

# Les preuves sans mots

(2ème partie)

Xavier Viennot

LaBRI, CNRS, Bordeaux

avec

Gérard H.E. Duchamp (violon)

LIPN, Sorbonne Universités

Metz 2012

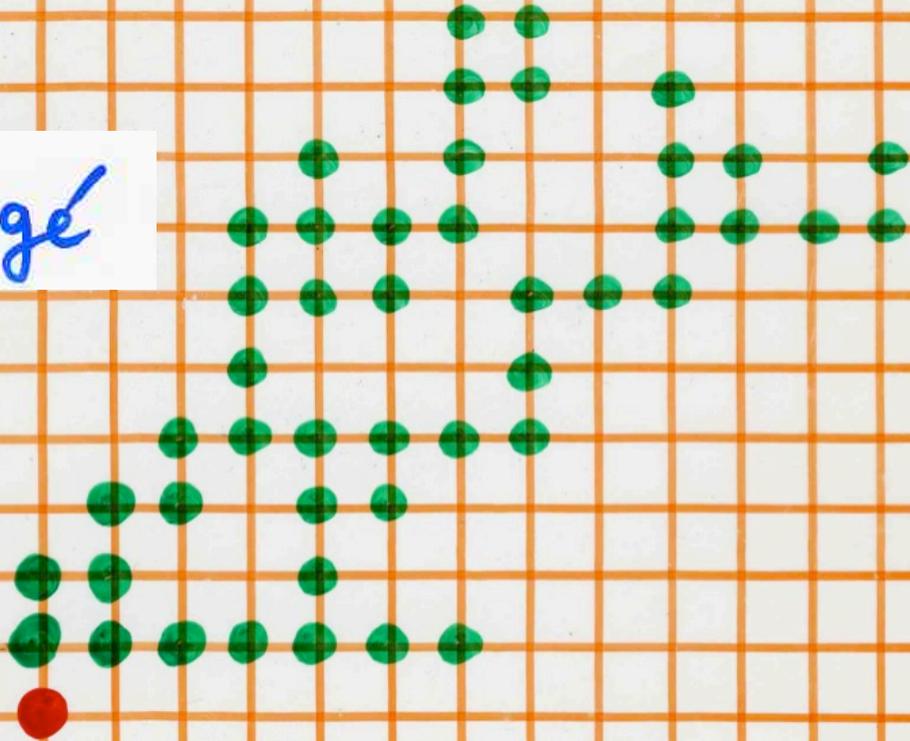
Journées nationales APMEP

30 octobre 2012

animaux dirigés

animal

dirigé





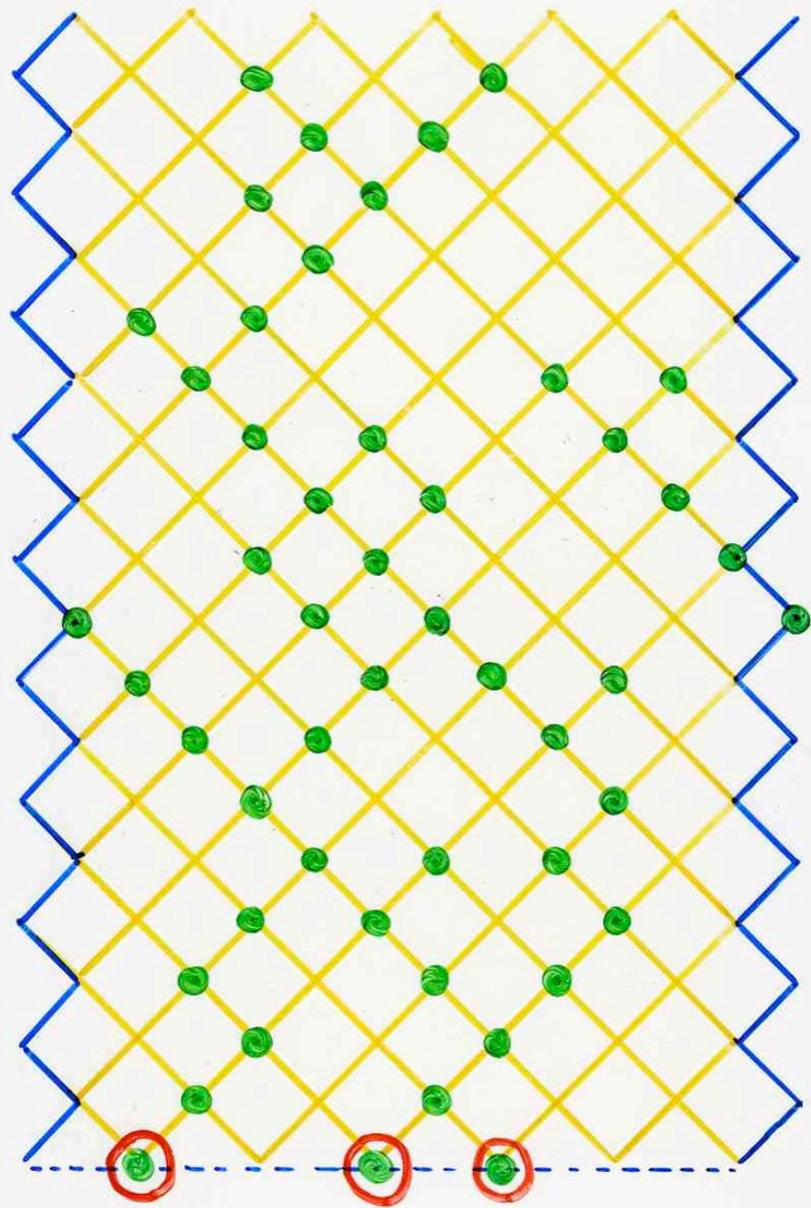
$a_n$  = nombre d'animaux  
dirigés  
ayant  $n$  points

percolation

animal

percolation dirigée

animal  
dirigé



$$b_n^{\leq k} = \frac{1}{k} \sum_{p=0}^{k-1} (-1)^p \sin \alpha_p \prod_{i=1}^{k-1} \left( \frac{\sin(i + \frac{1}{2}) \alpha_p}{\sin \frac{\alpha_p}{2}} \right)^{N_i} (1 + 2 \cos \alpha_p)^{n-1}$$

animals  
 circular strip  
 width  $k$

$$\alpha_p = \frac{2p+1}{2k} \pi$$



B. Derrida



J. Vannimenus



J.P. Nadal

une conjecture .....

