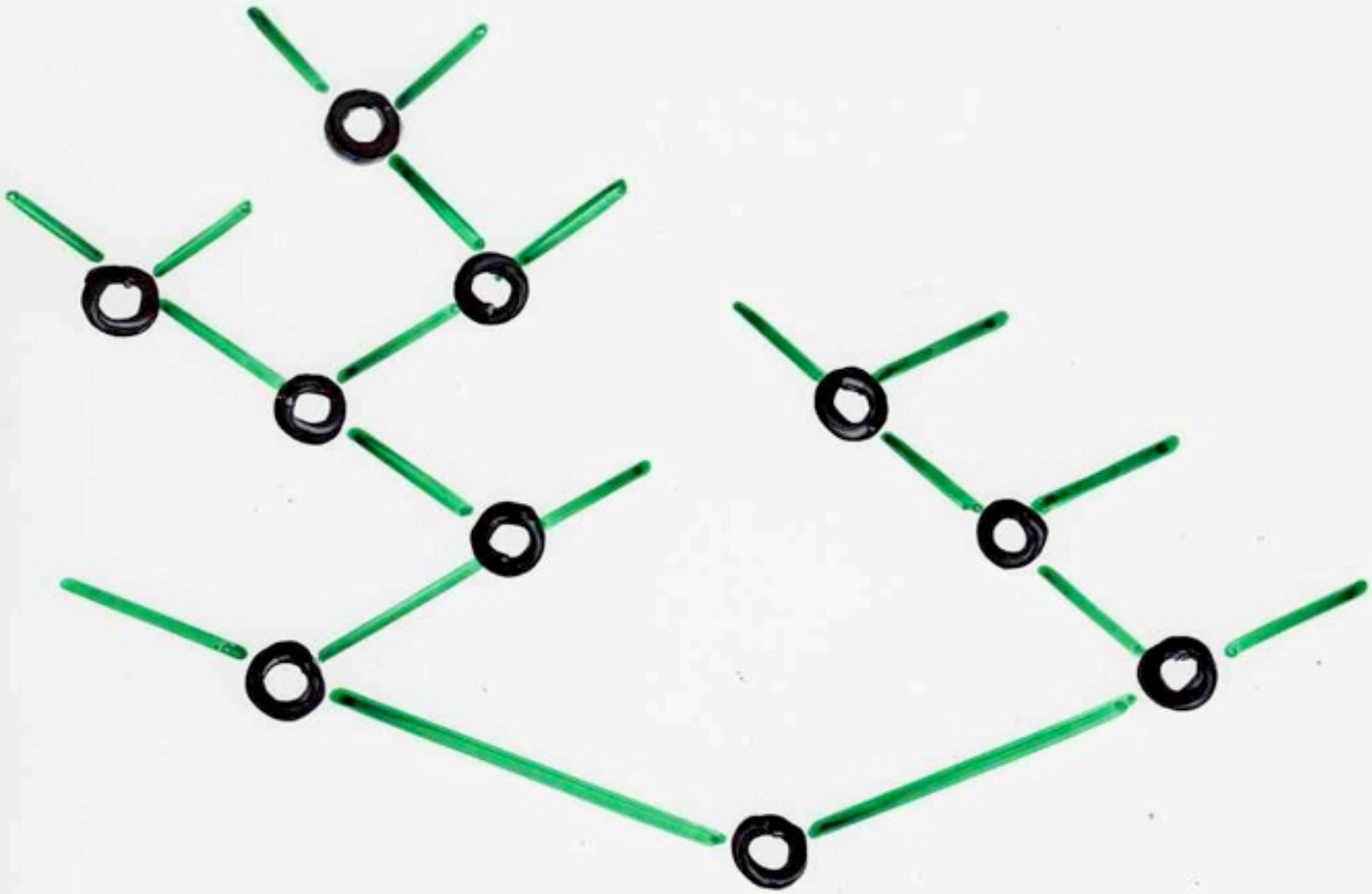
Complures iam a Geometris excogitatae sunt methodi aequationum algebraicarum radices, vel accurate, vel proxime saltem, determinandi: omnes autem fere postulant, vt valores radicum, quae quaeruntur,

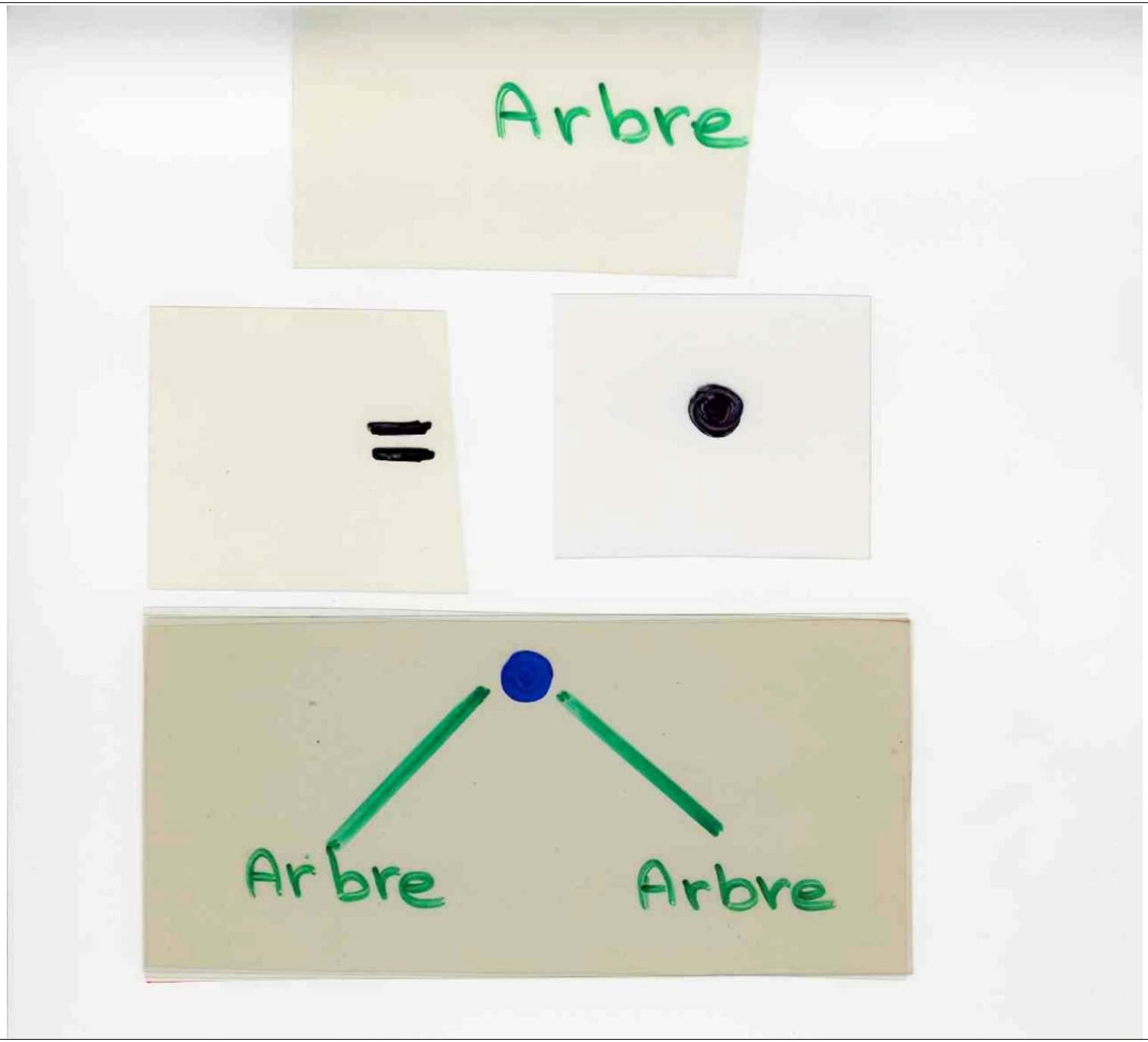
$$\frac{1}{n+1}\binom{2n}{n}$$

au 20 ème siècle ...

les arbres binaires






Arbre
=
Arbre
Arbre

C_n = nombre

d'arbres binaires

ayant n

sommets internes

(et donc $n+1$ feuilles)