médaille Boltzman 2010

$a_n$  = nombre d'animaux  
dirigés  
ayant  $n$  points



mathématiques expérimentales

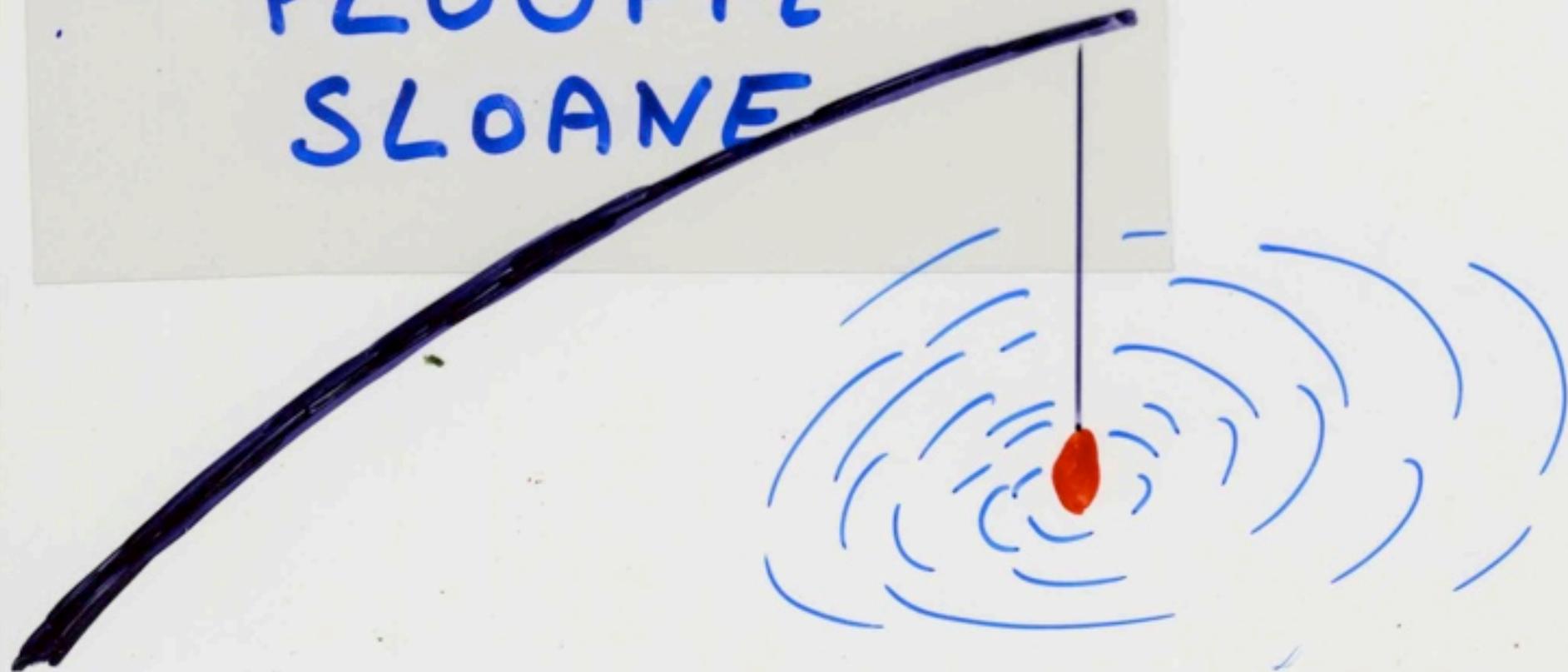
$a_n = 1, 2, 5, 13, 35, 96, 267, \dots$

deviner une formule .....

animation au violon

pêche à la ligne.

PLOUFFE  
SLOANE



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**THE  
ENCYCLOPEDIA  
▼▼▼ OF ▼▼▼  
INTEGER  
SEQUENCES**

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**N. J. A. Sloane**

*Mathematical Sciences Research Center  
AT&T Bell Laboratories  
Murray Hill, New Jersey*

**Simon Plouffe**

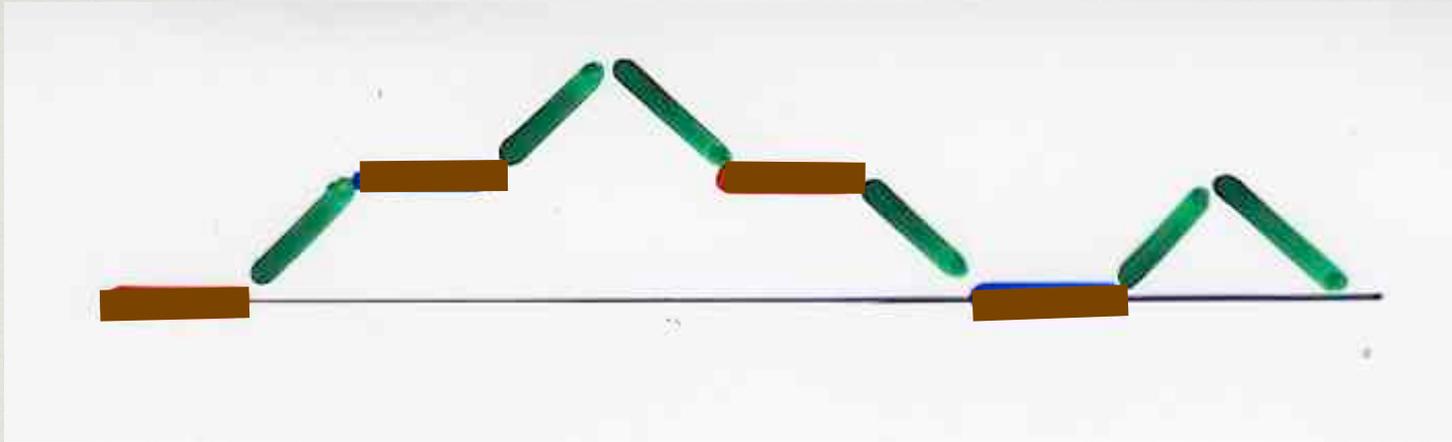
*Département de Mathématiques et d'Informatique  
Université du Québec à Montréal  
Montréal, Québec*



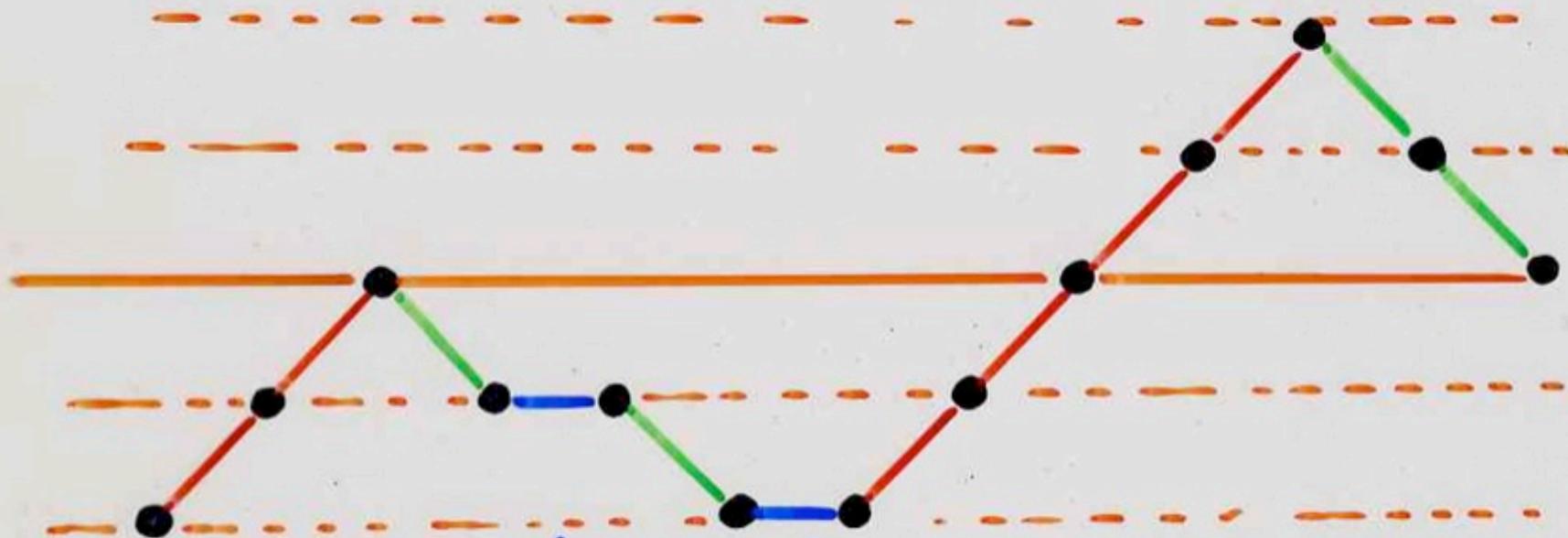
**ACADEMIC PRESS**

San Diego New York Boston London Sydney Tokyo Toronto





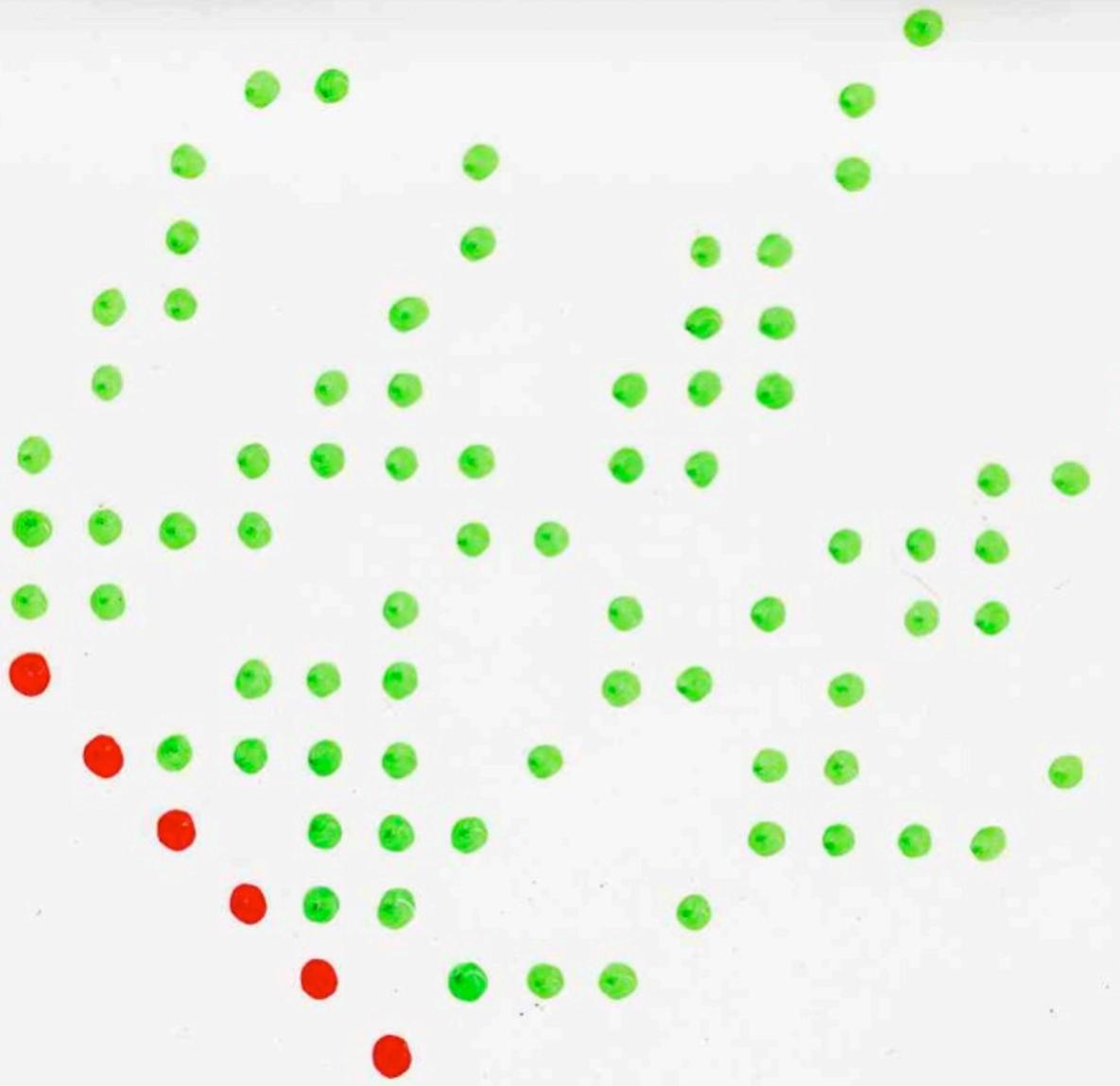
chemin de Motzkin



facteur gauche (ou de début)  
d'un chemin de Motzkin

animal dirigé  
à source compacte

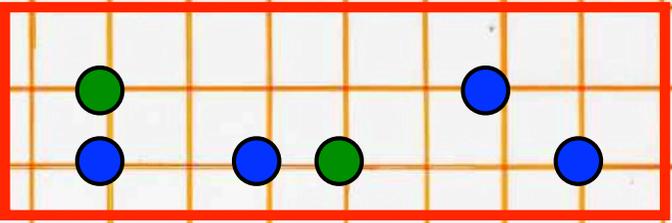
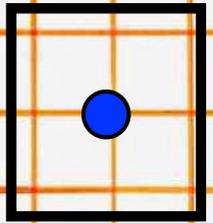
D. Gouyou-Beauchamps  
X. V. (1984)



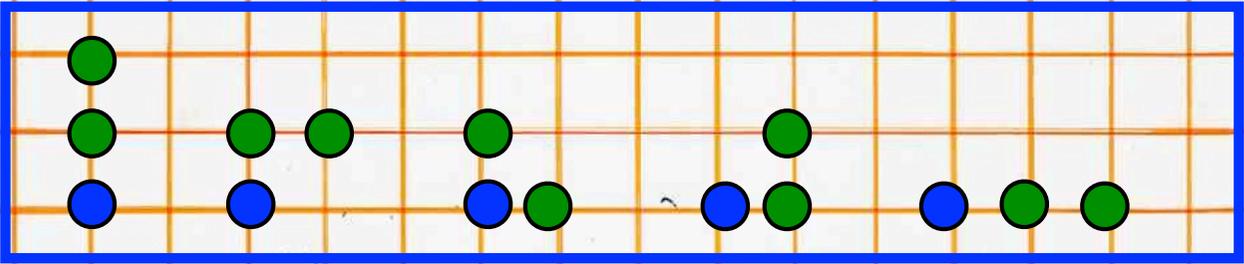
The diagram shows a collection of dots on a white background. Most dots are green, forming a large, irregular shape that resembles a stylized animal or a cluster of points. There are seven red dots located on the left side of the green cluster, arranged in a roughly vertical line. The dots are scattered, with some forming small groups and others being isolated.

animal dirigé  
à source compacte

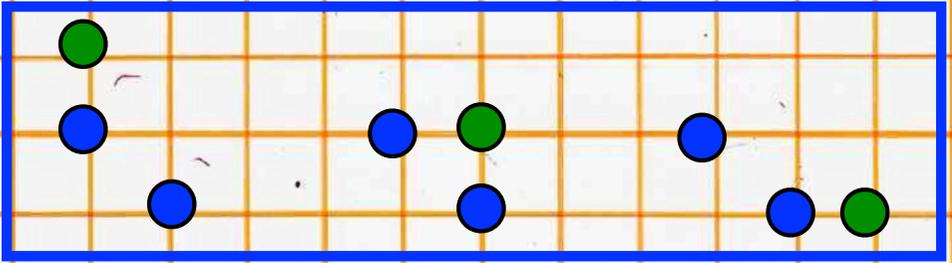
1



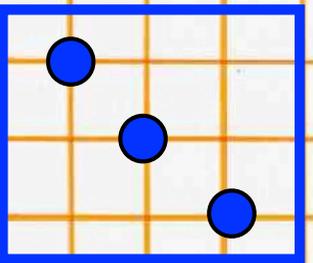
3



5



3



1

9

1, 3, 9, 27, 81, ...