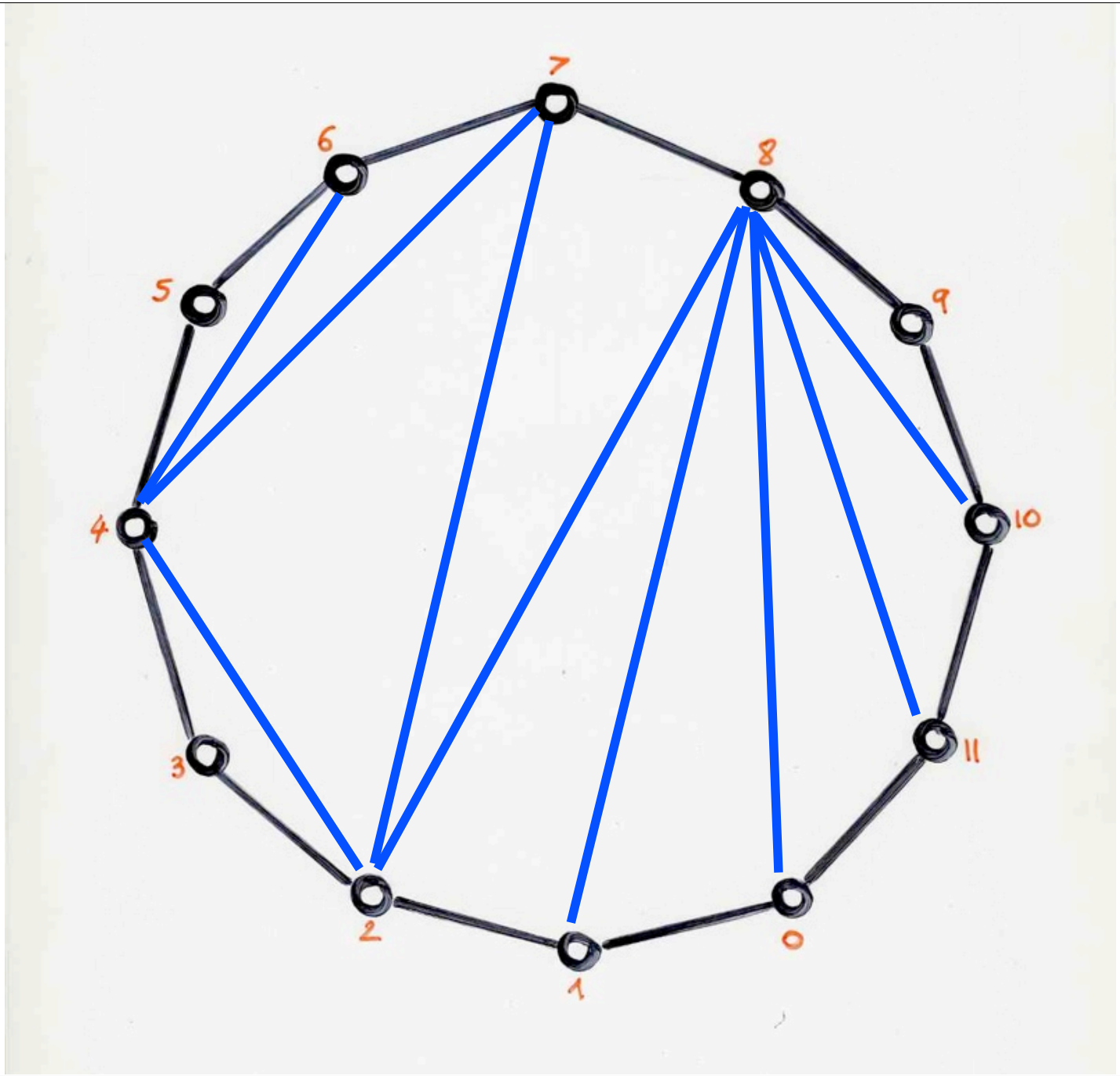
<u>nombre de Catalan</u>

recurrence

$$C_{n+1} = \sum_{i+j=n} C_i \, C_j$$

$$C_0 = 1$$

des triangulations aux arbres binaires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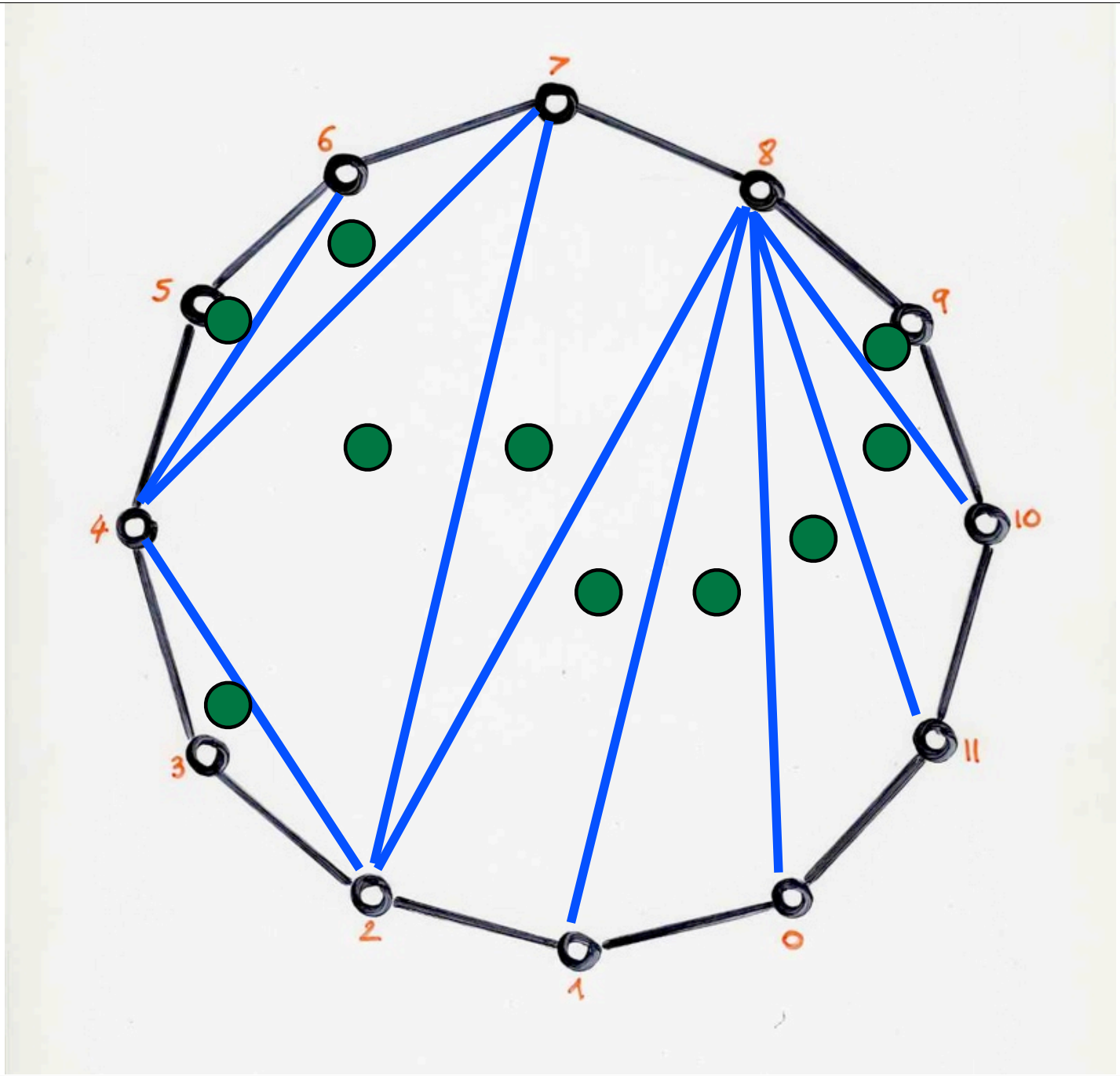
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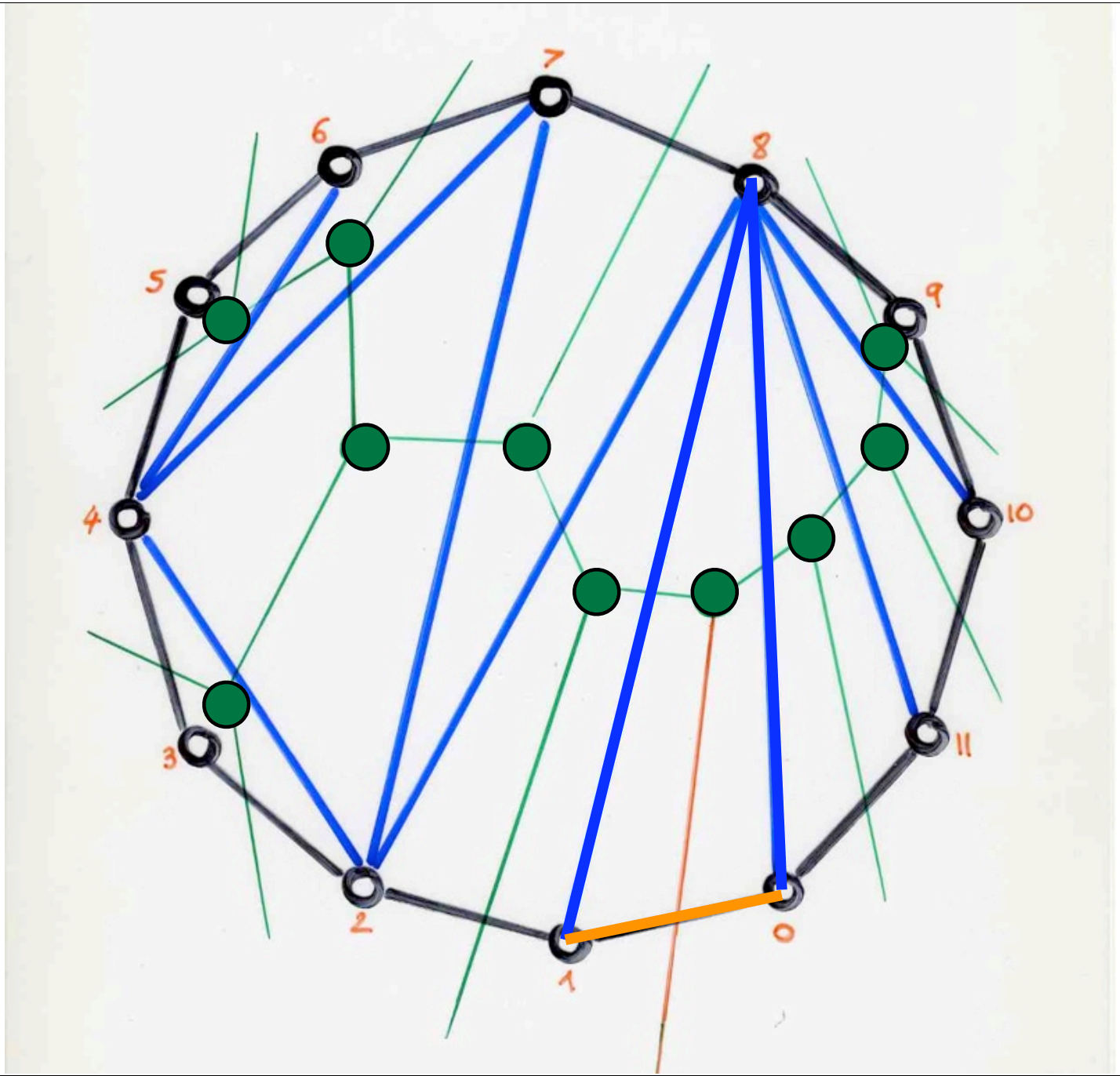
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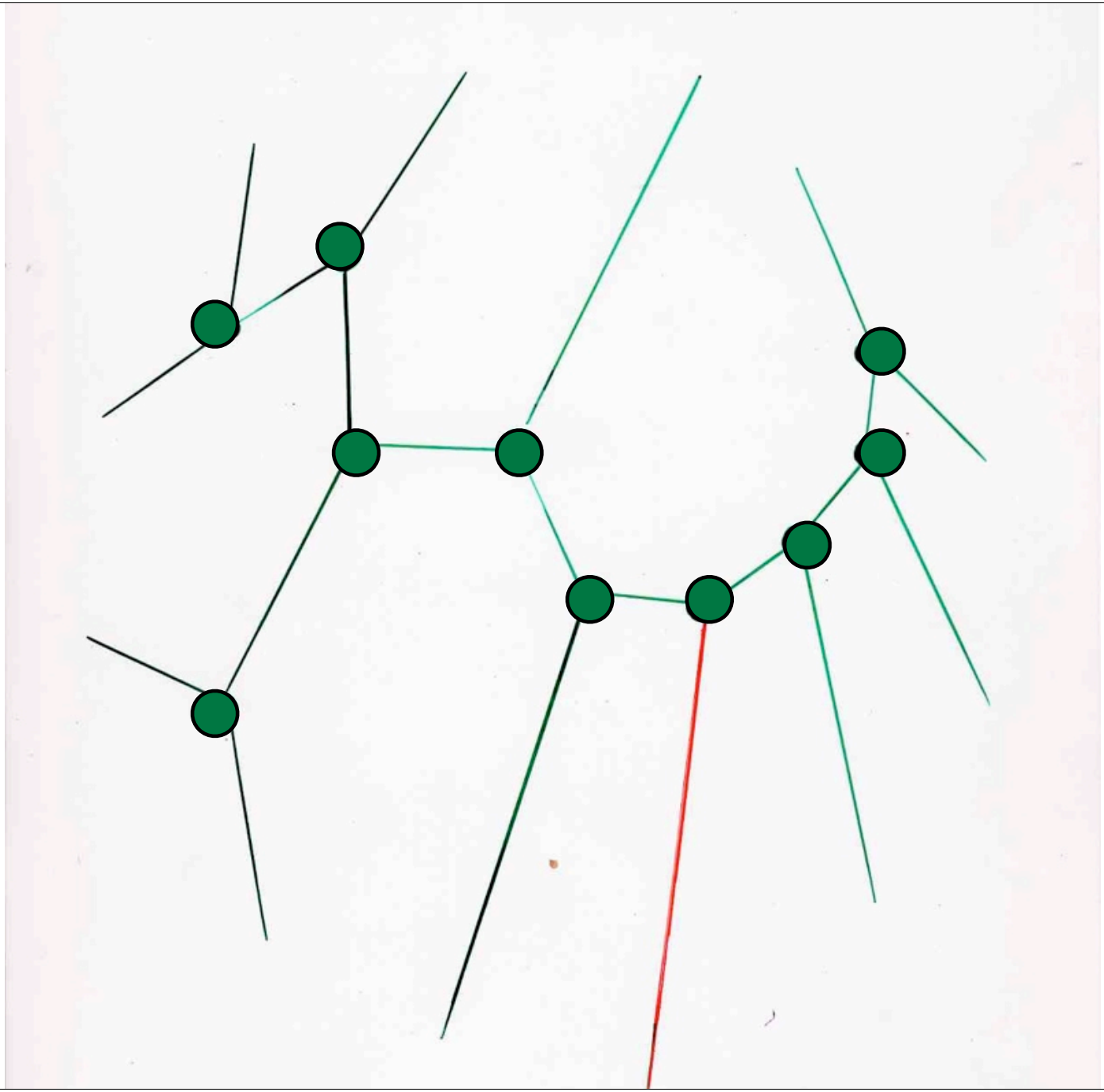
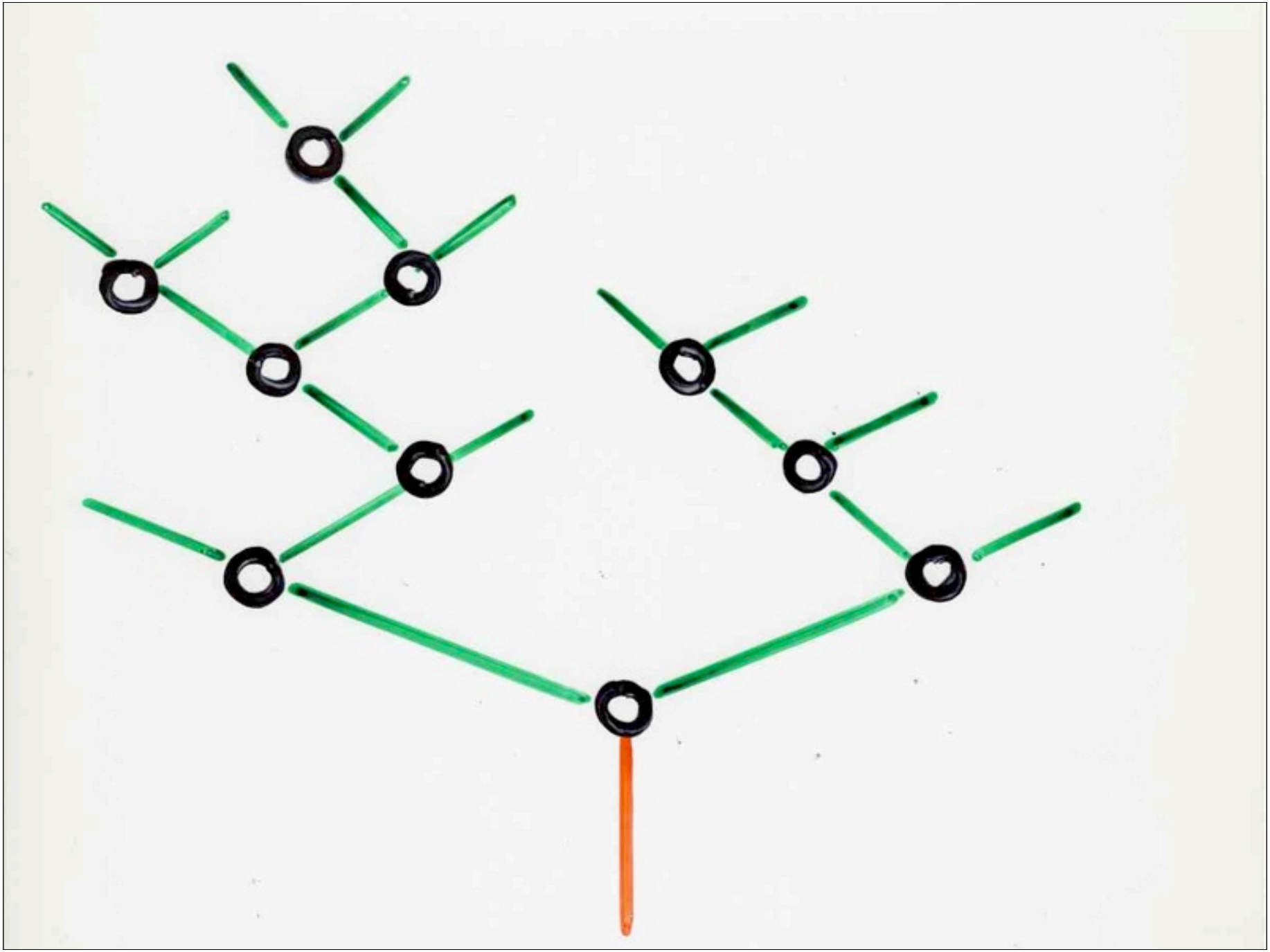
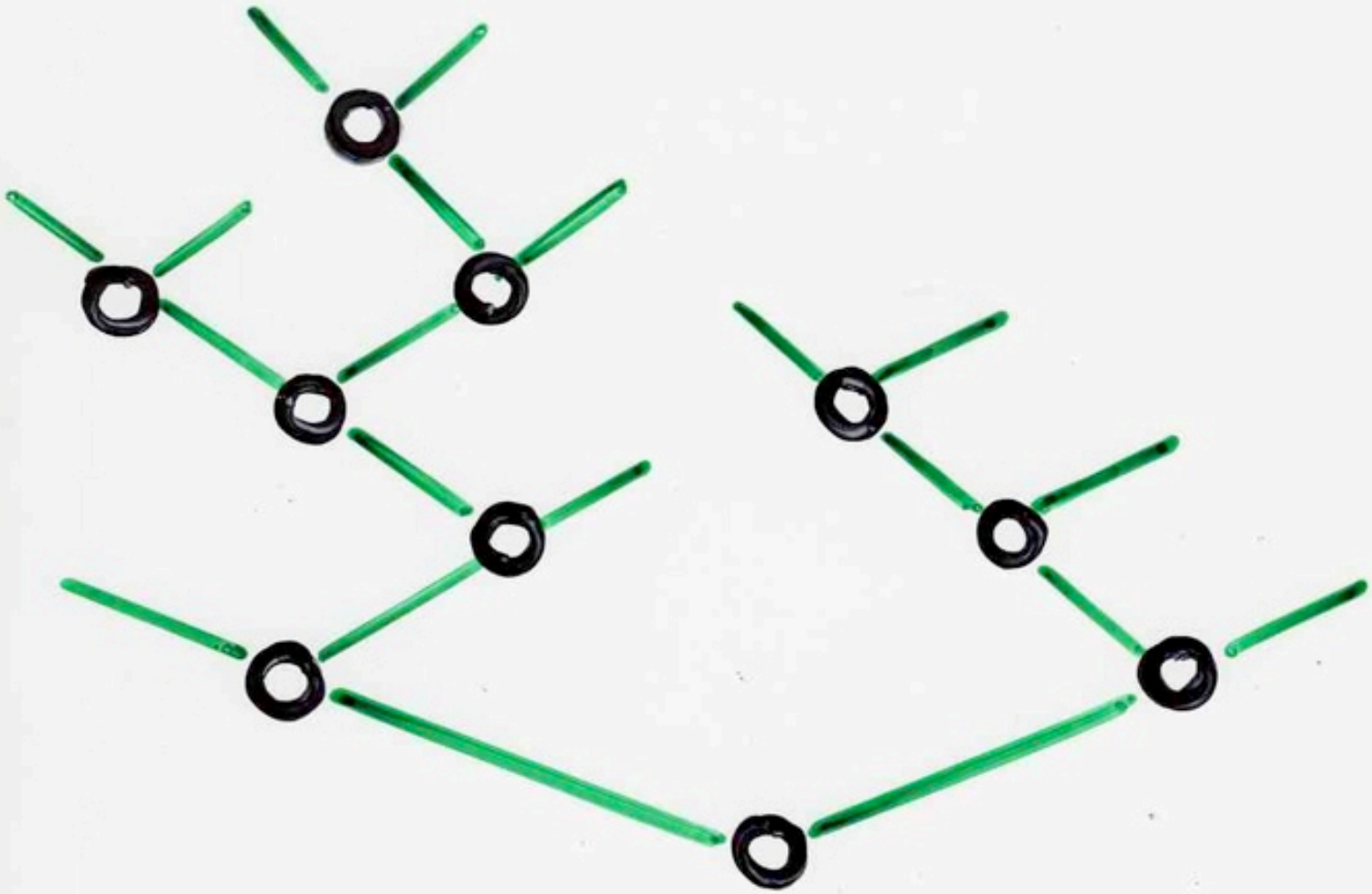
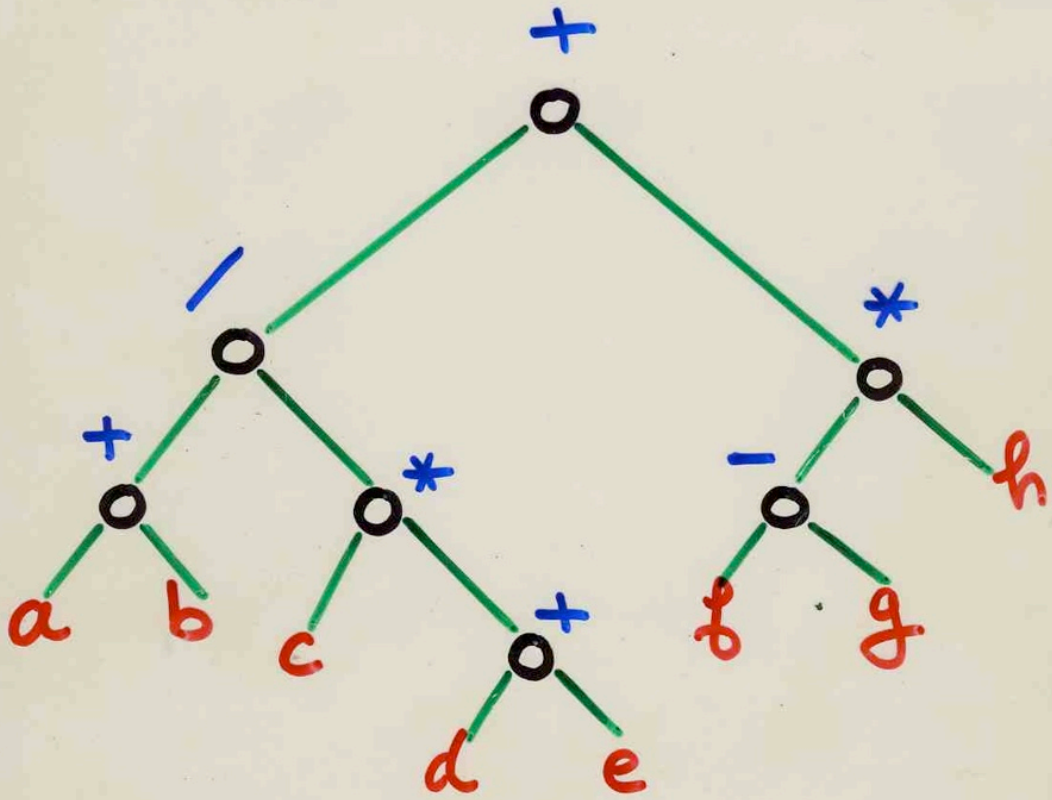