1, 3,  $3^2$ ,  $3^3$ ,  $3^4$ ,  $3^5$ ,  $3^6$ ,

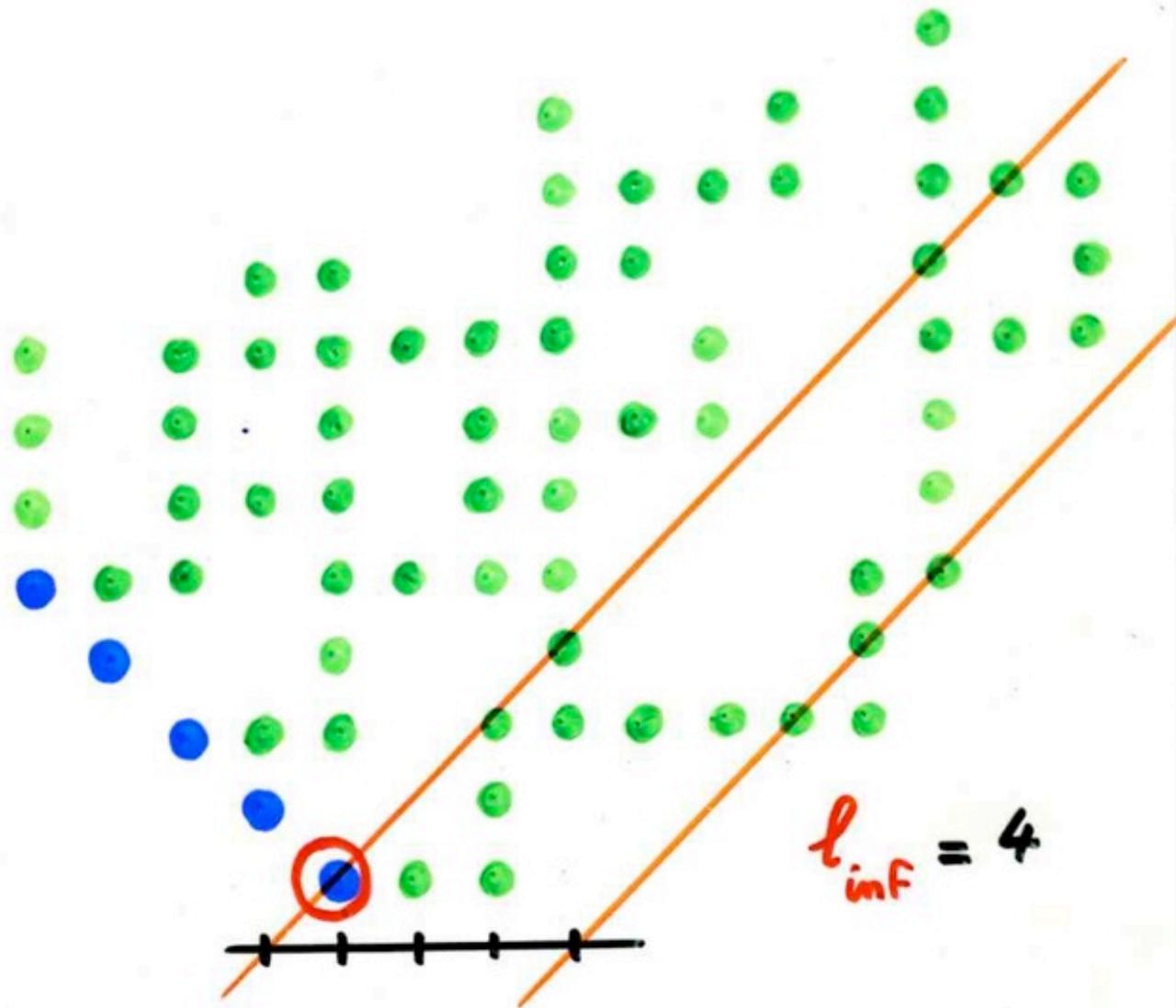


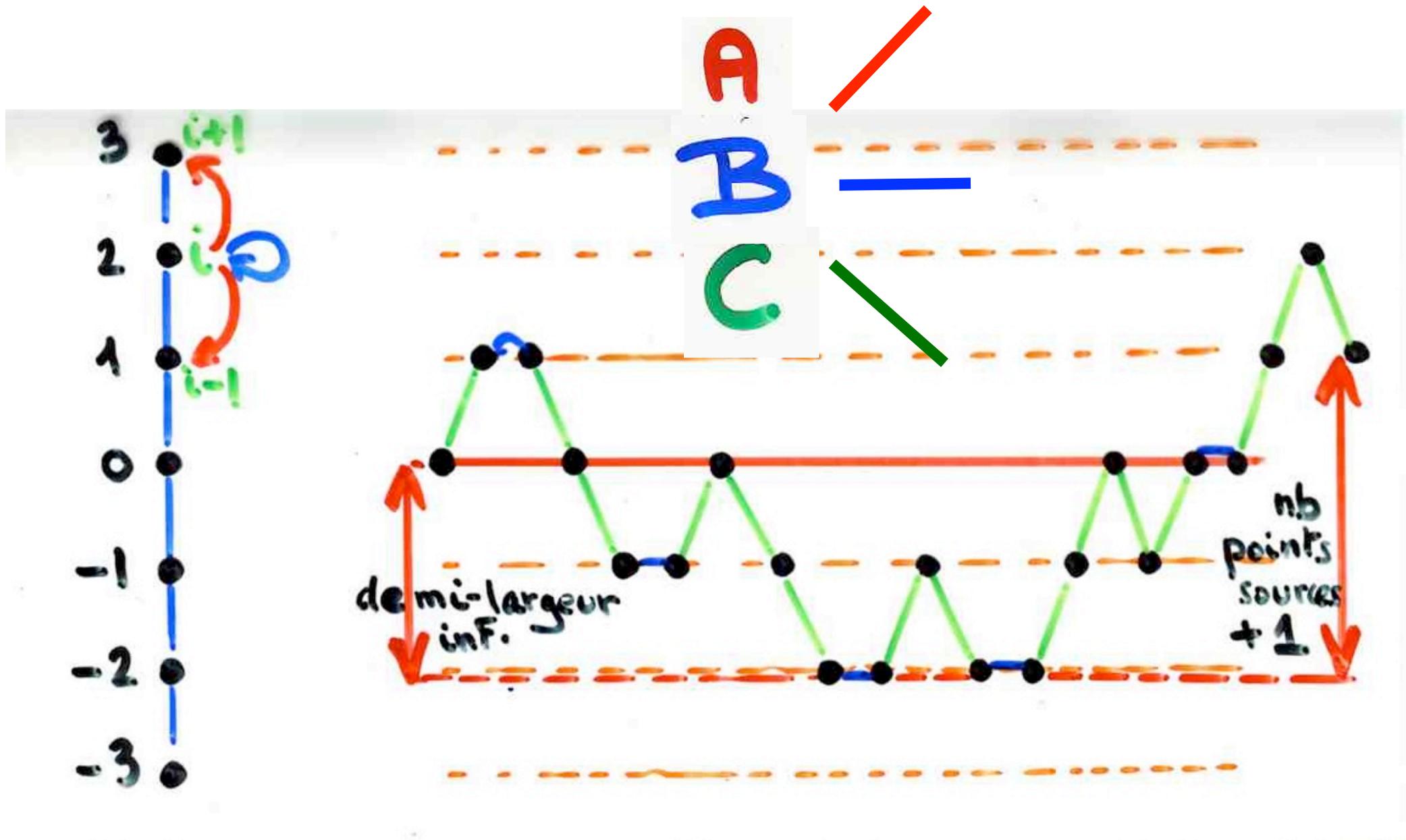
Le nombre d'animaux dirigés  
à source compacte  
ayant  $n+1$  points est