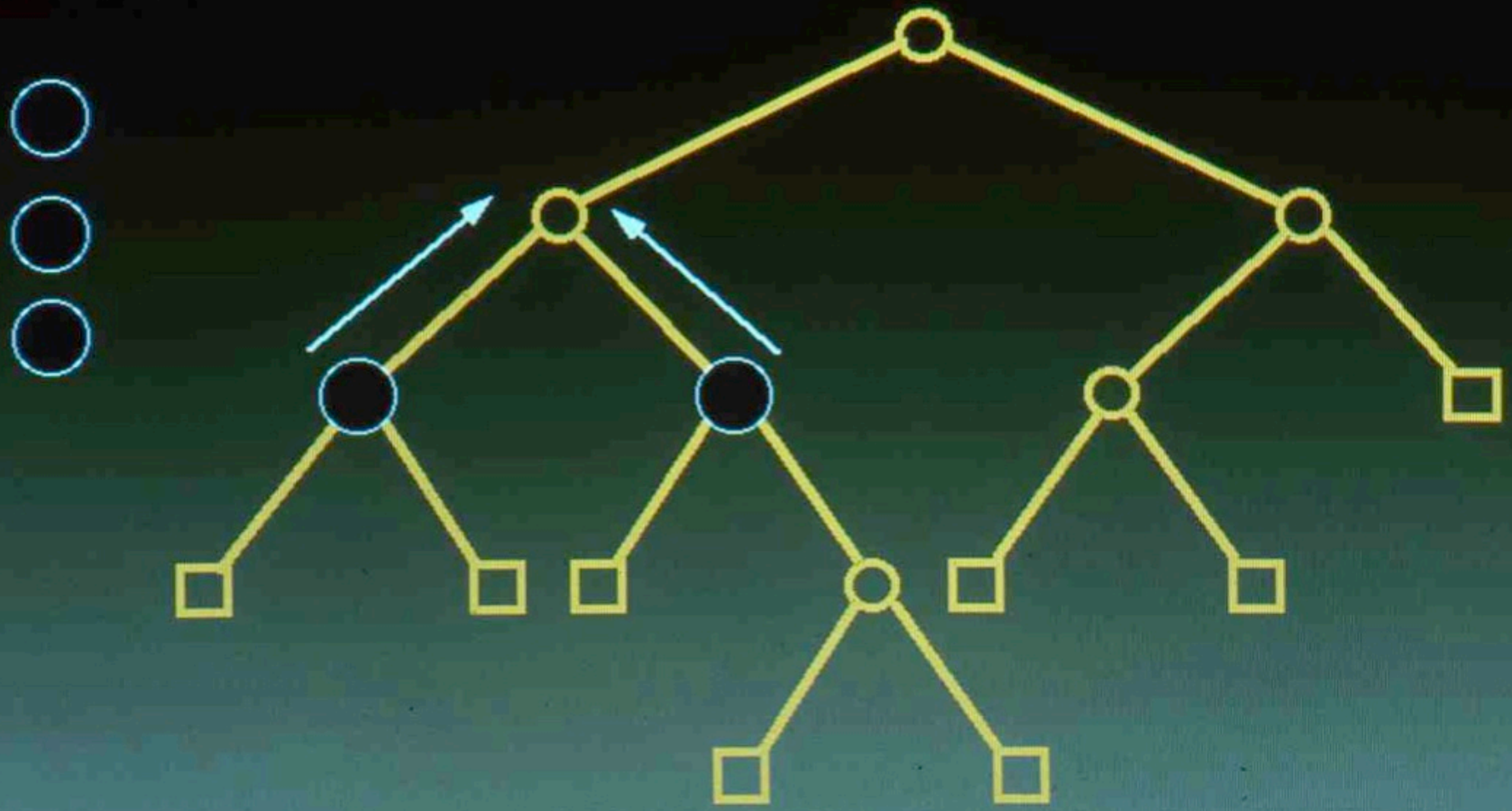
calcul d'une expression arithmétique

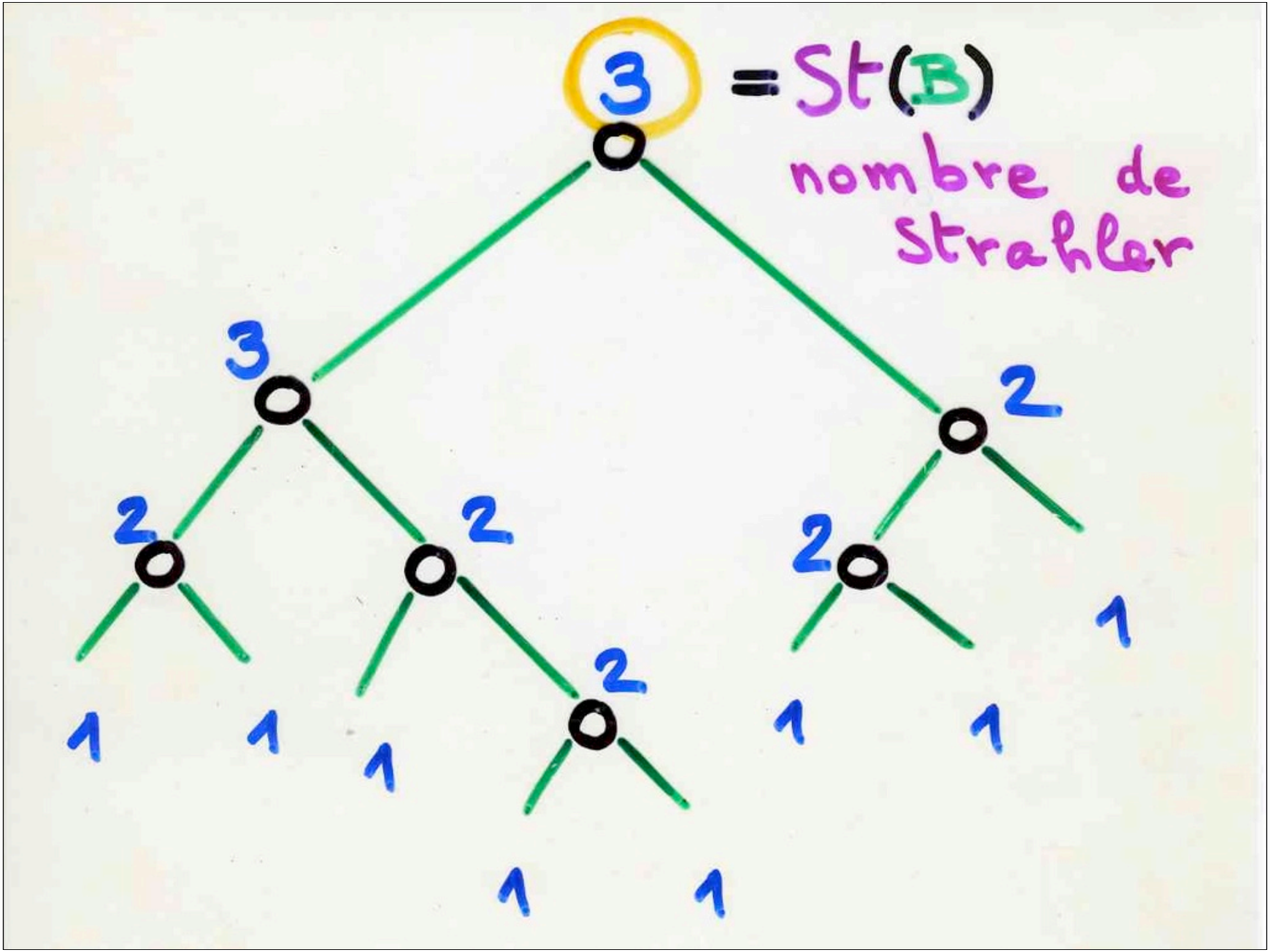
$$\frac{(a+b)}{c\,(d+e)} + (f-g)h$$

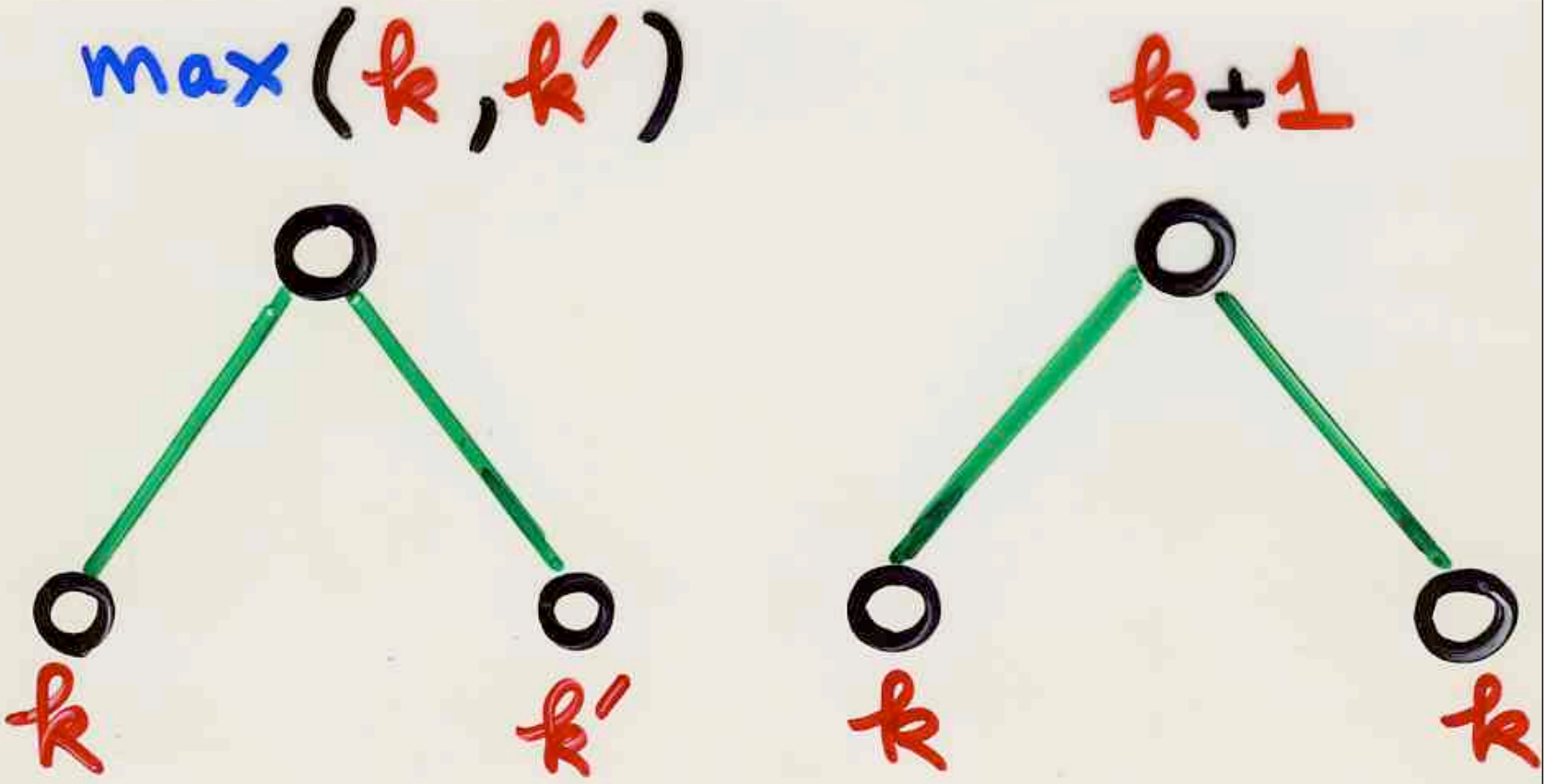


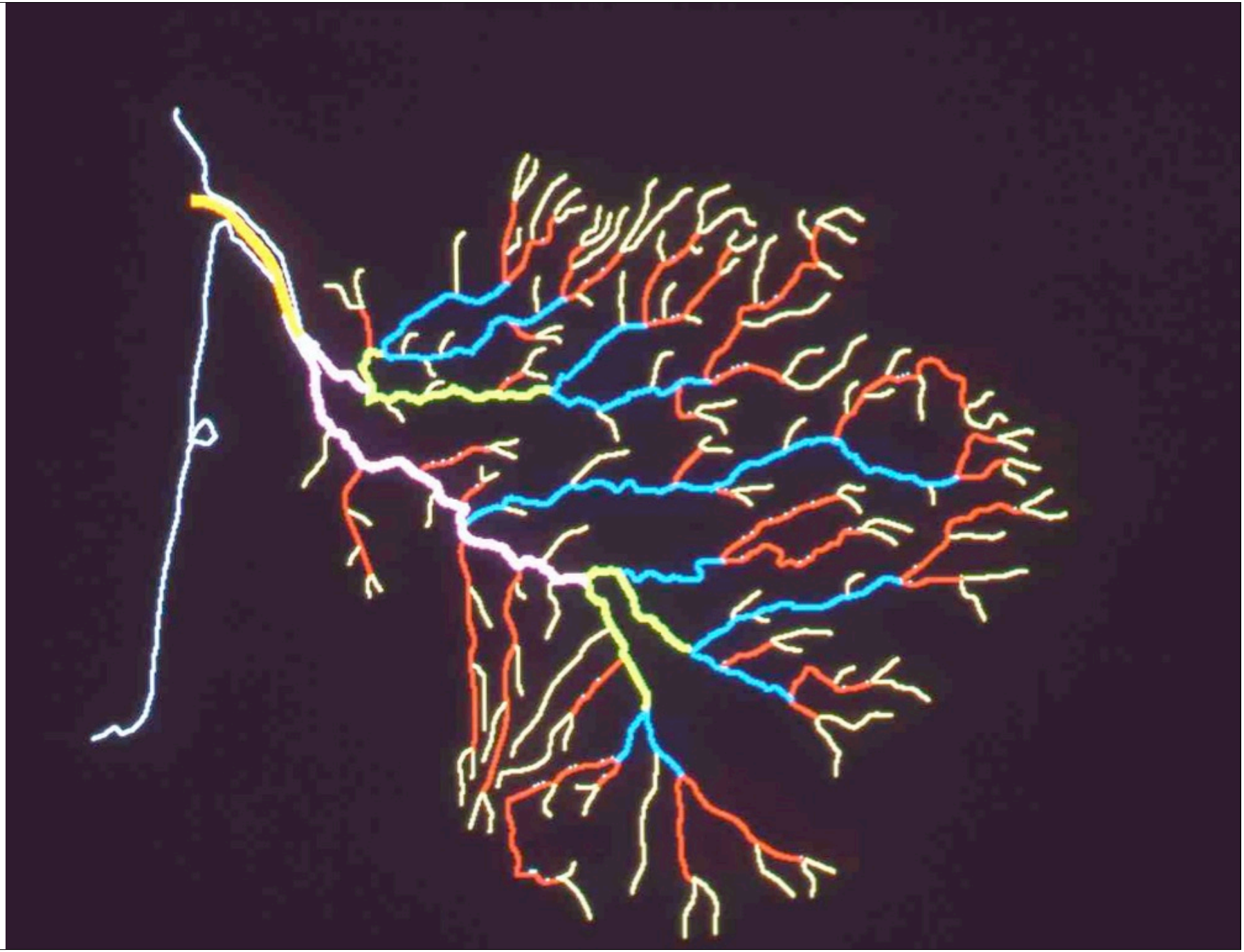
$$\frac{(a+b)}{c\,(d+e)} + (f-g)\,h$$

Pebbles problem

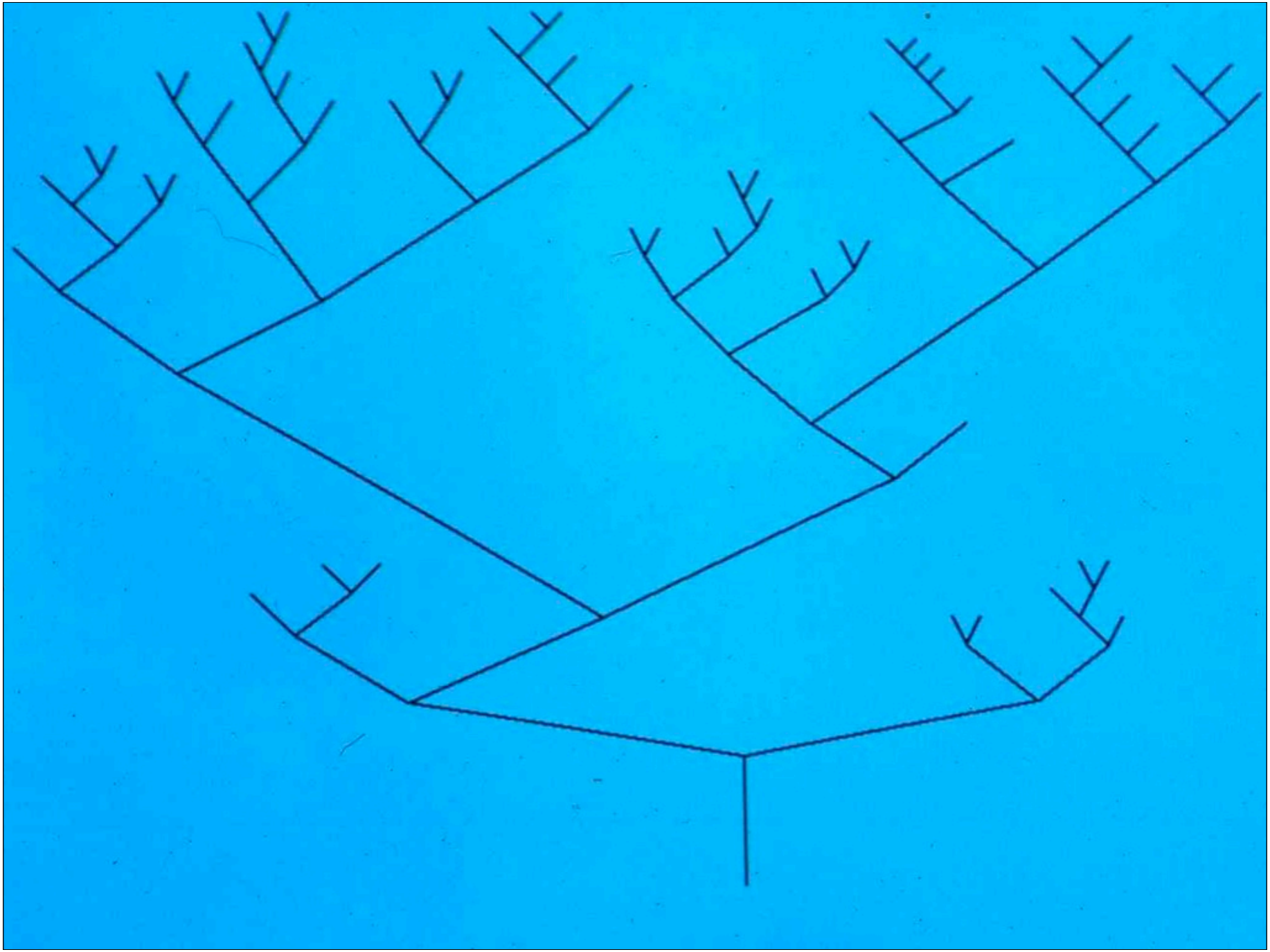


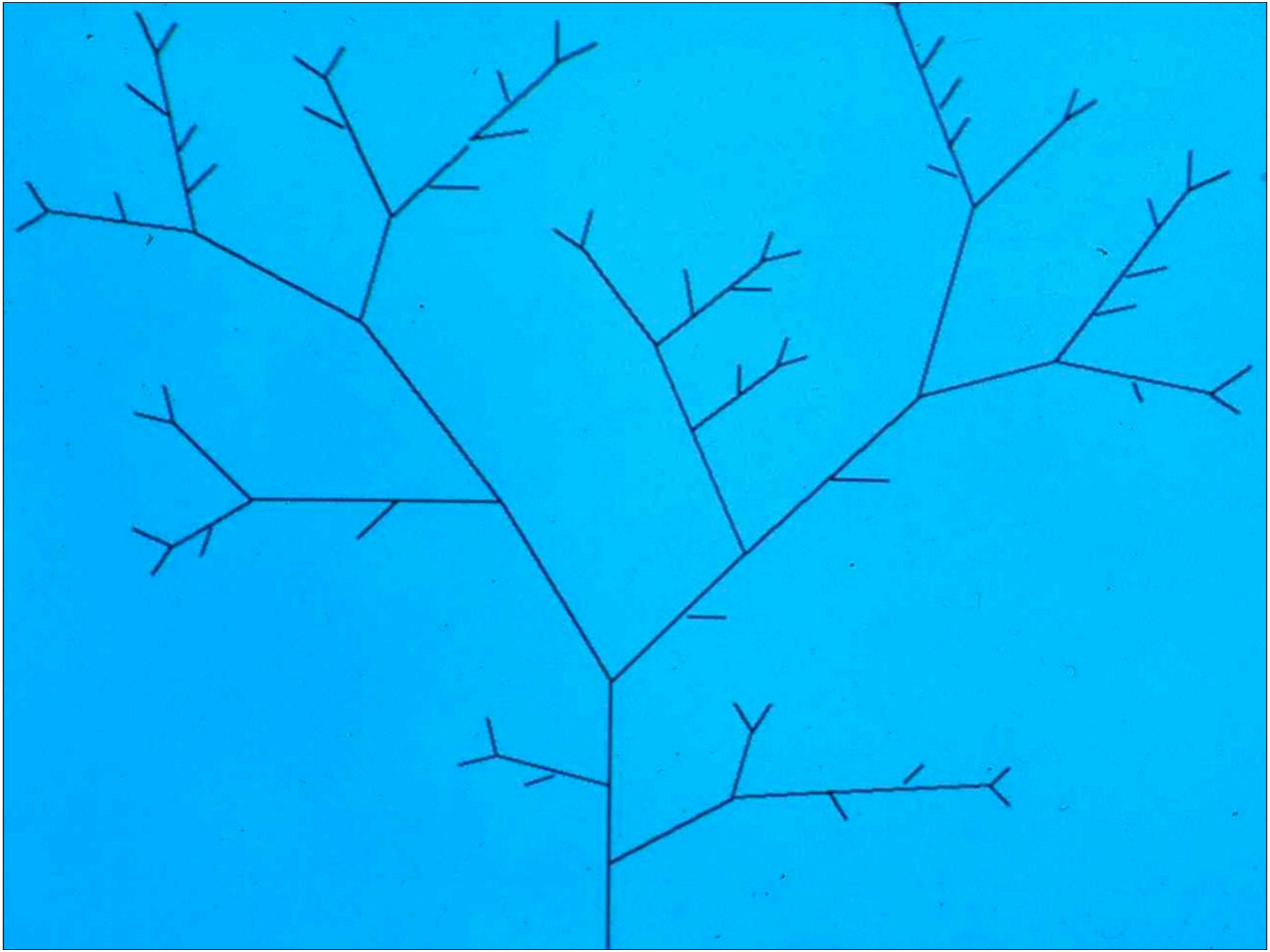

3
= St(B)
nombre de Strahler
3
2
2
2
2
1
2
1
1
1
1
1
1
1


max(k, k′)
k
k′
k+1
k
k



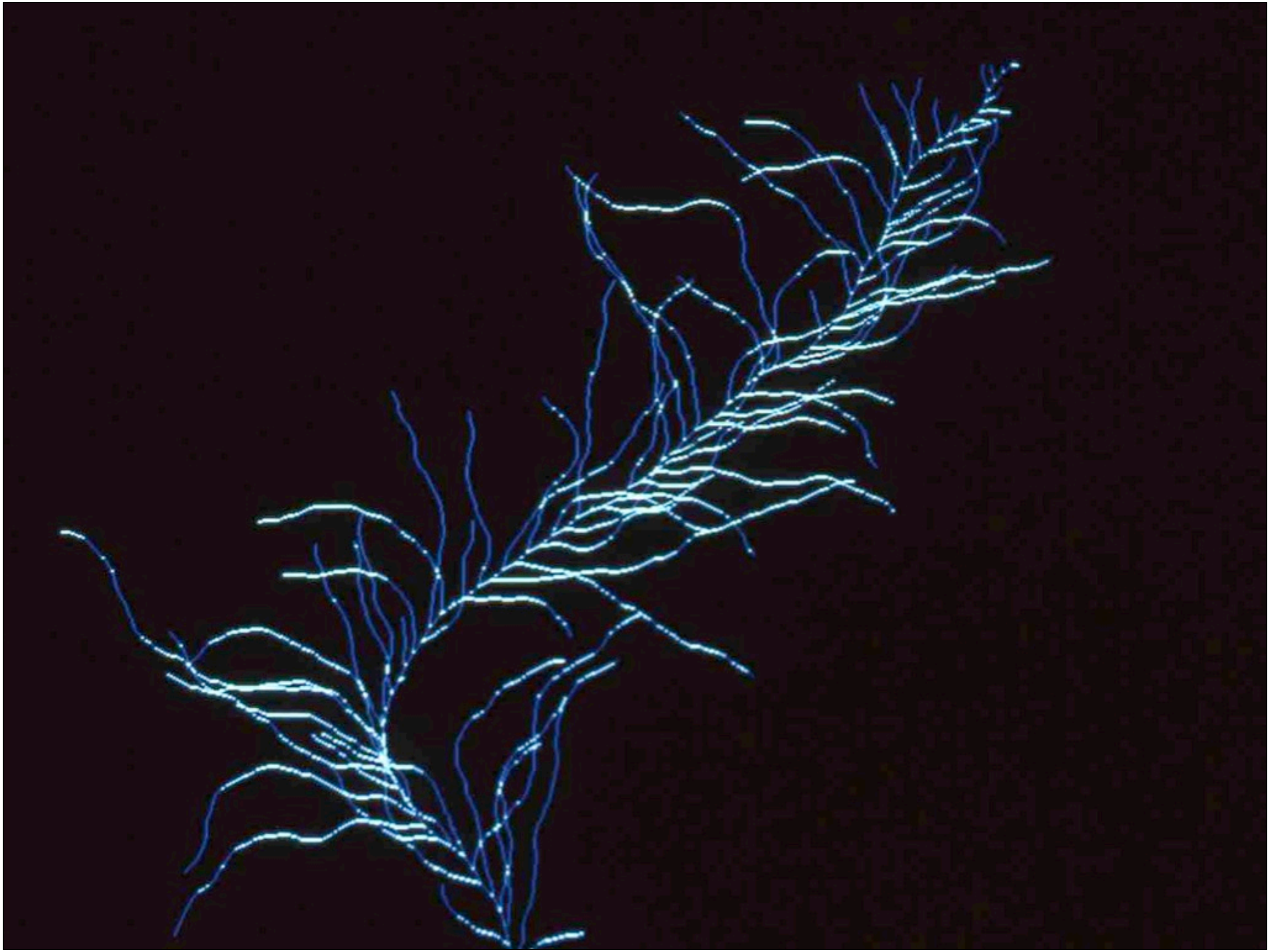
dessine moi un arbre ....





synthèse d'images d'arbres


ASA



S'il existe une quelconque beauté dans ces images de synthèse d'arbres, elle n'est que le pâle reflet de l'extraordinaire beauté des mathématiques se cachant derrière les algorithmes générant ces images.