$$3^n$$



recherche expérimentale  
d'une bijection  
avec trois opérateurs A, B, C





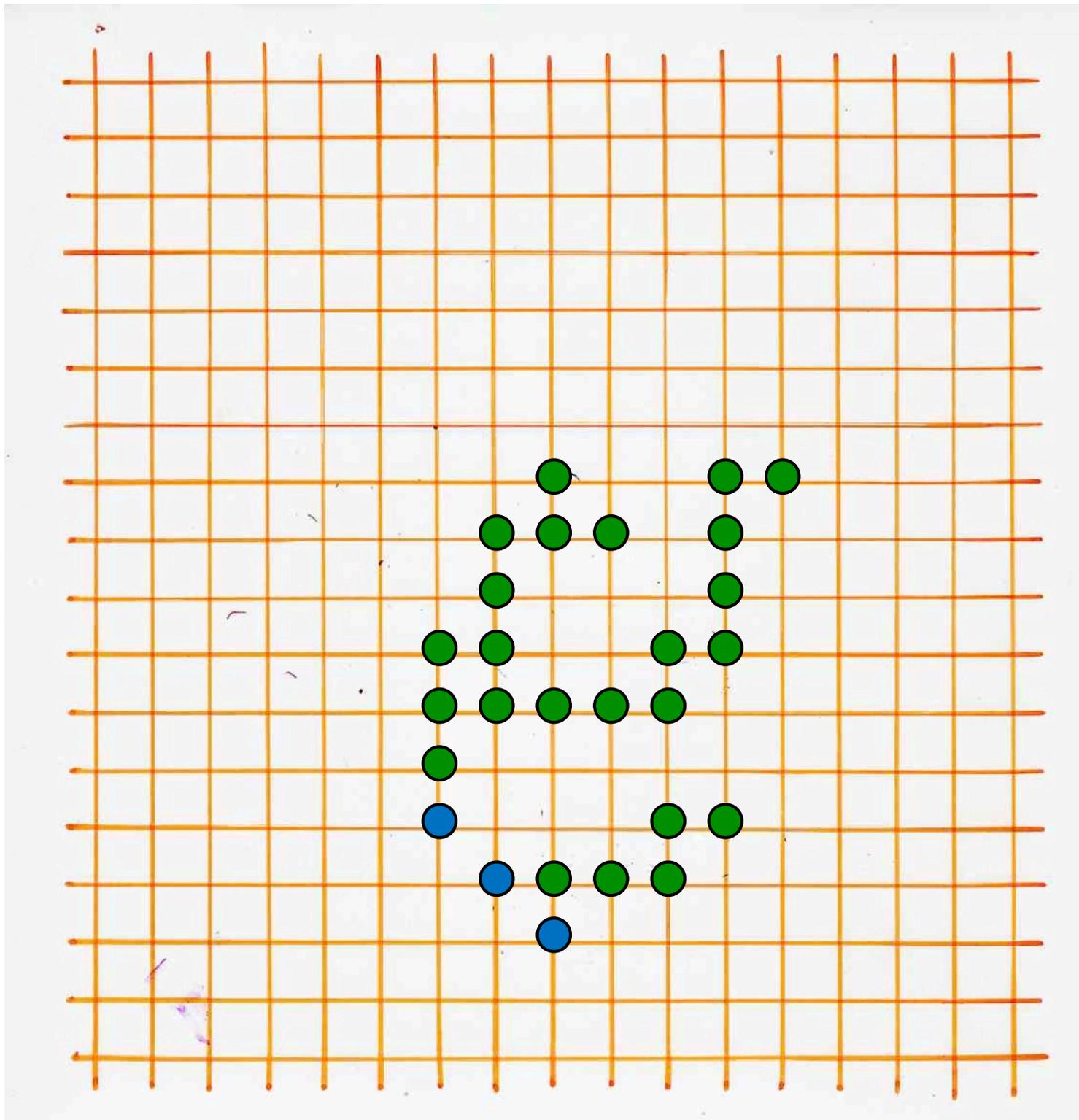
A  
B  
C

de mi-largeur  
inf.

nb points  
sources  
+ 1

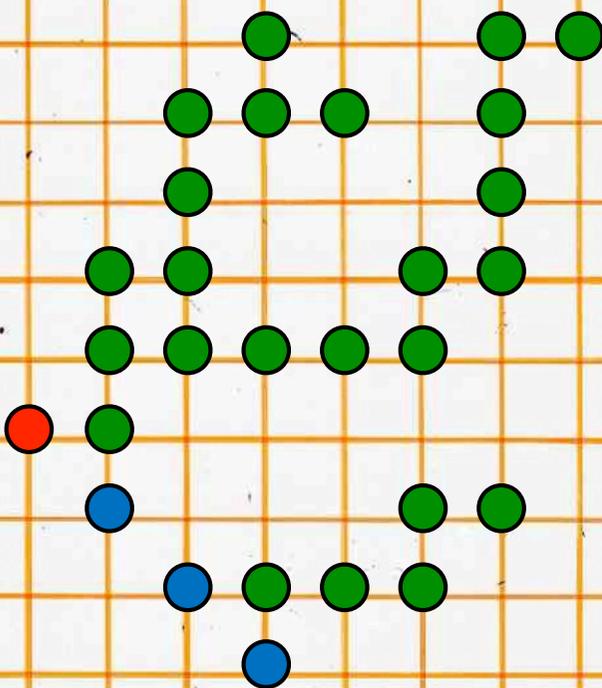
une bijection avec  
trois opérateurs  $A$ ,  $B$ ,  $C$

animation au violon



opérateur

A

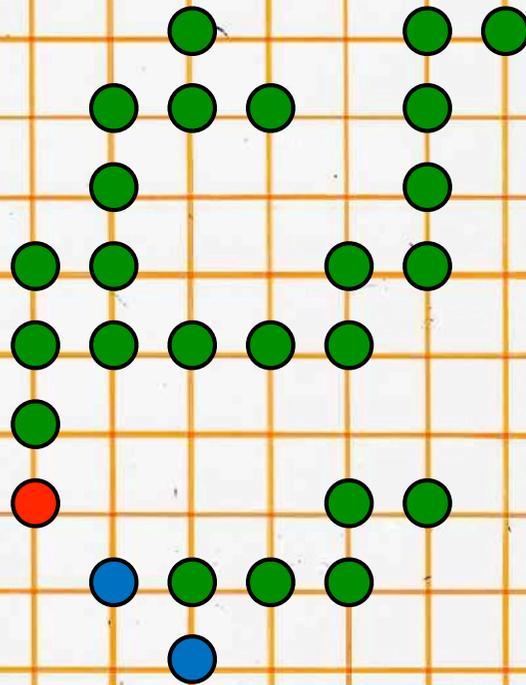




opérateur

C

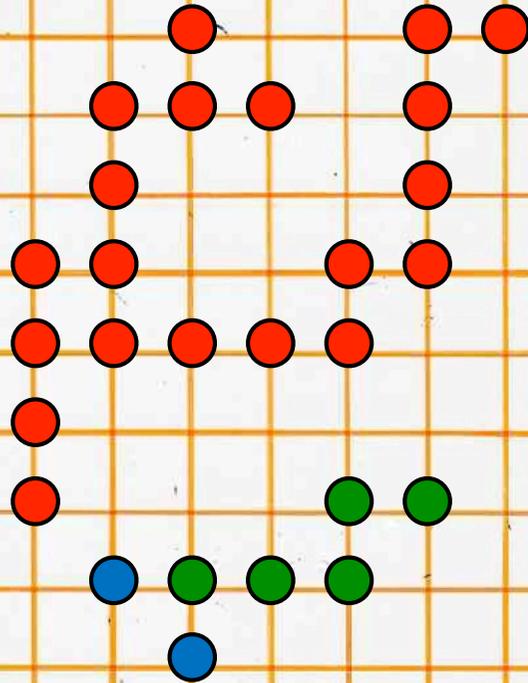
sous-animal  
issu de ●



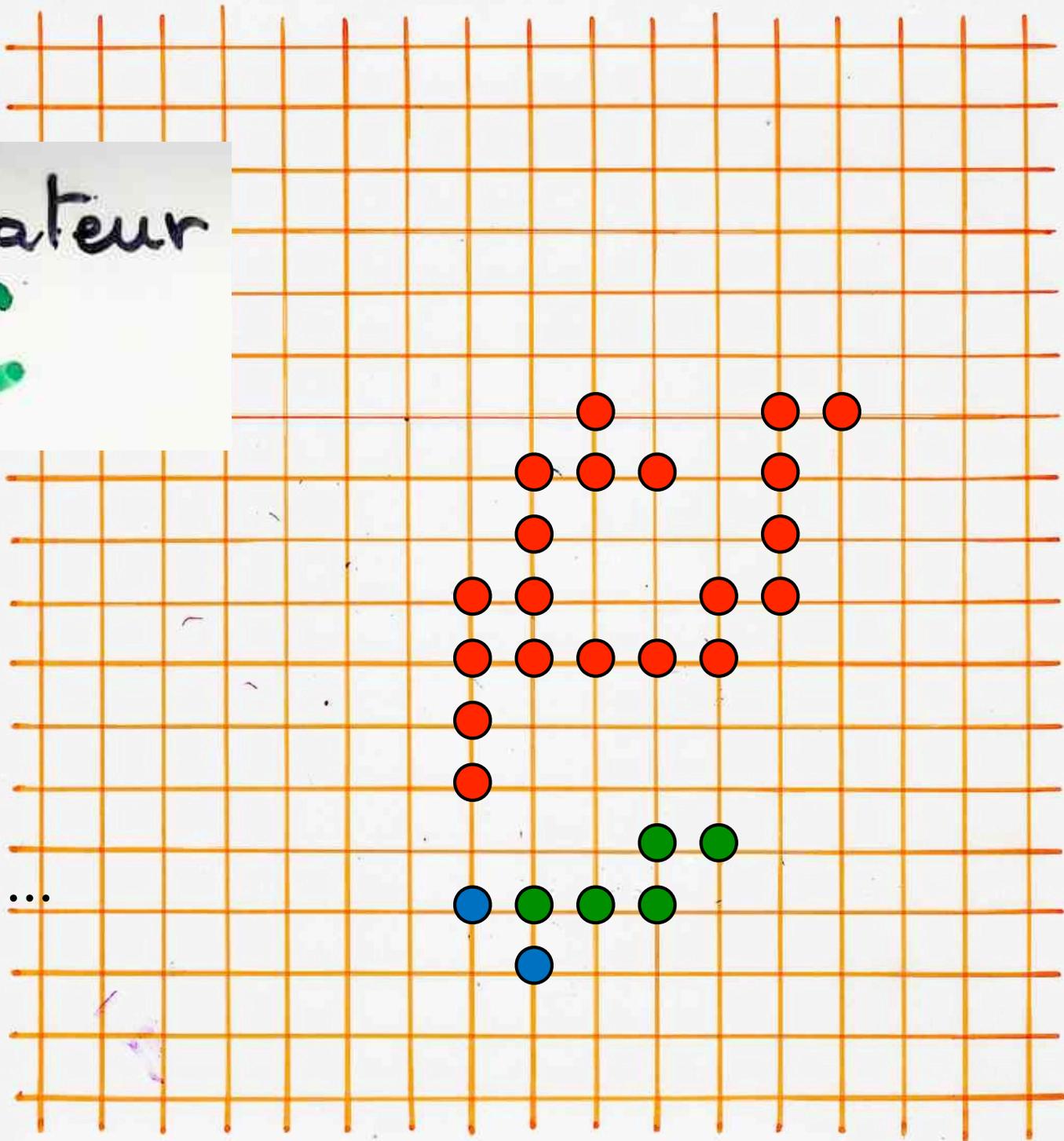
opérateur

C

sous-animal  
issu de ●



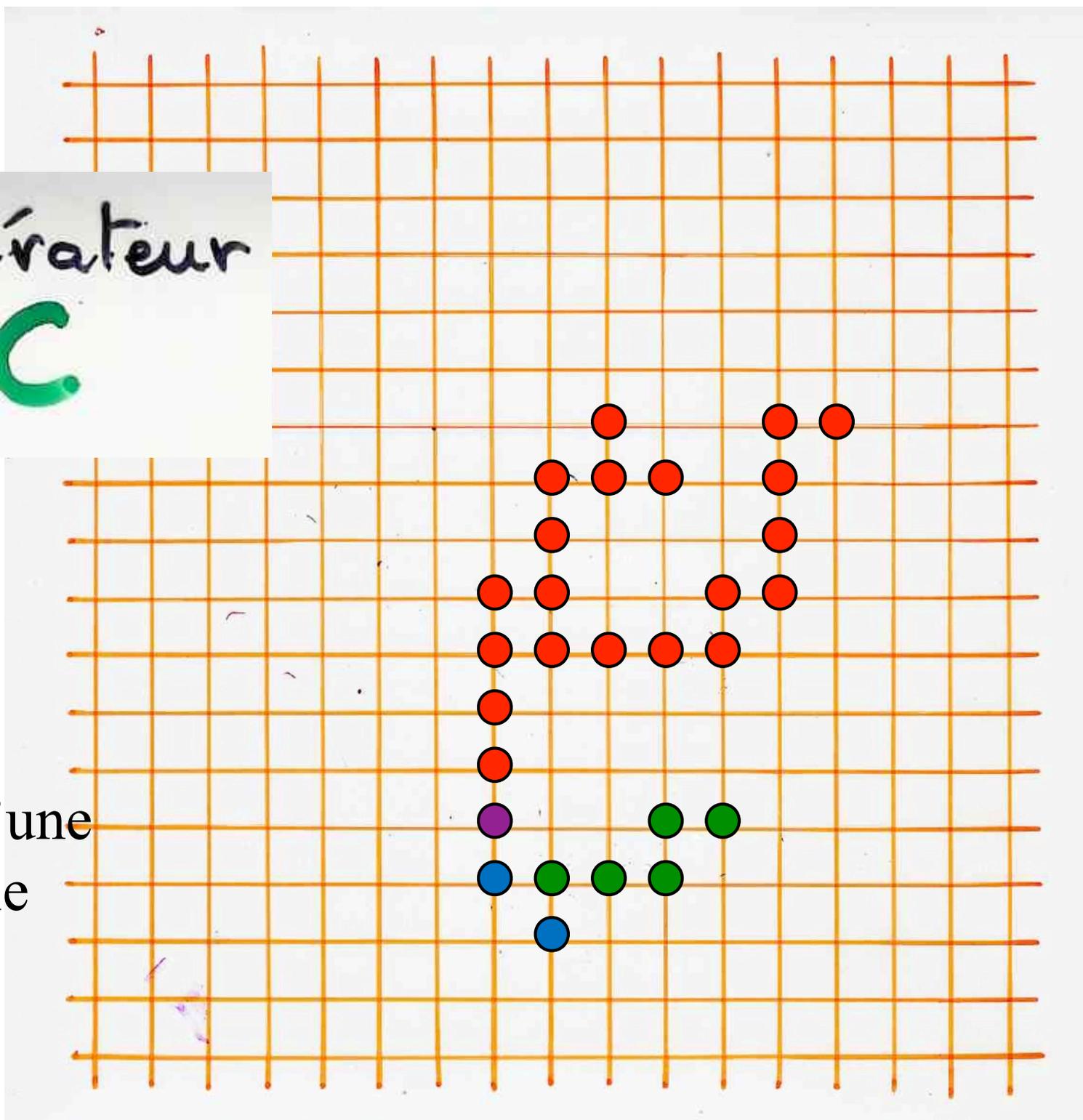
opérateur  
C



glissement ...