Nombre de Strahler moyen
d'un arbre linaire ayant n sommets

$A_n =$

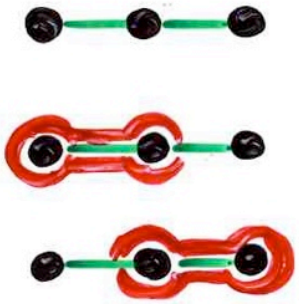
Nombre de Strahler moyen
d'un arbre binaire ayant n sommets

$$A_n = \log_4 n +$$

Nombre de Strahler moyen
d'un arbre binaire ayant n sommets

$$A_n = \log_4 n + f(\log_4 n) + \dots$$

périodique



x^3

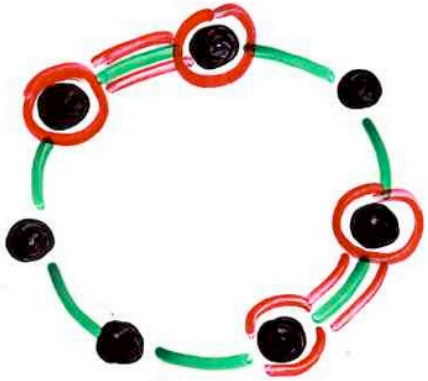
$-x$

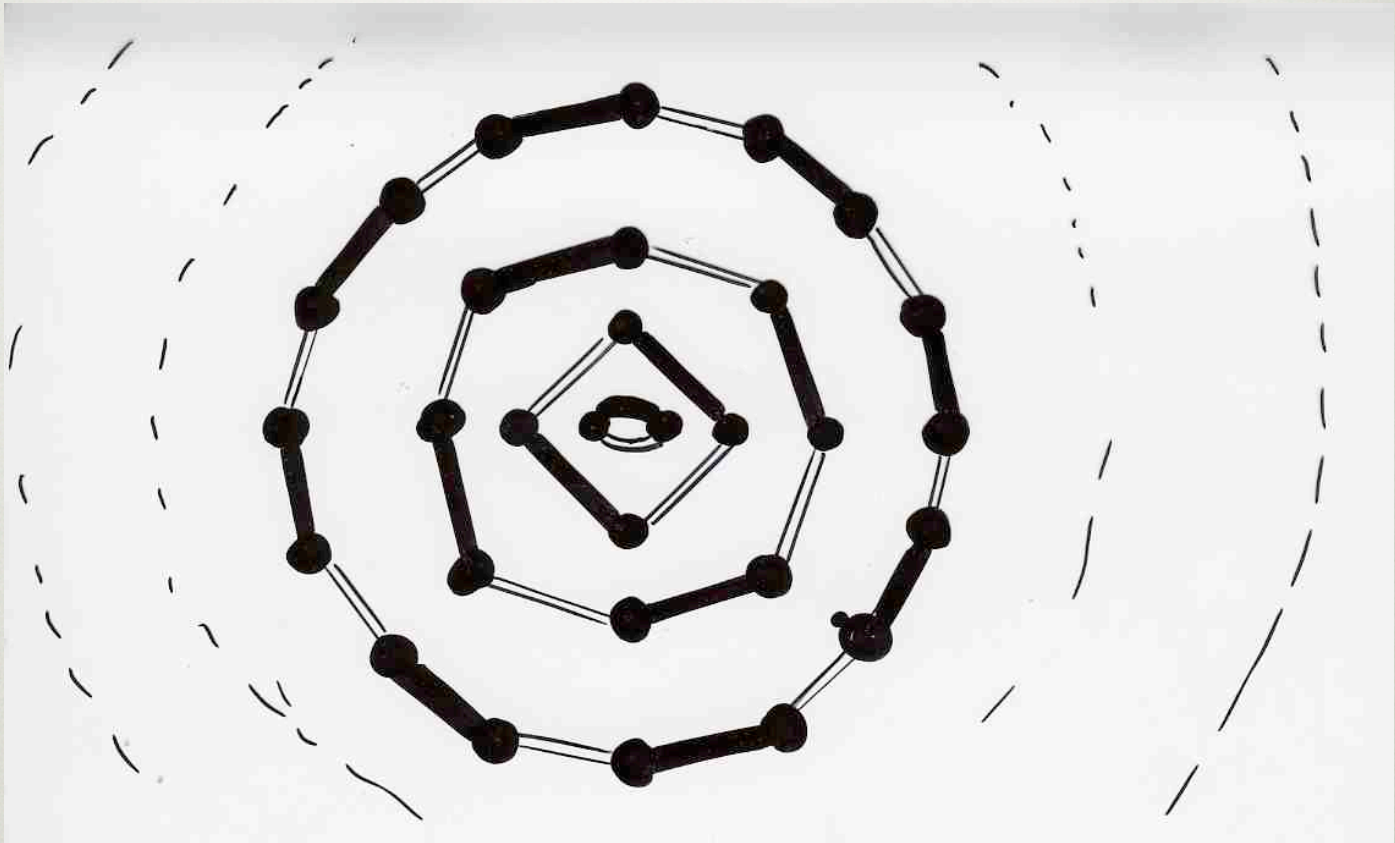
$-x$

$$F_3(x) = x^3 - 2x$$

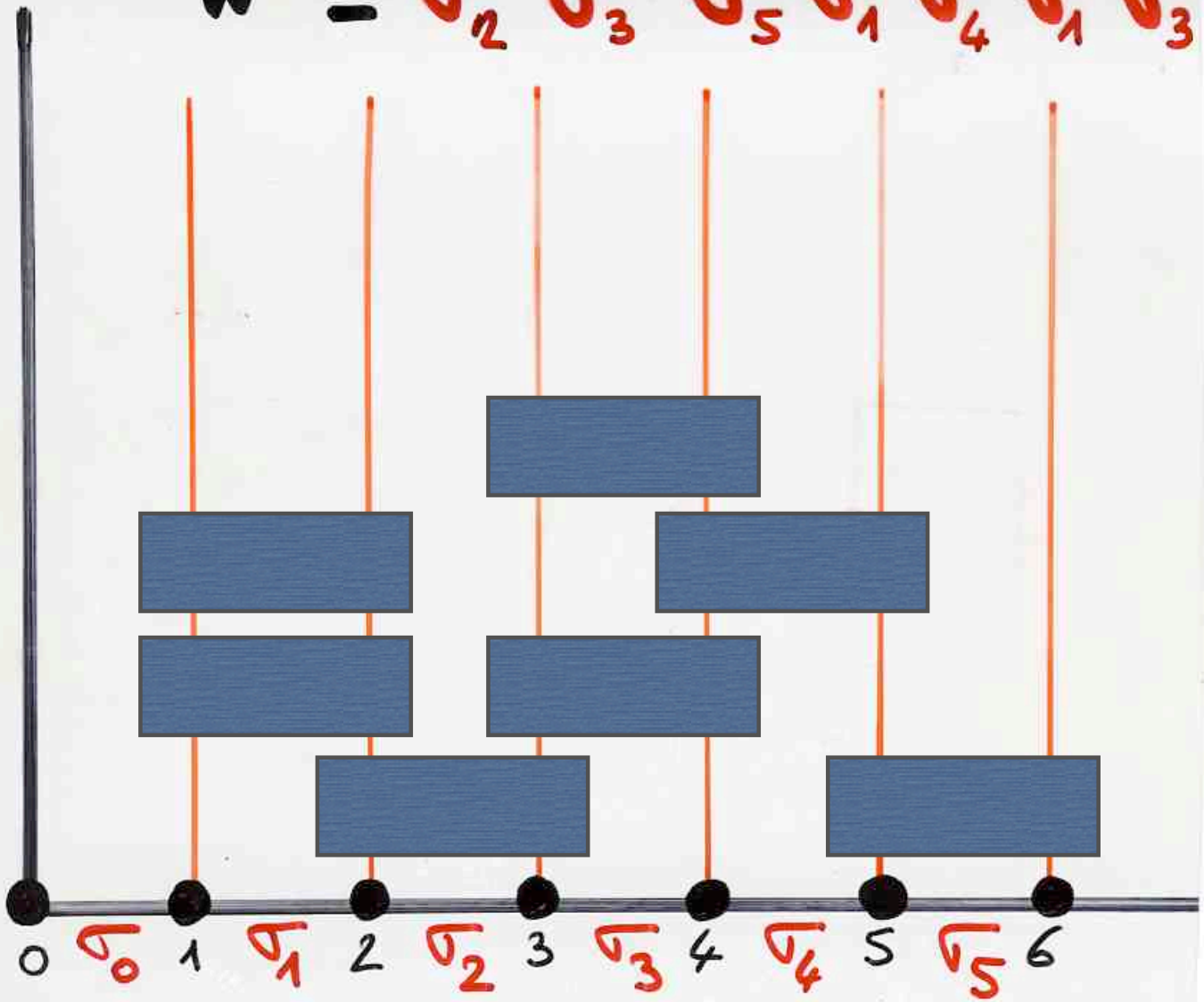
$$\sin(4\theta) = \sin\theta\left[8\cos^3\theta - 4\cos\theta\right]$$