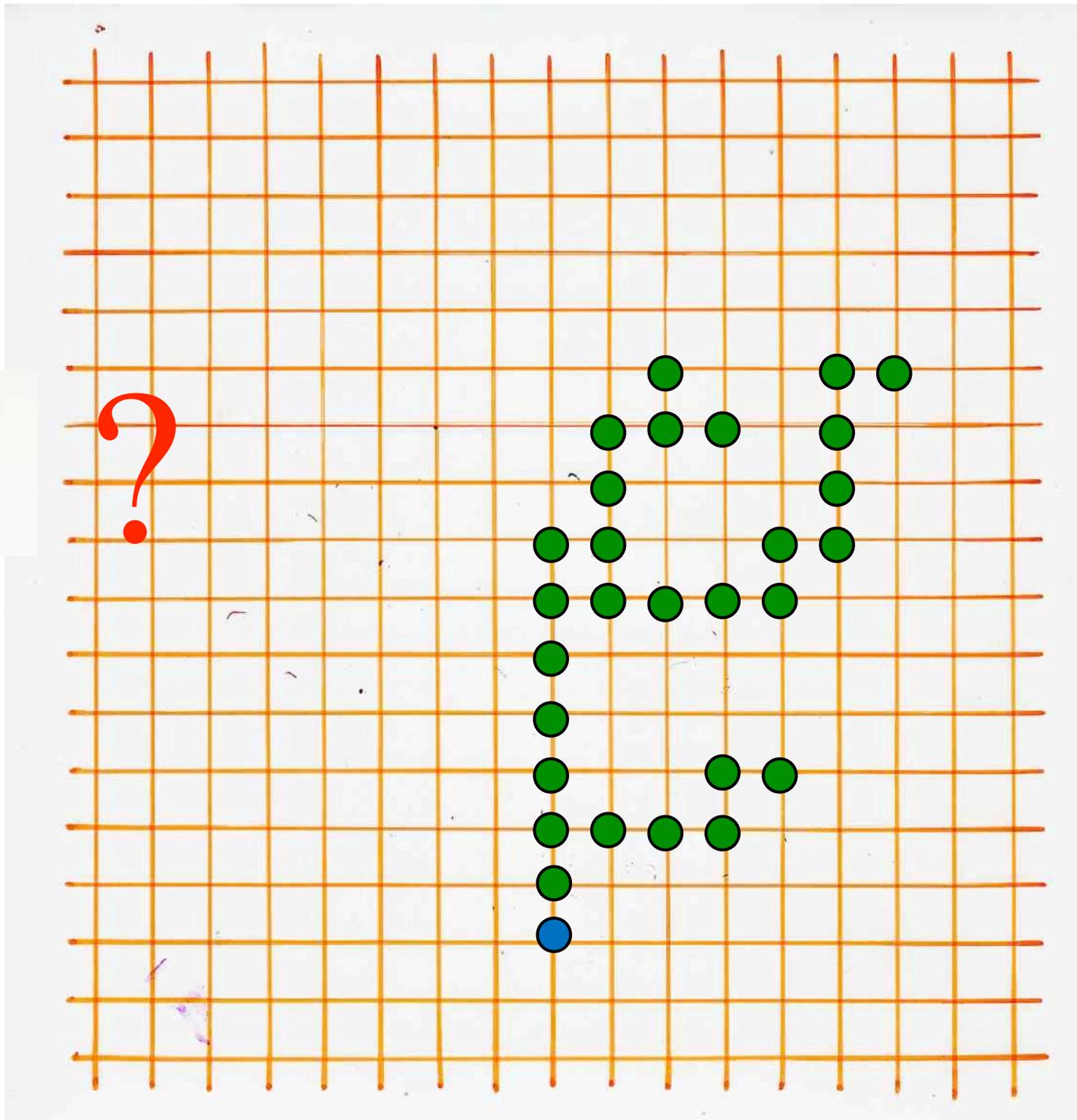
opérateur  
C

ajout d'une  
cellule





et  
encore