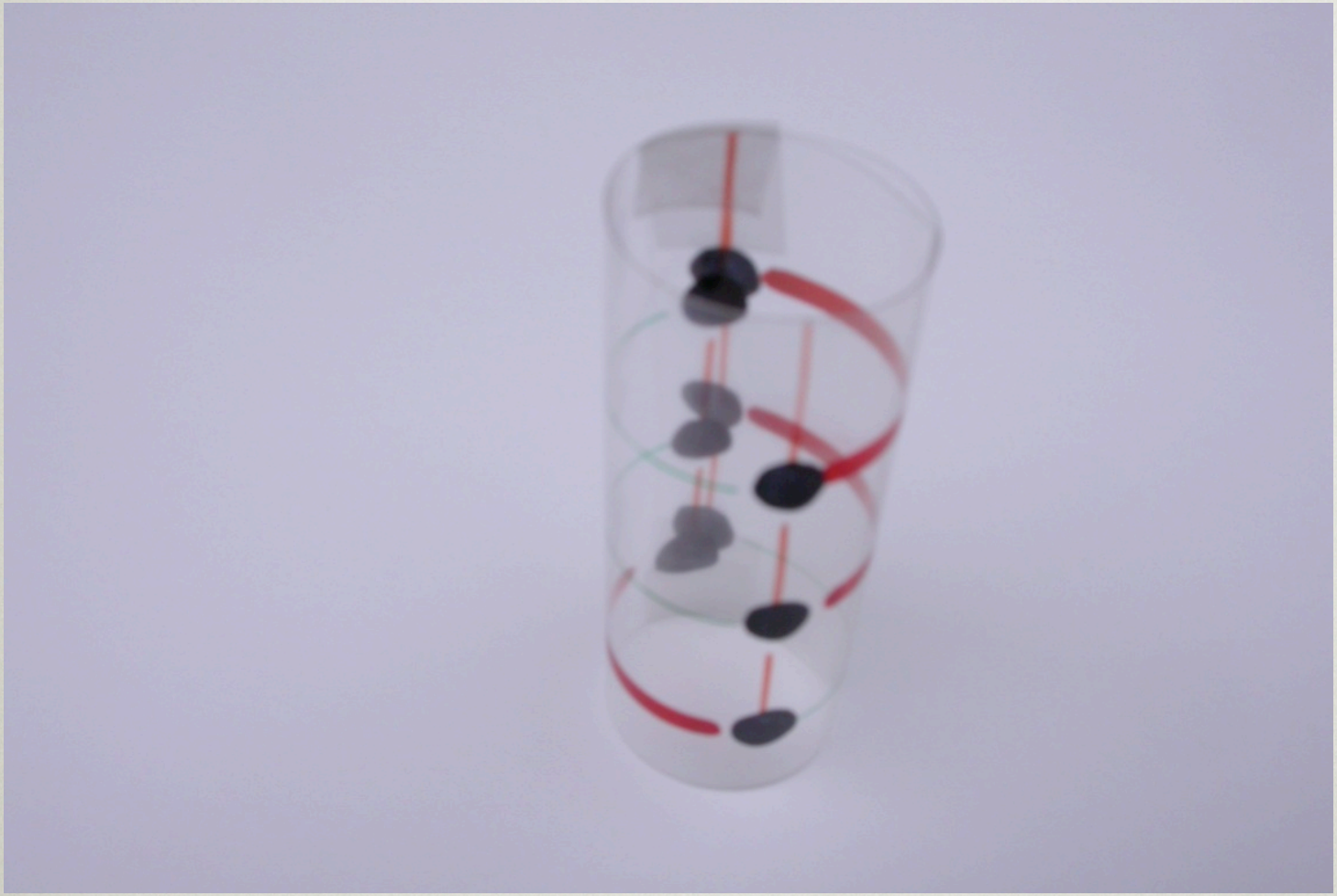
$$\cos(n\theta) = T_n(\cos\theta)$$

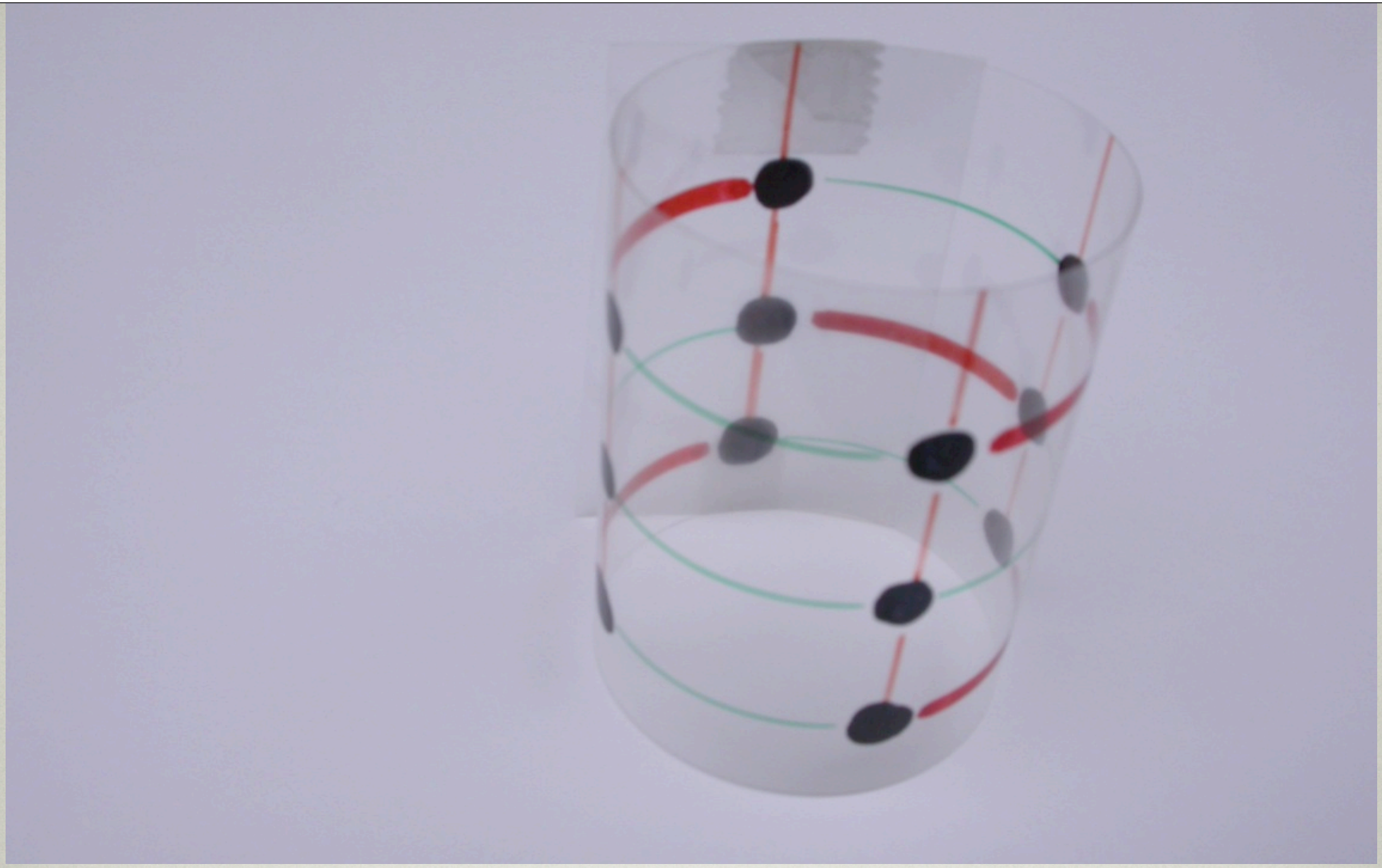


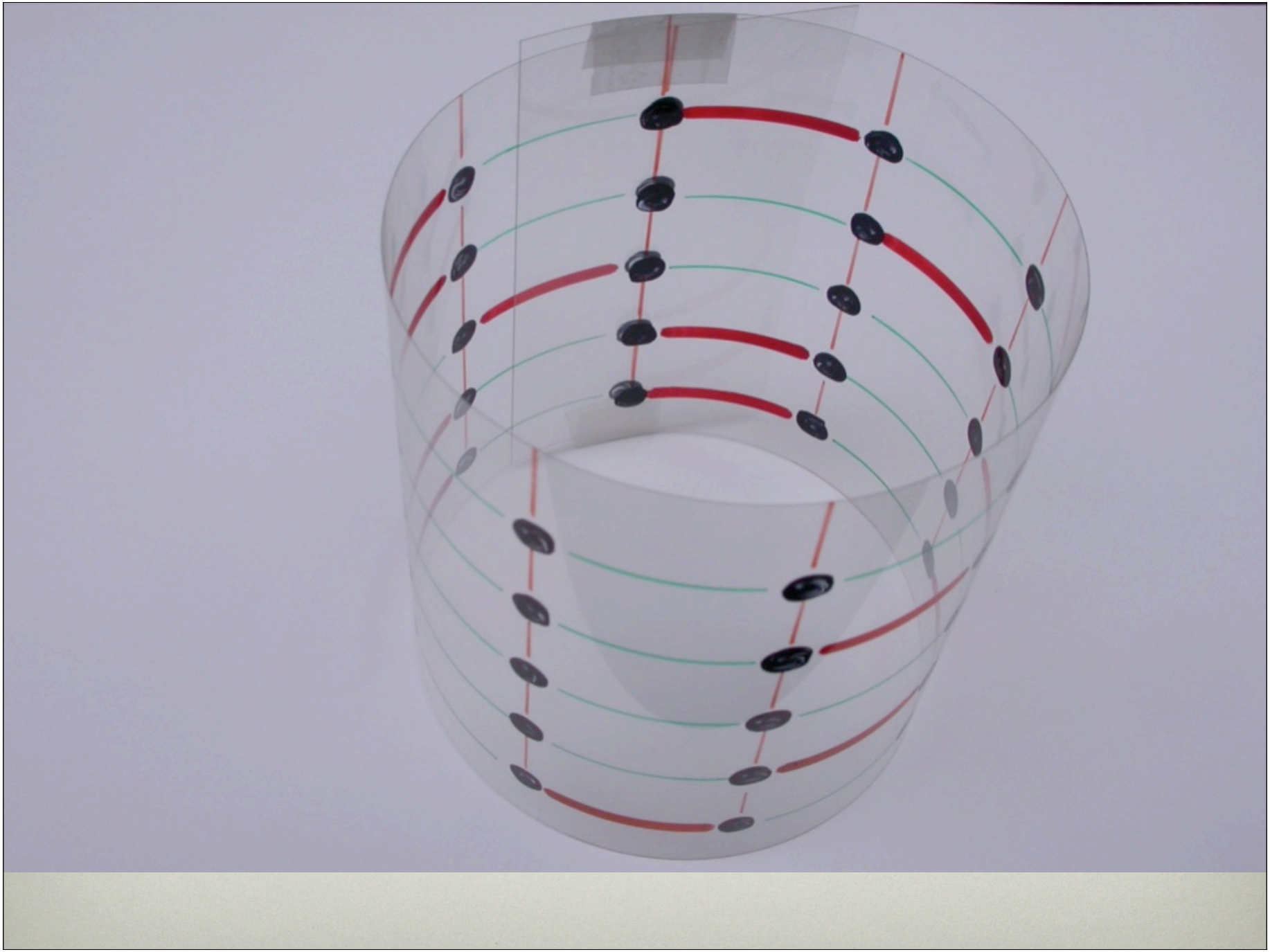


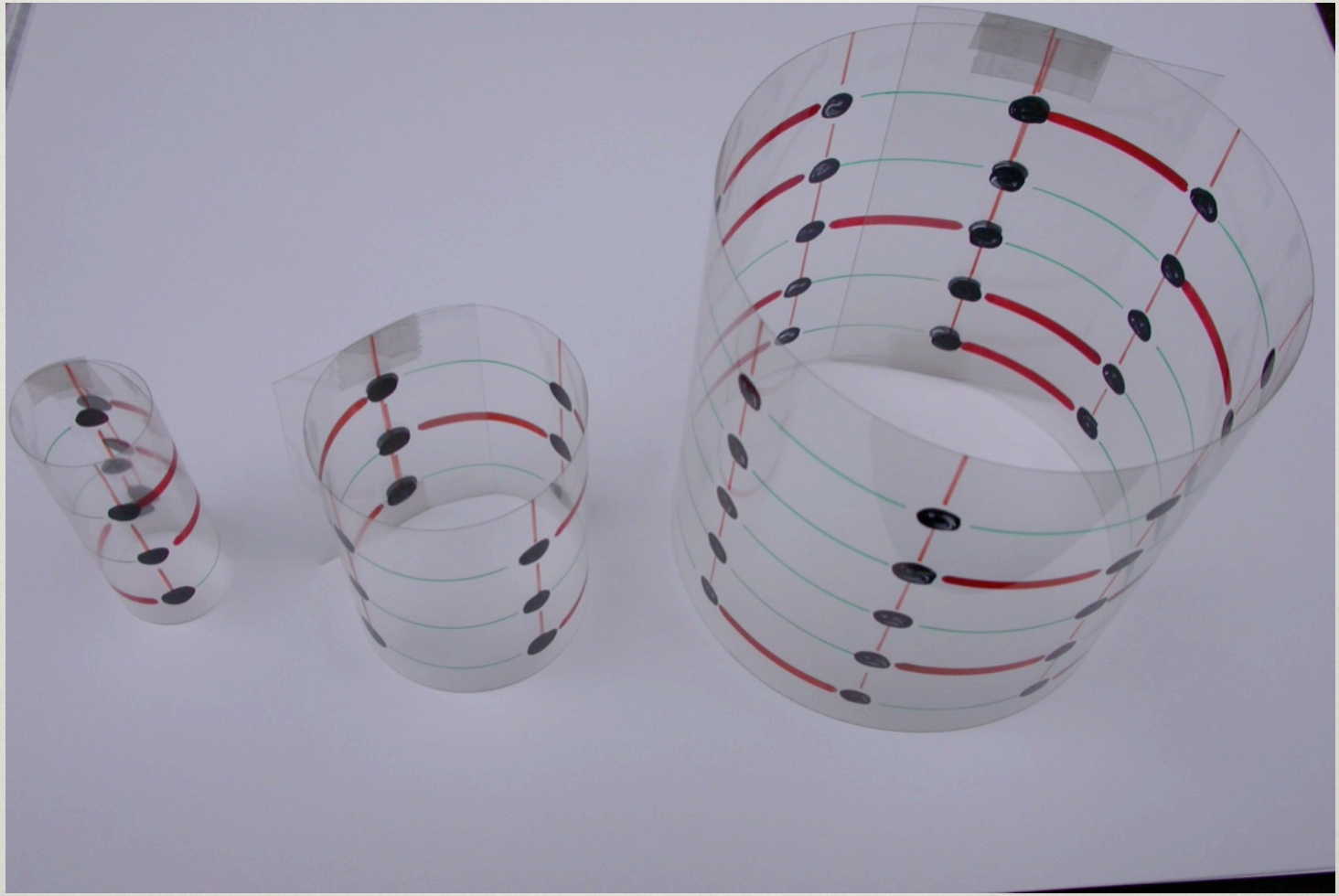
$$w = \sigma_2\,\sigma_3\,\sigma_5\,\sigma_1\,\sigma_4\,\sigma_1\,\sigma_3$$

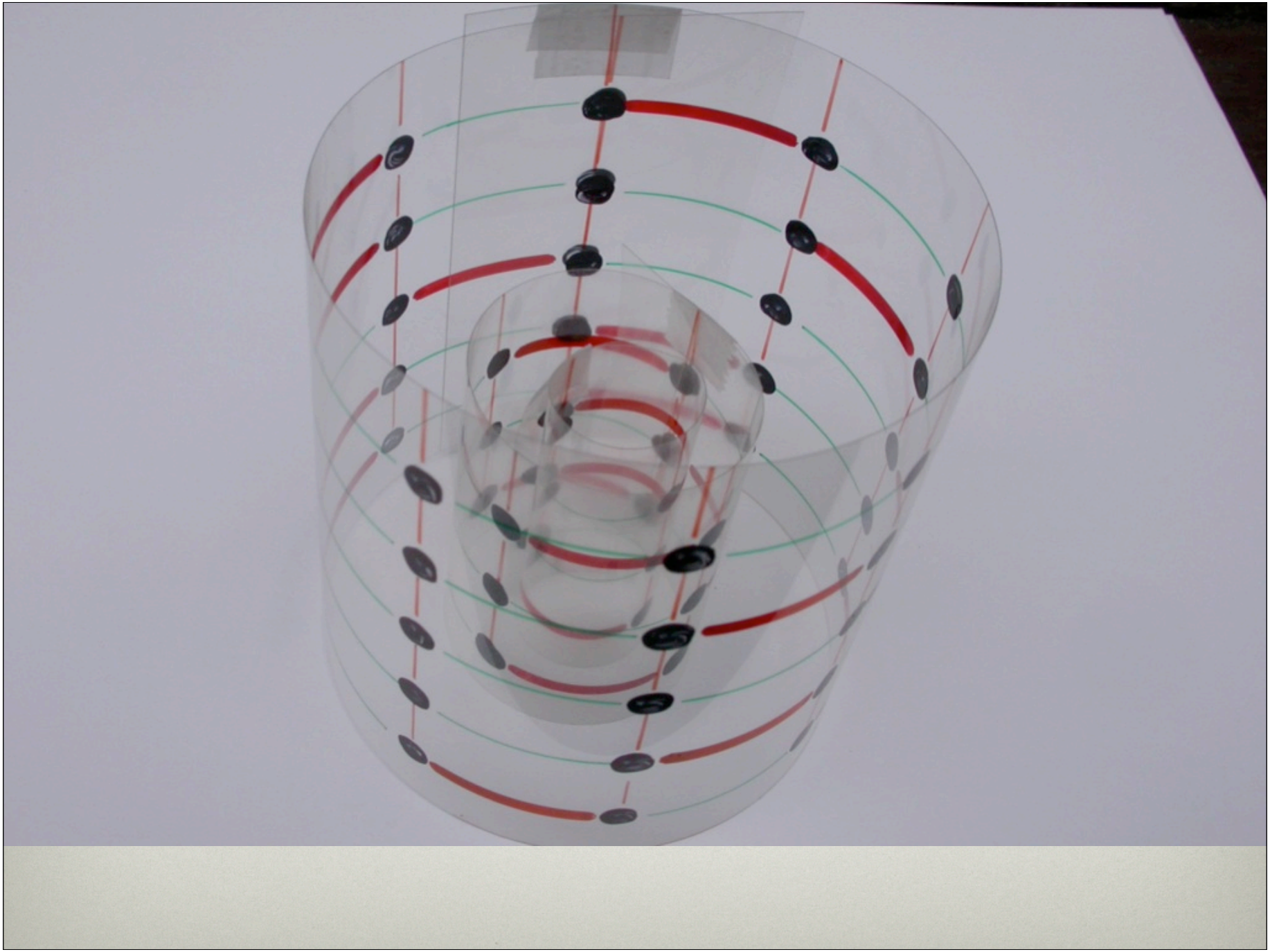


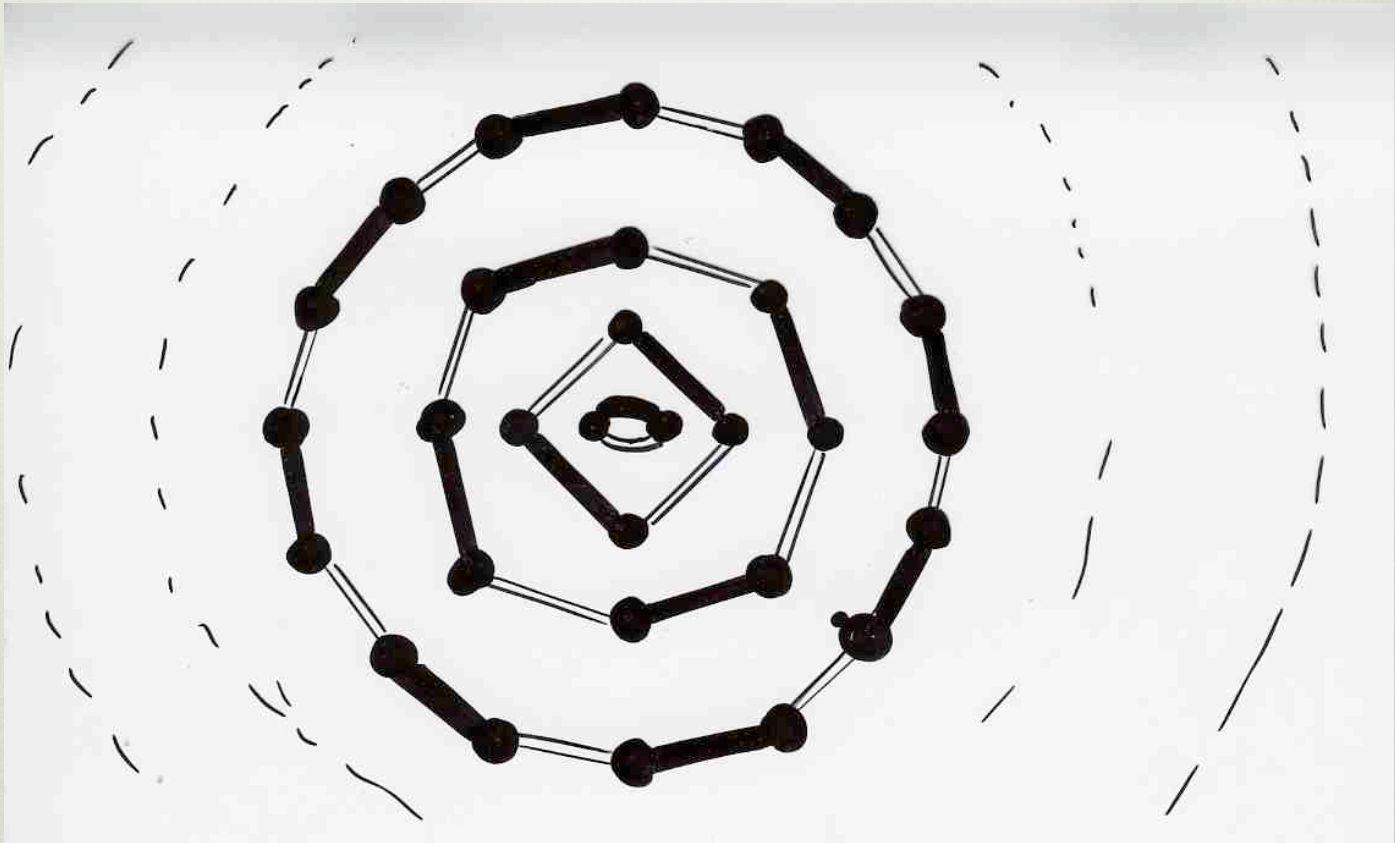


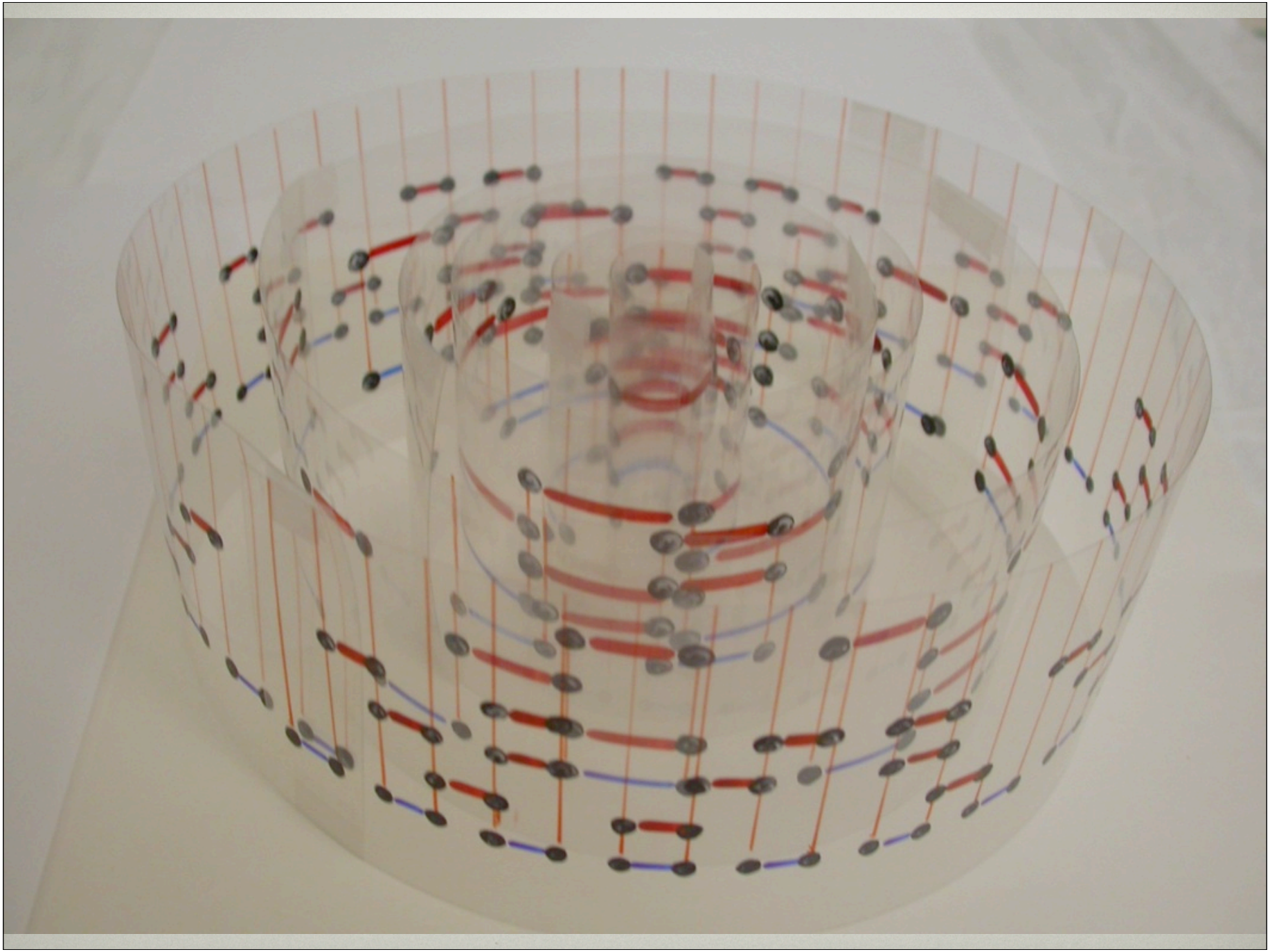




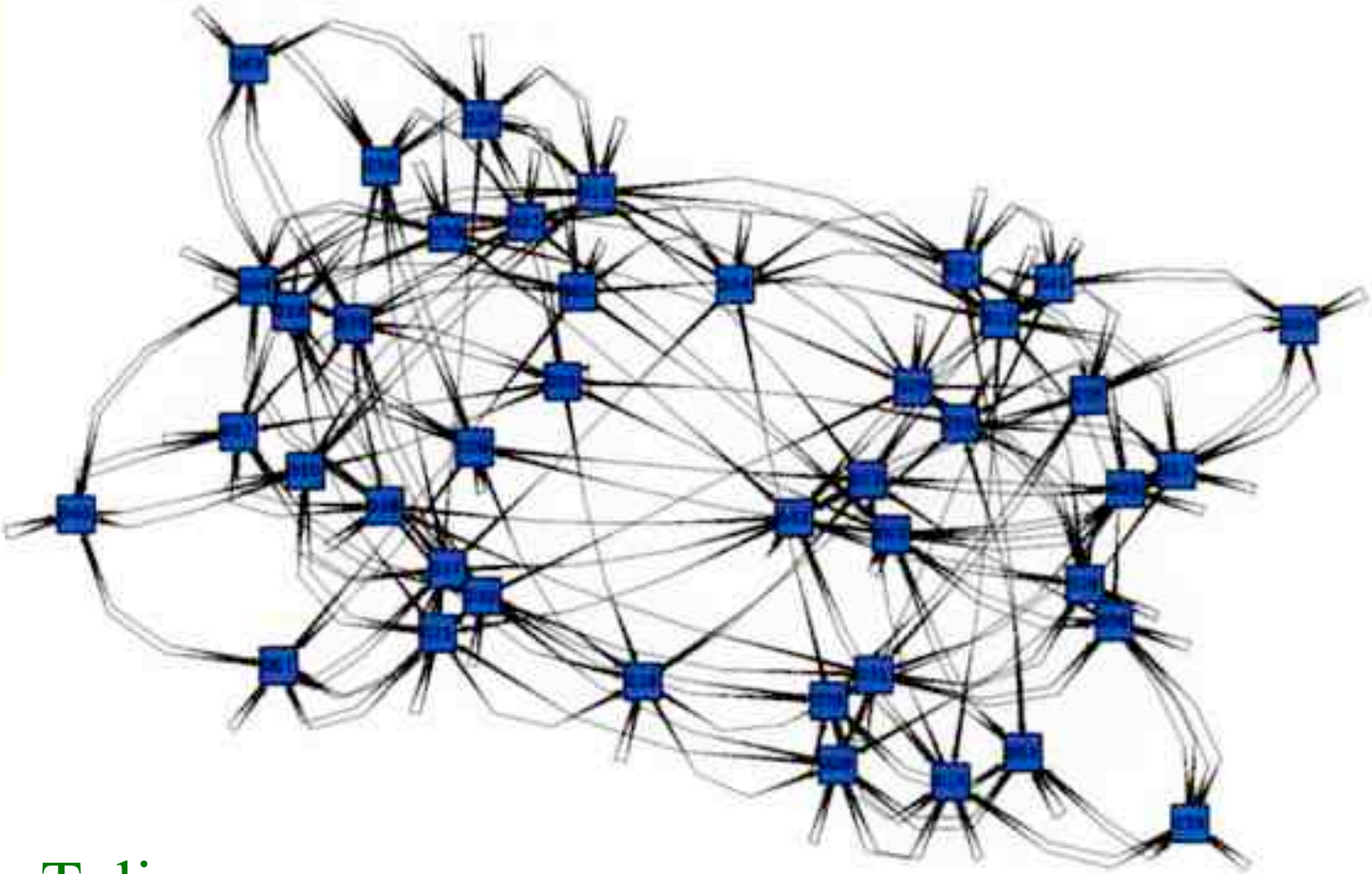








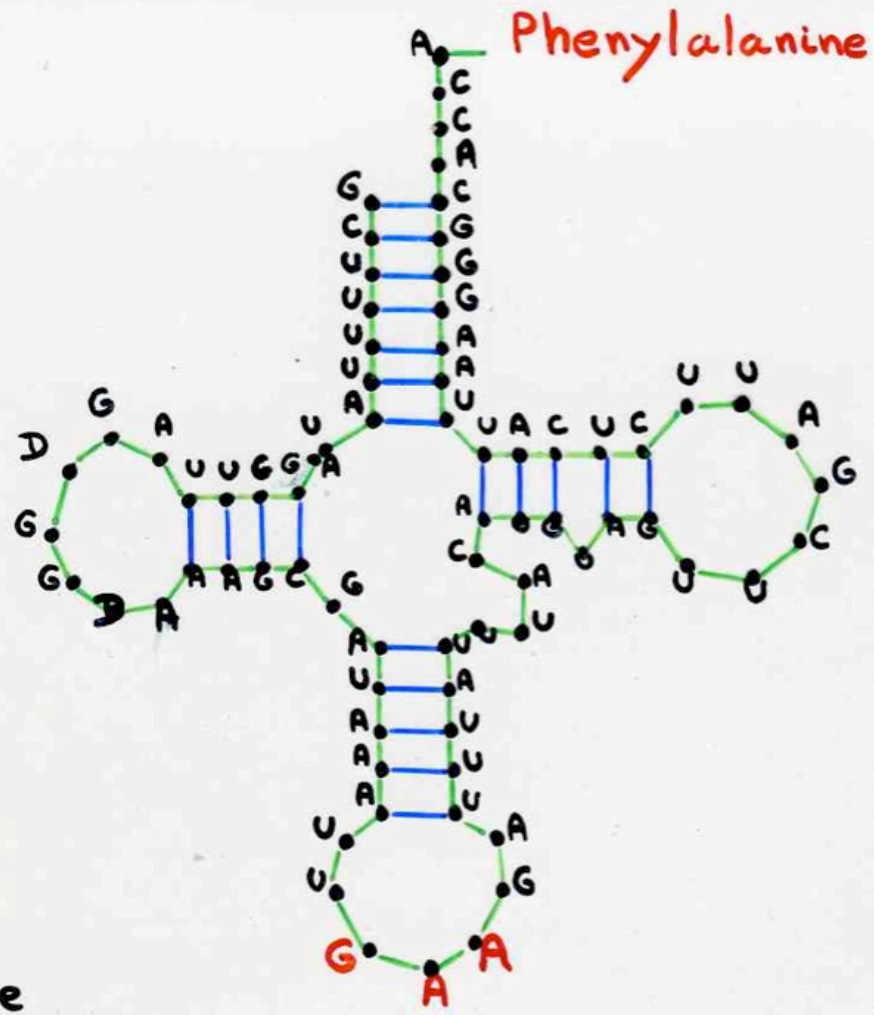
visualisation de l'information ...



Tulip

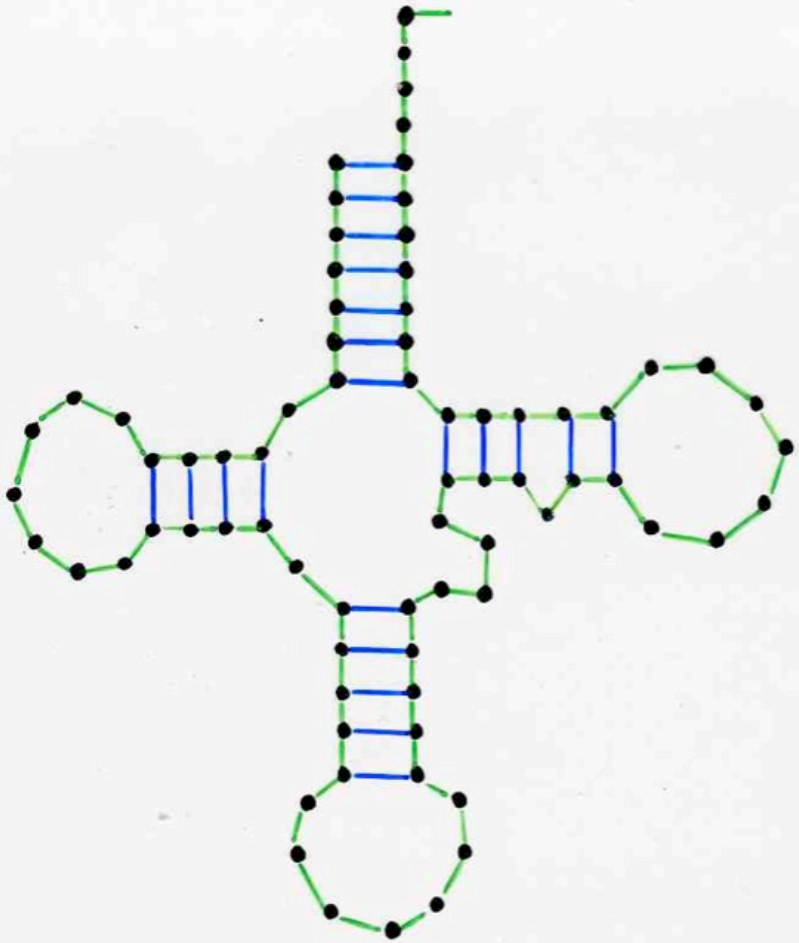
D. Auber

des arbres dans les molécules ...

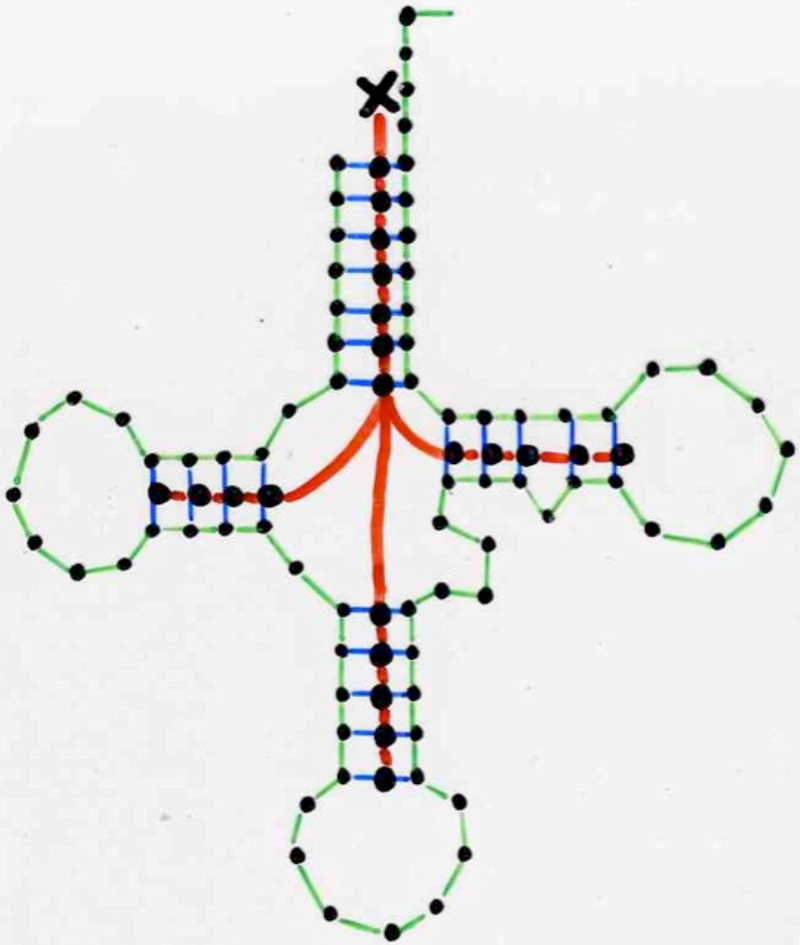


A denine
U racyle
G uanine
C ytosine

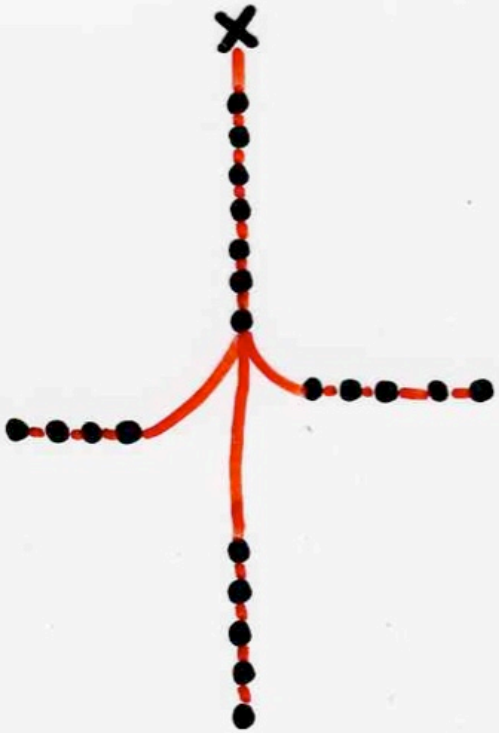
t ARN Phe



tARN Phe



tARN Phe


X

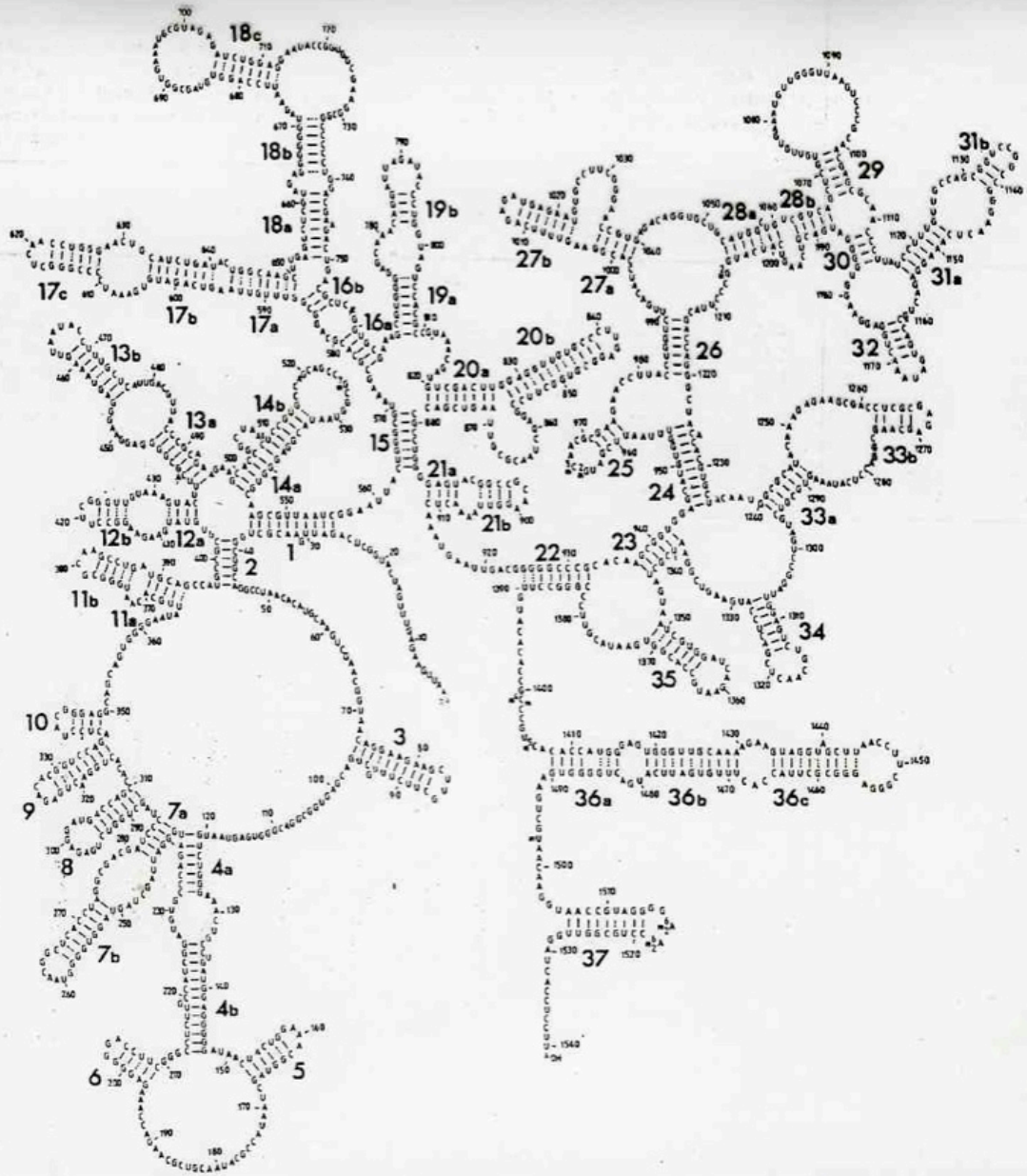


Fig. 1. *Secondary structure model of the 16-S RNA from* E. coli. This model has been fully described elsewhere [18]. The various secondary structure motifs are numbered for reference. Base-pairings 2 and 23 are included in this up-dated scheme and slight modifications have been introduced into helices 18b and 33b

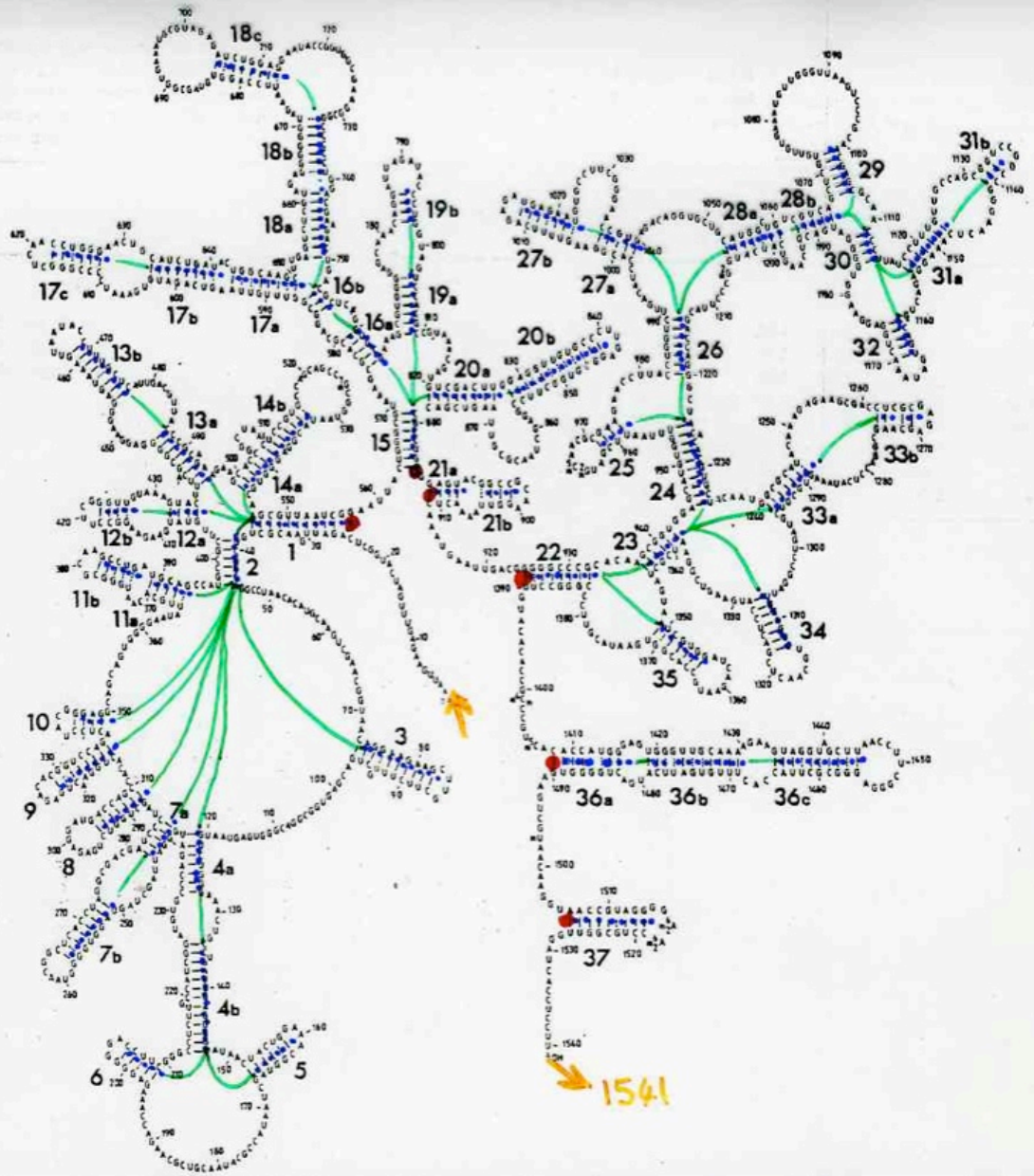


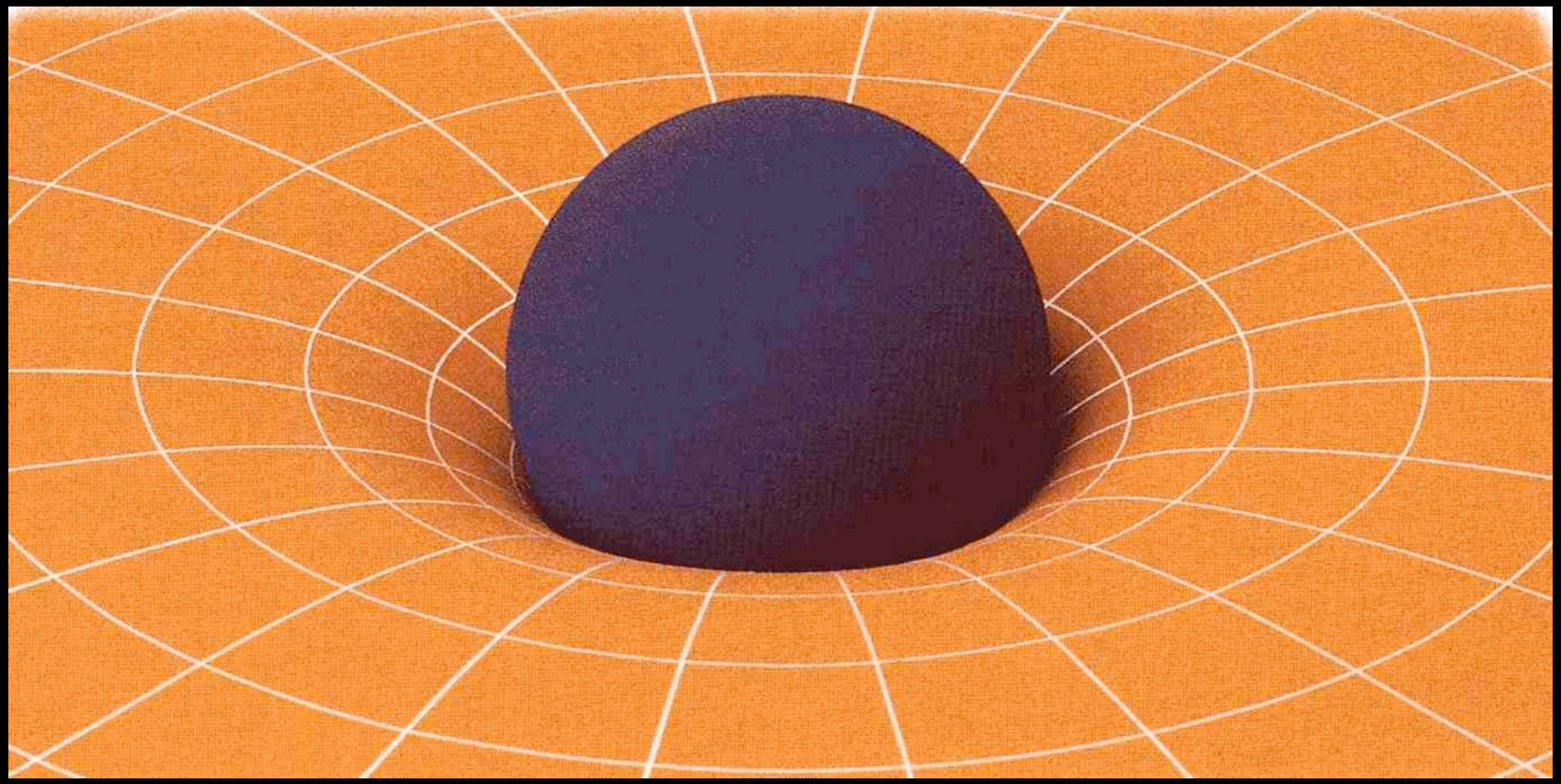
Fig. 1. *Secondary structure model of the 16-S RNA from* E. coli. This model has been fully described elsewhere [18]. The various secondary structure motifs are numbered for reference. Base-pairings 2 and 23 are included in this up-dated scheme and slight modifications have been introduced into helices 18b and 33b

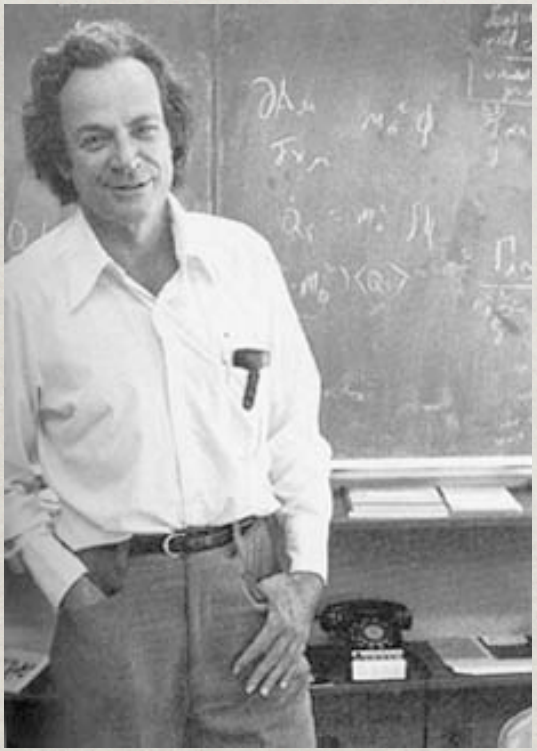
Electrodynamique quantique

Richard Feynman

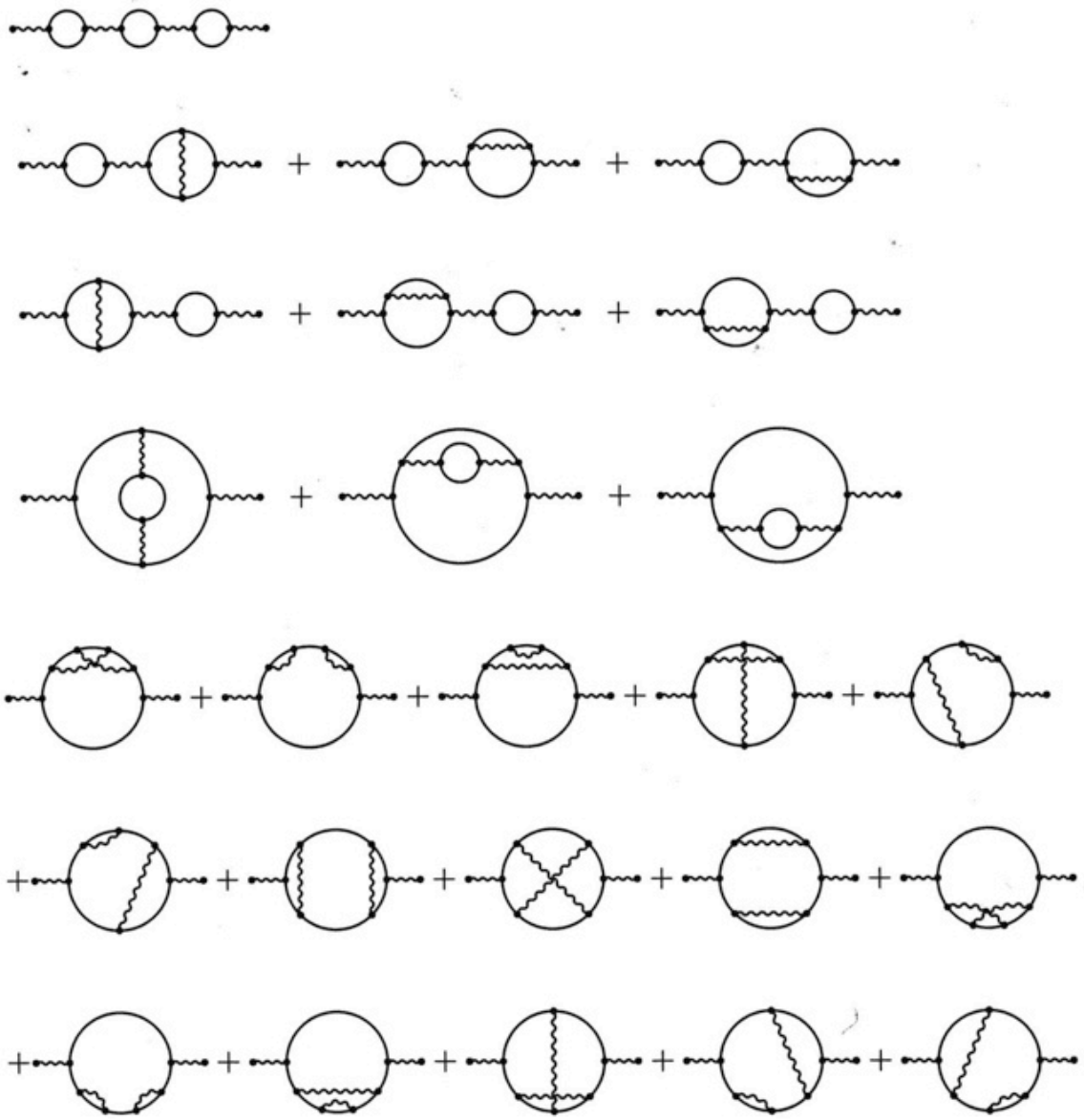
Renormalisation

Alain Connes





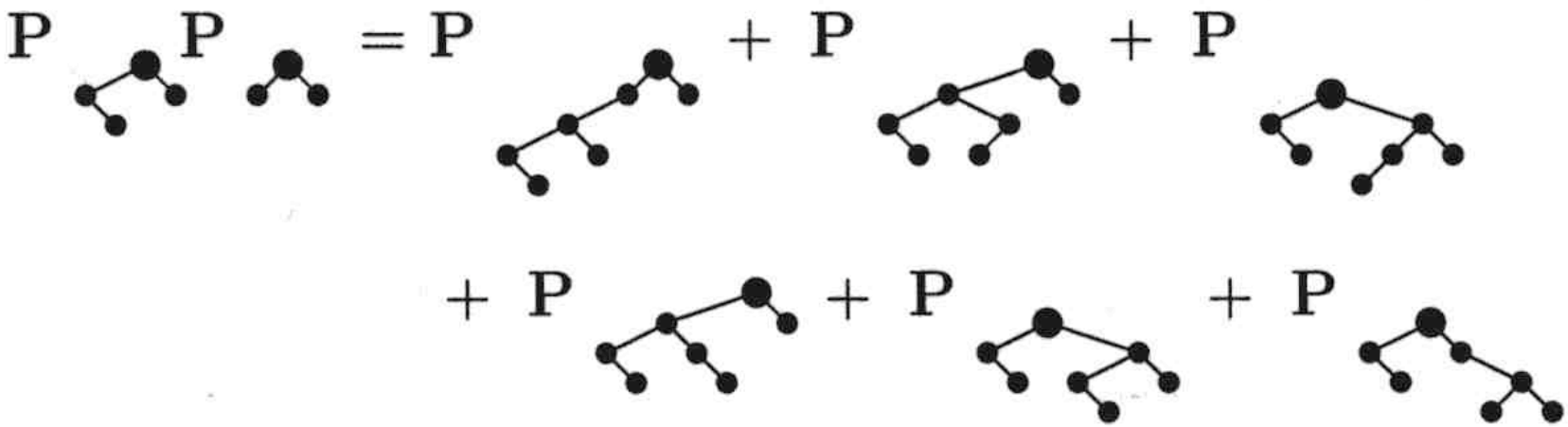
Richard Feynman



Alain Connes

Géométrie non commutative

Renormalisation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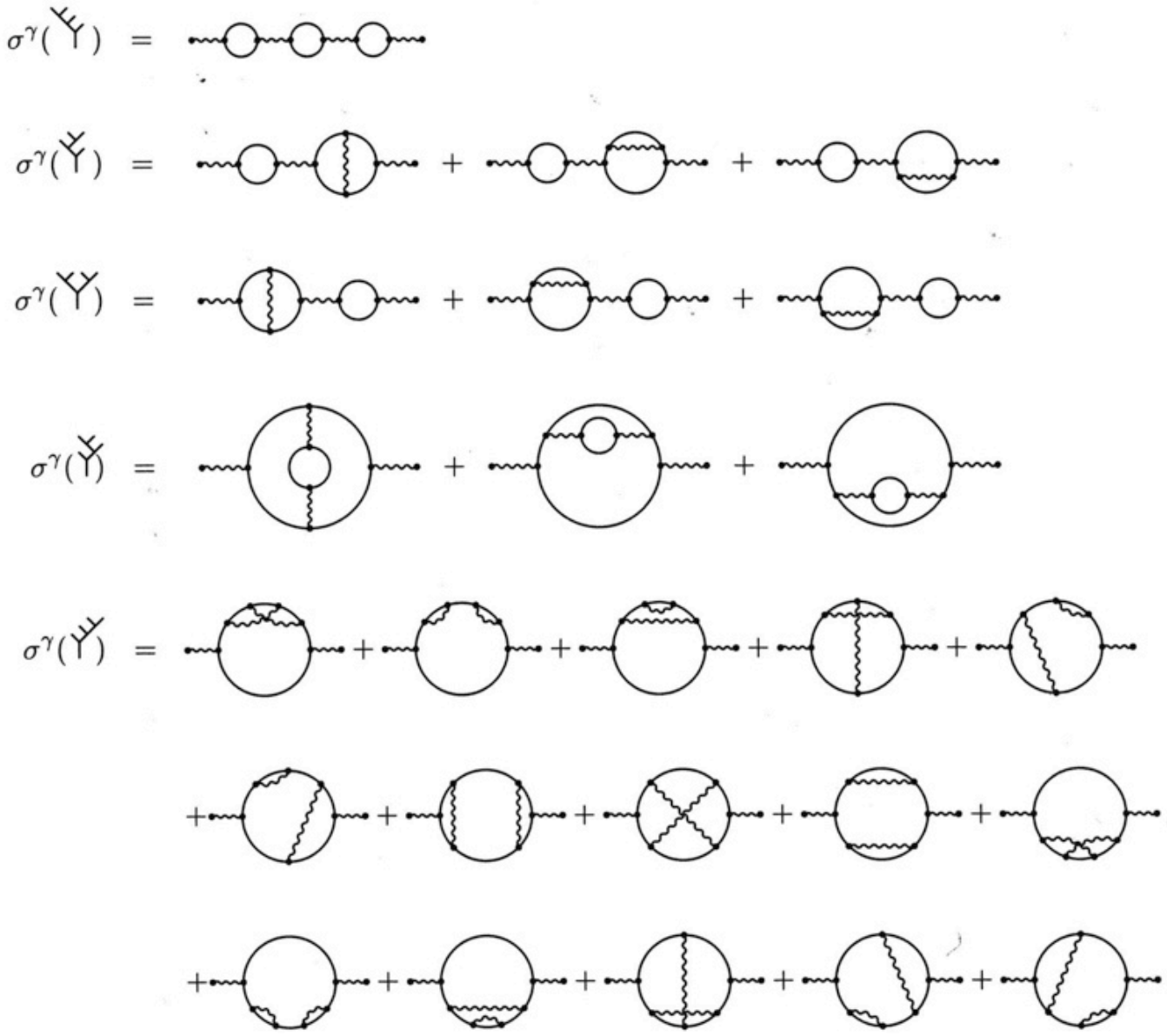
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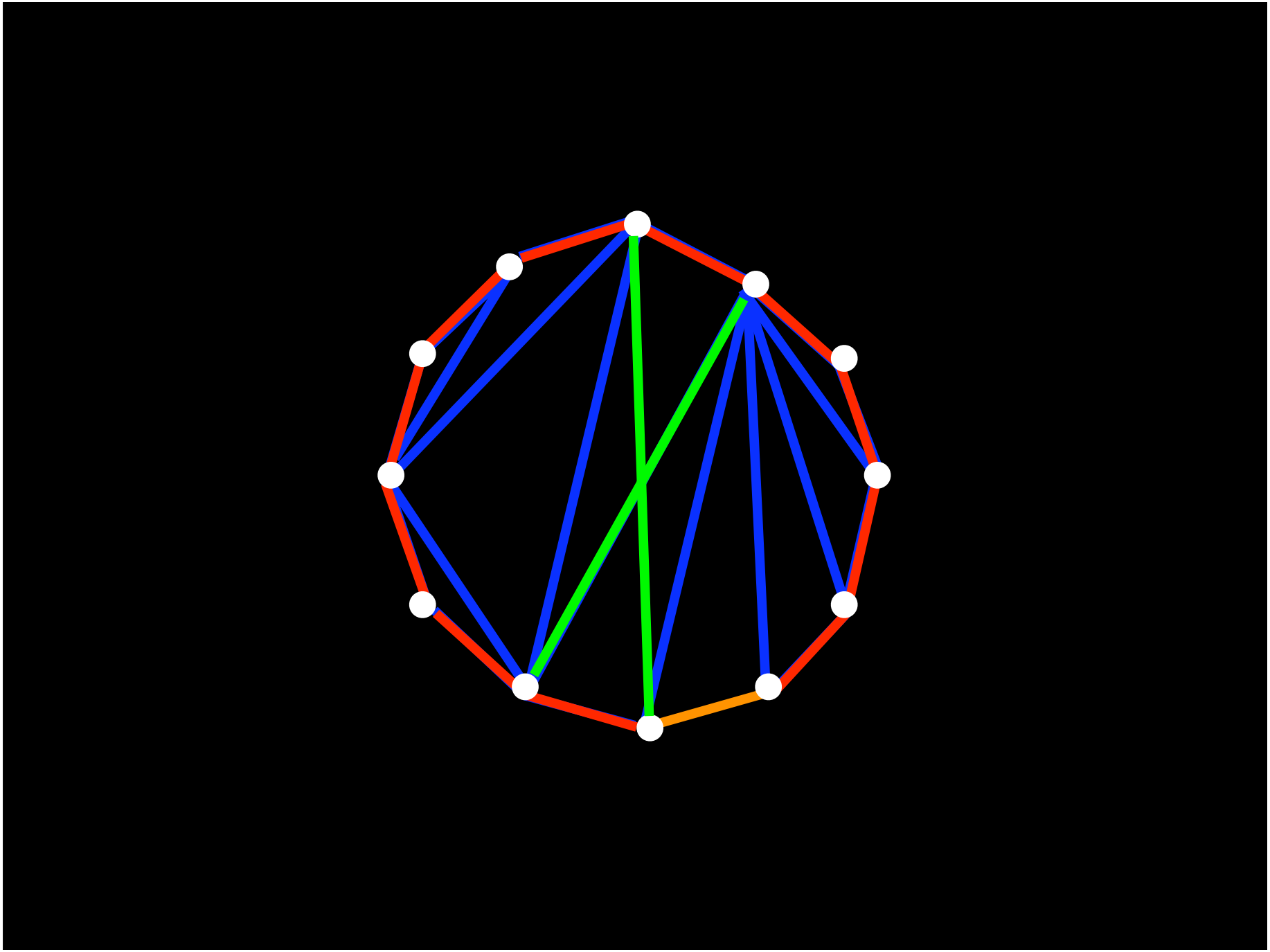
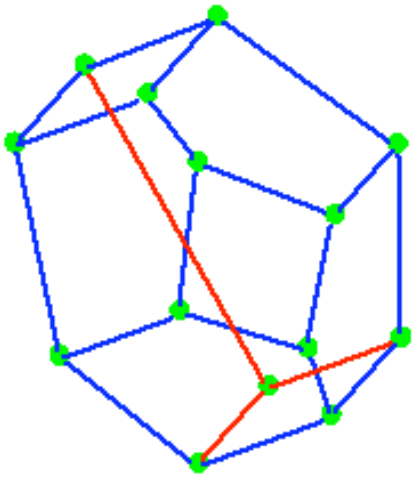


Physique combinatoire



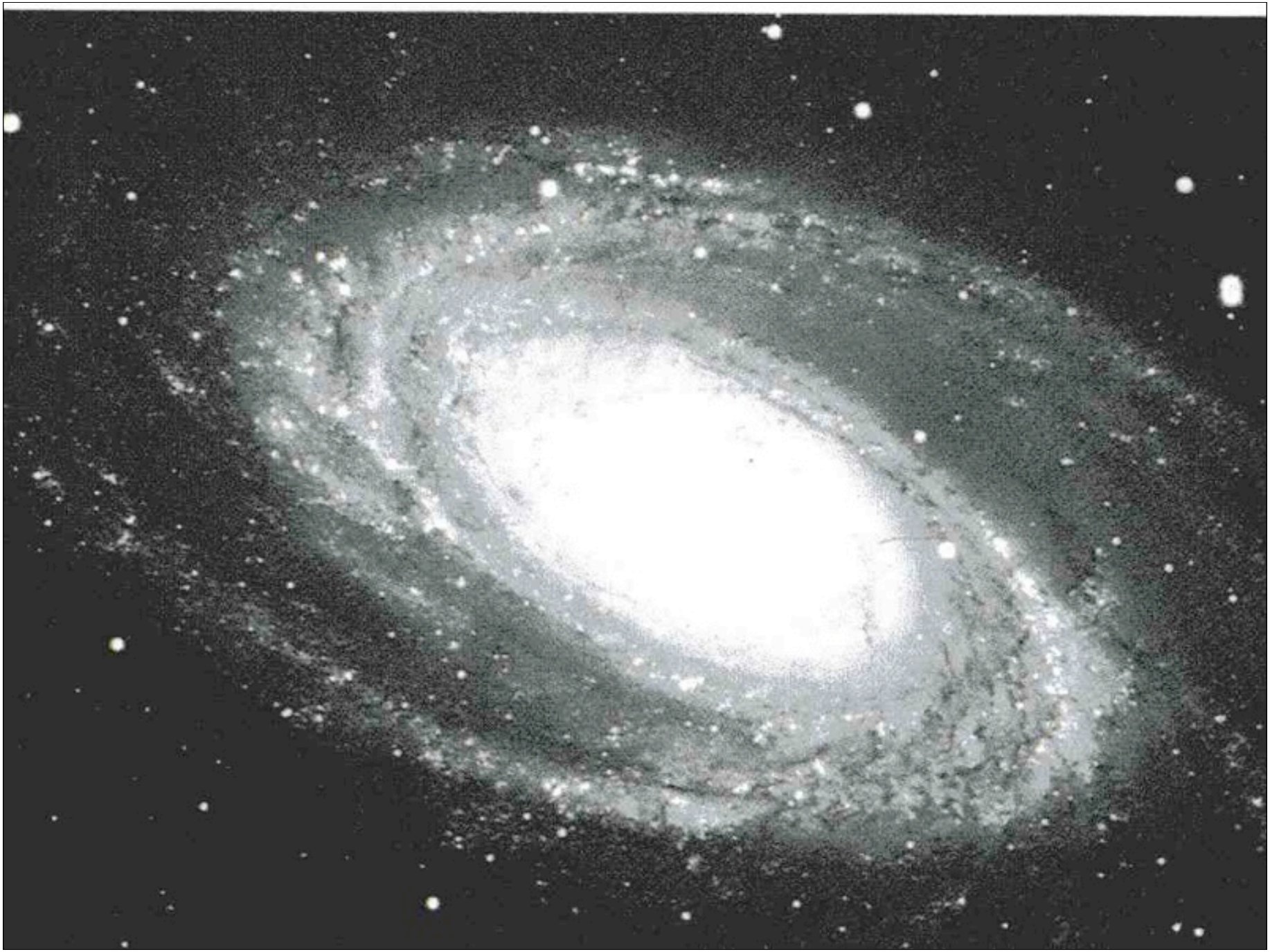
Leonhard
Euler

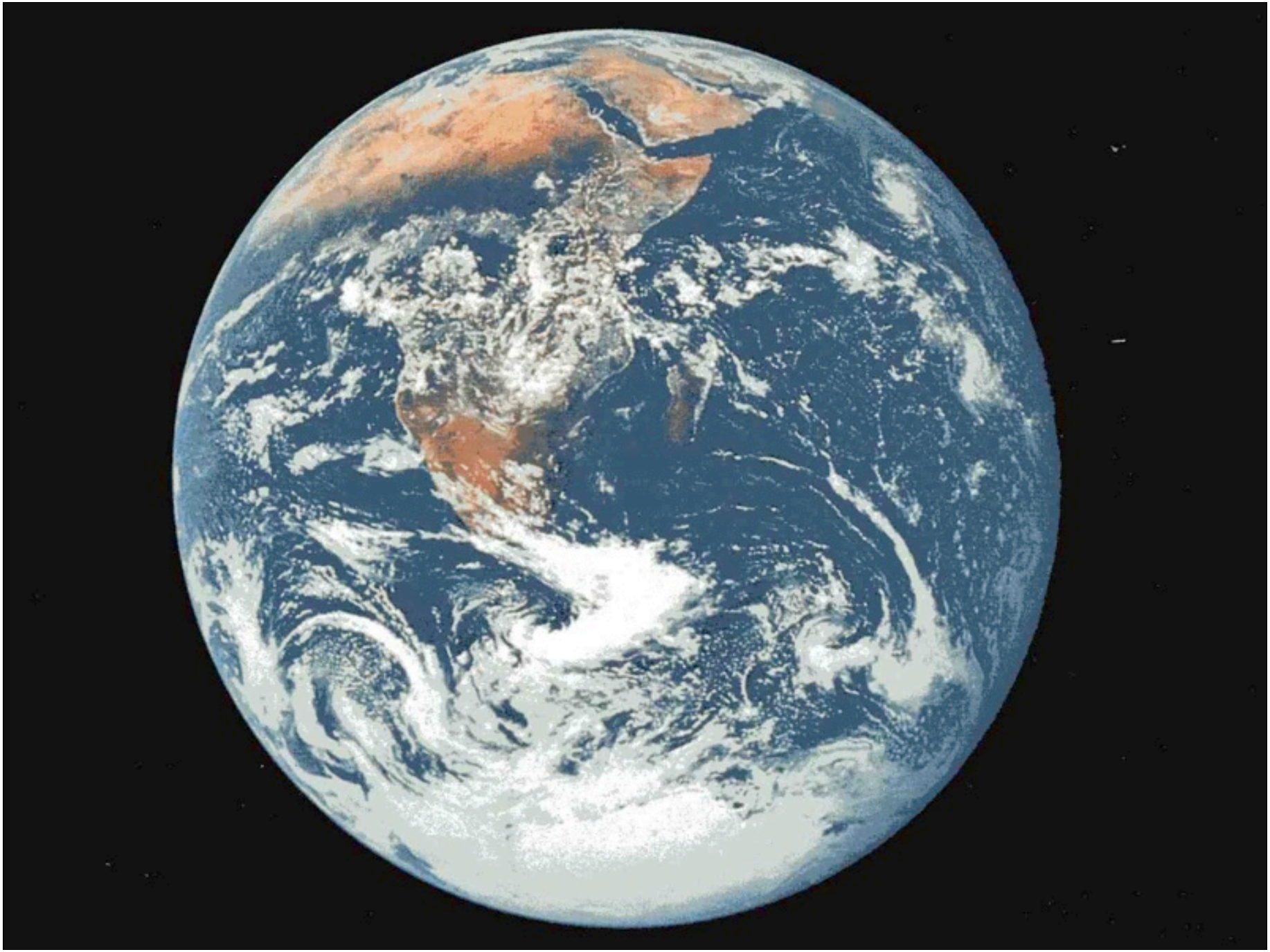
Il y a des arbres dans les étoiles,
des arbres dans les grains de lumière.

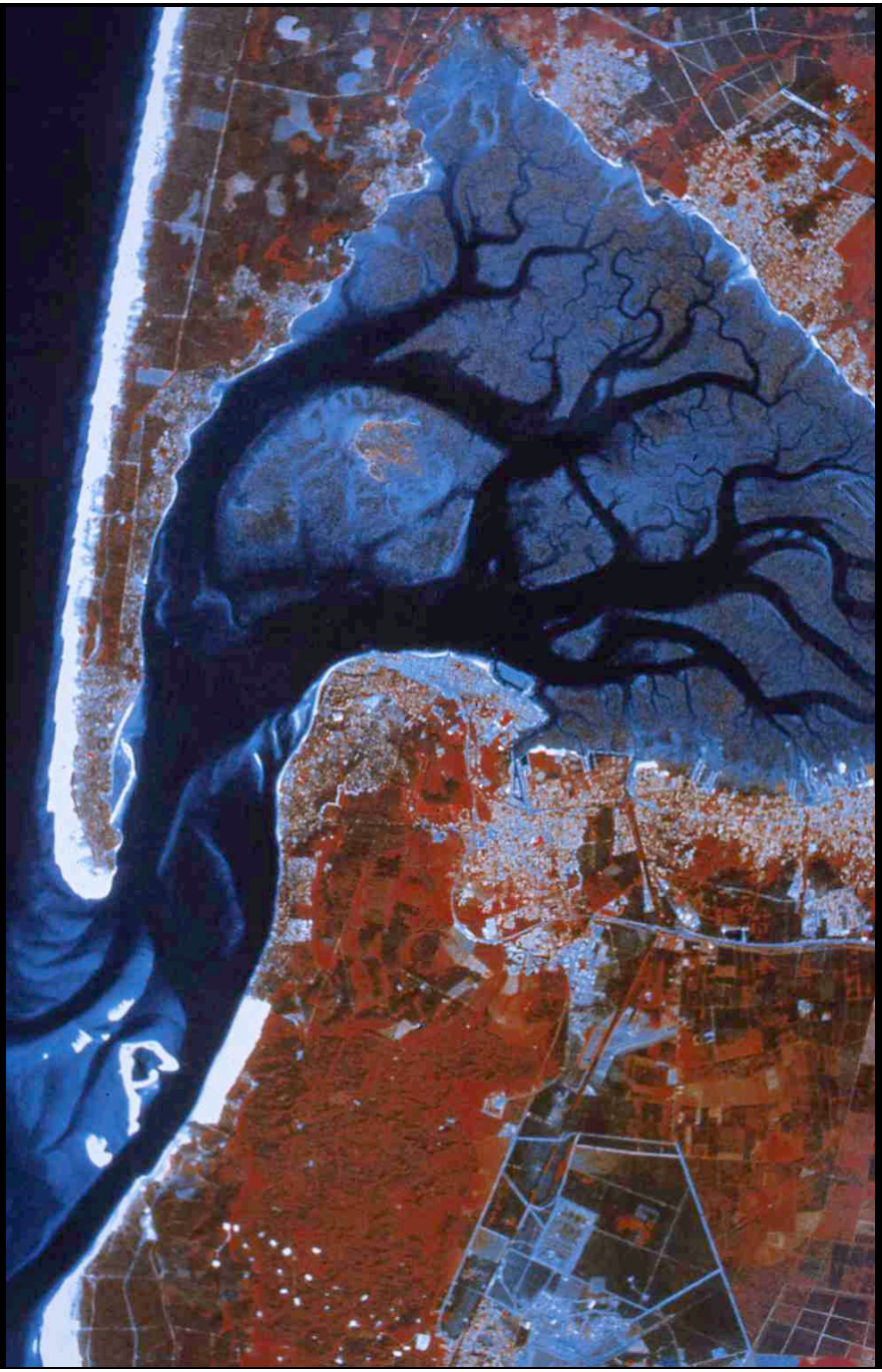
Les théories mathématiques s'interpellent,
s'entrecroisent, renaissent, se fondent entre elles.

Les grands Maîtres se parlent à travers les siècles
dans le jardin merveilleux des Mathématiques.

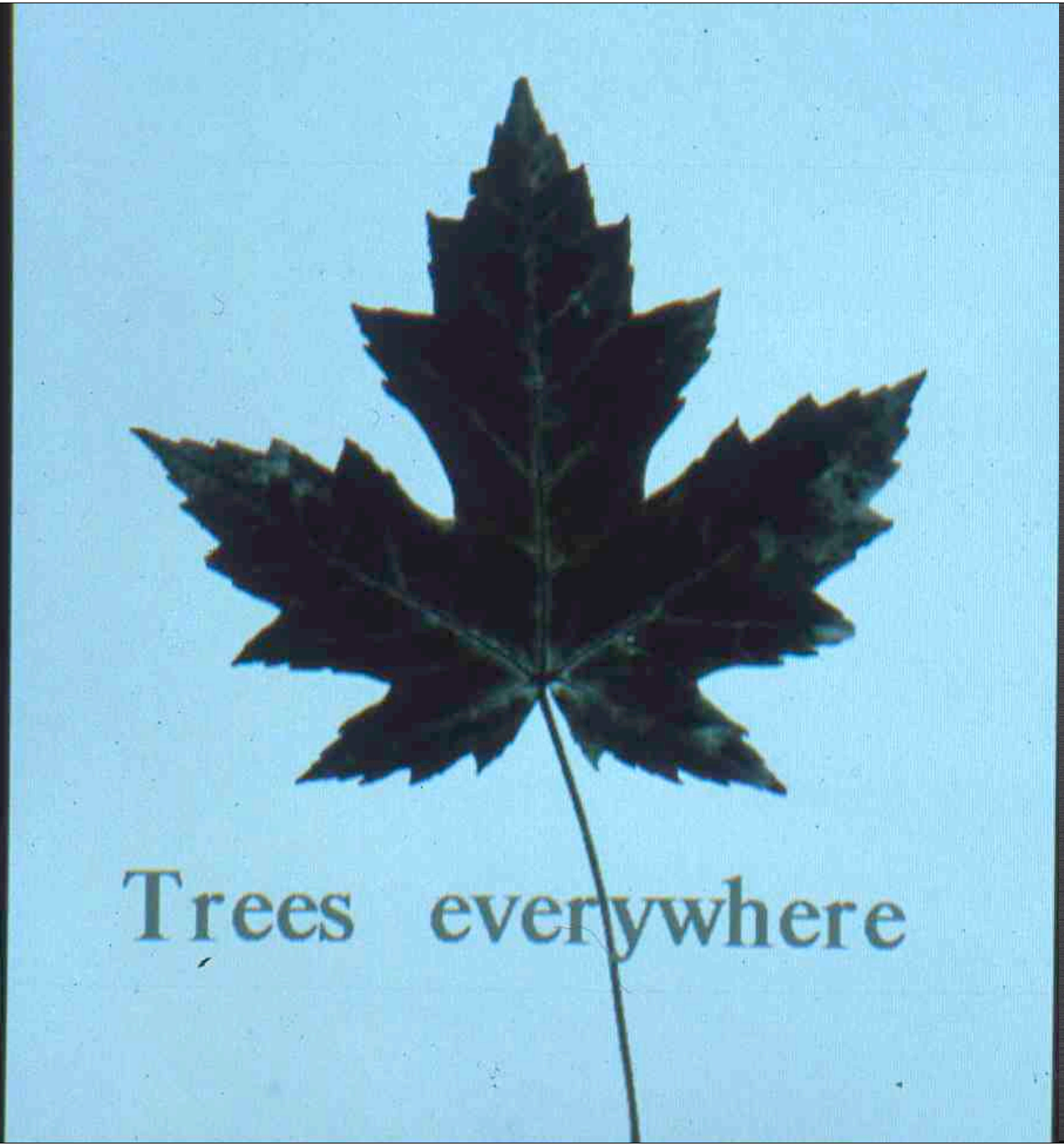
épilogue

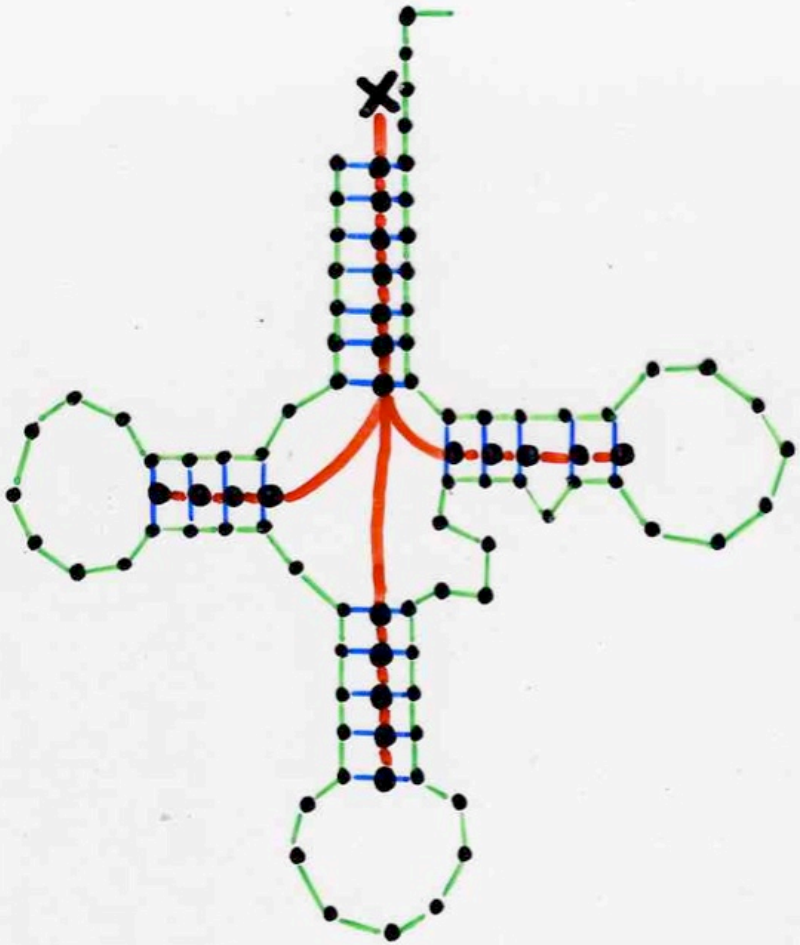




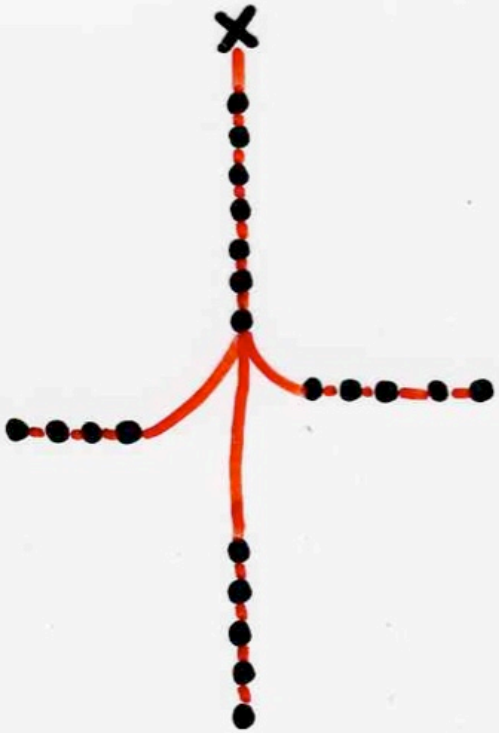



National Geographic


Trees everywhere



tARN Phe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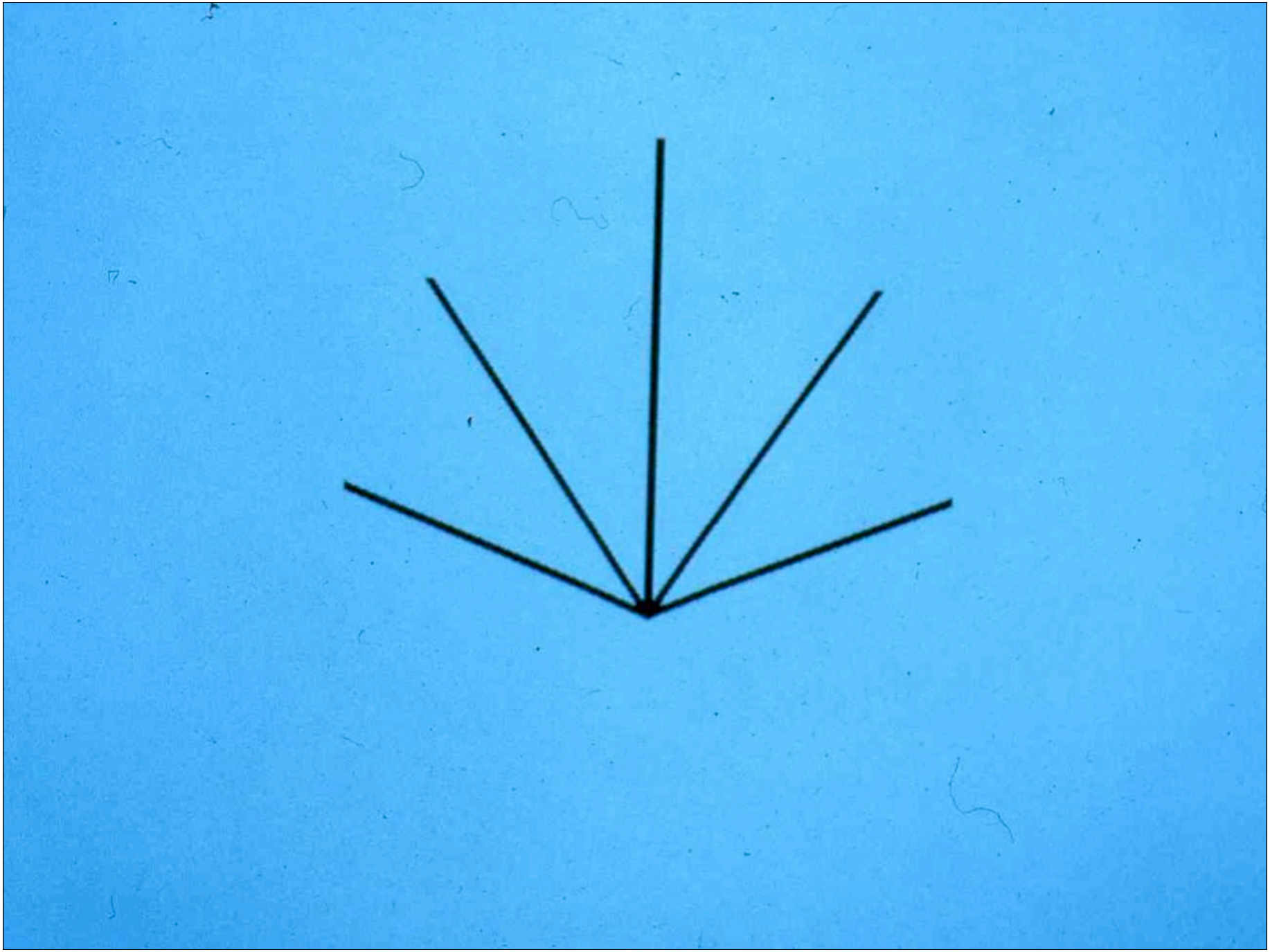

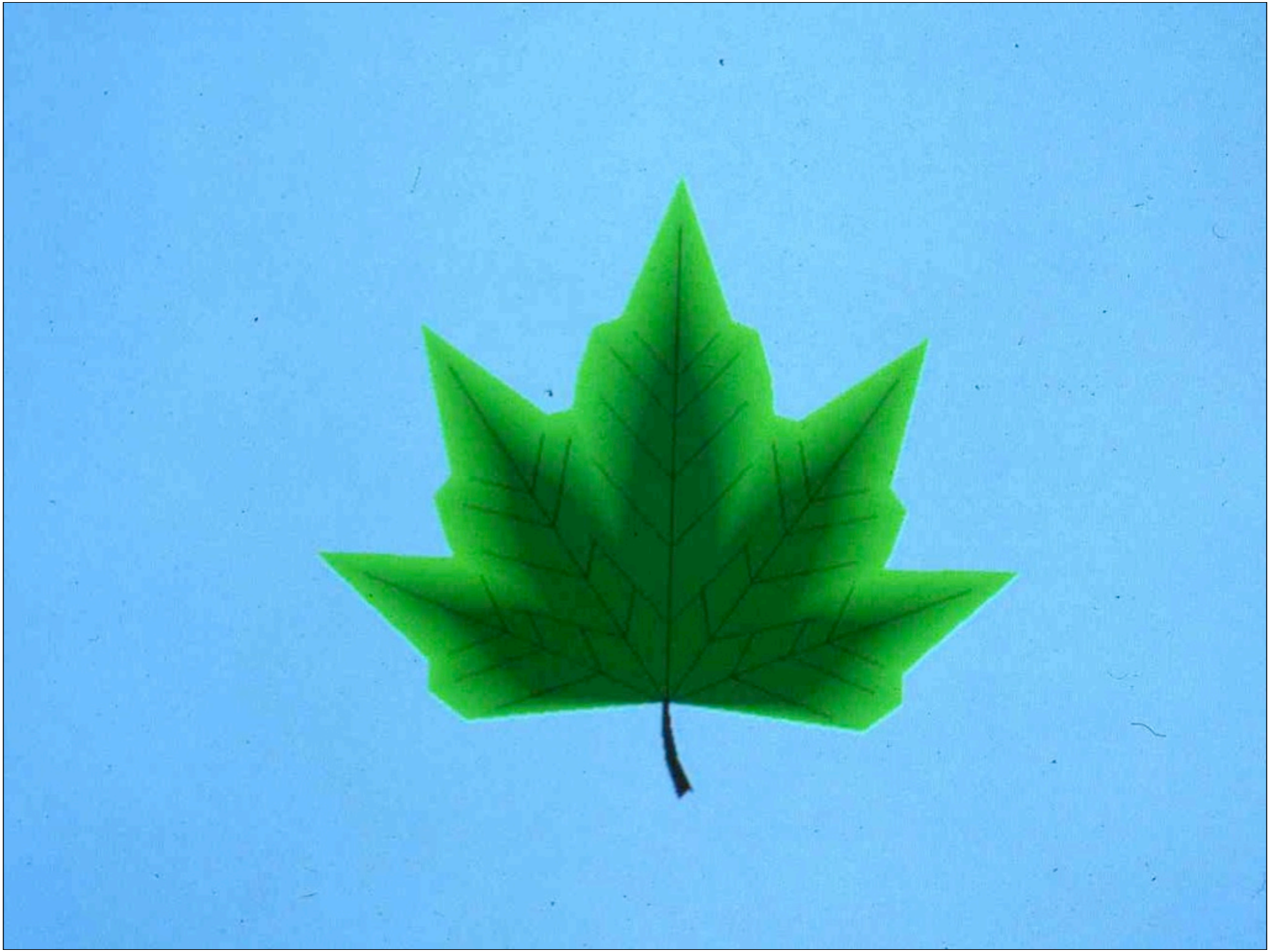

c'est la fin

Merci à tous !