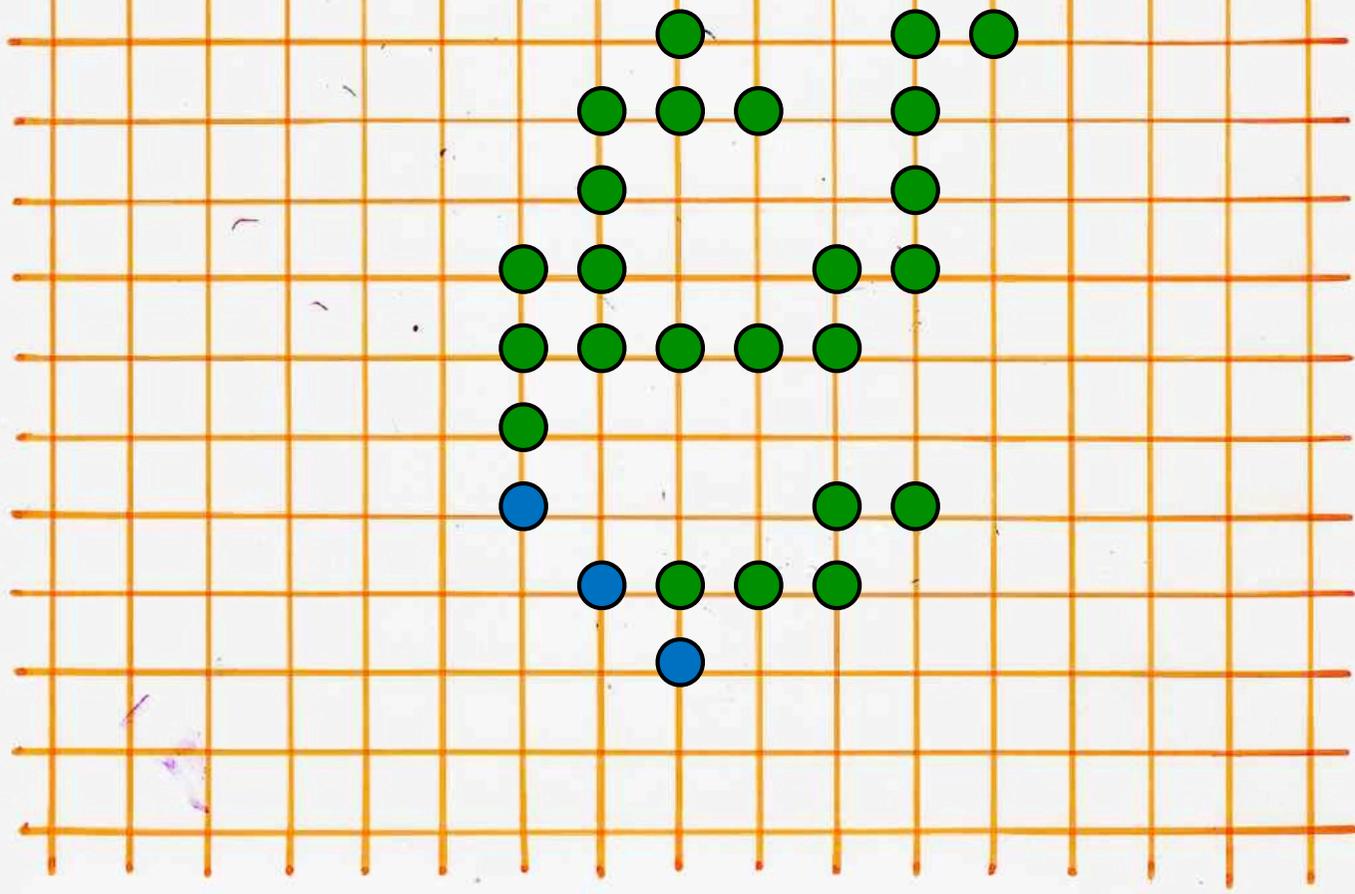




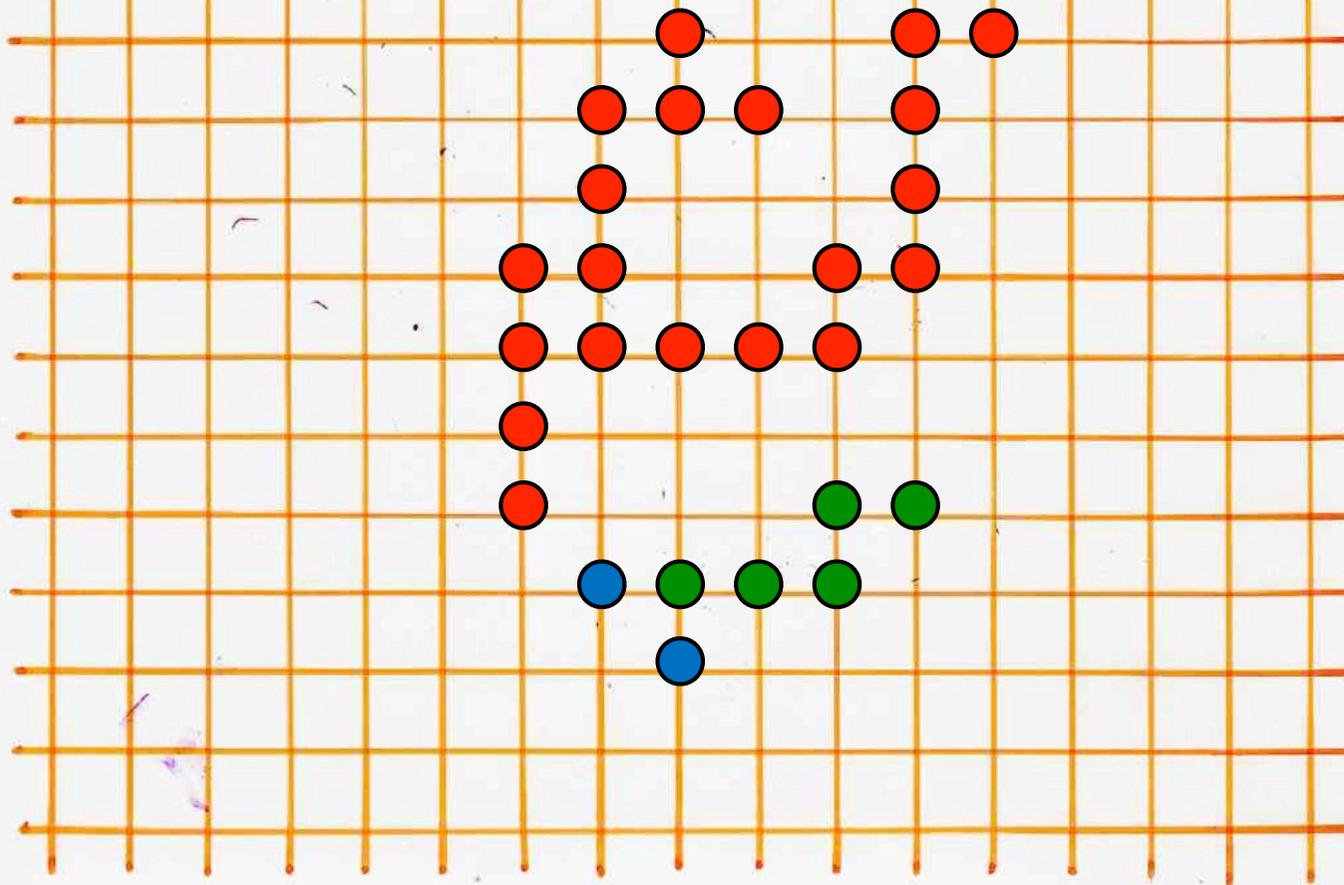
opérateur

B

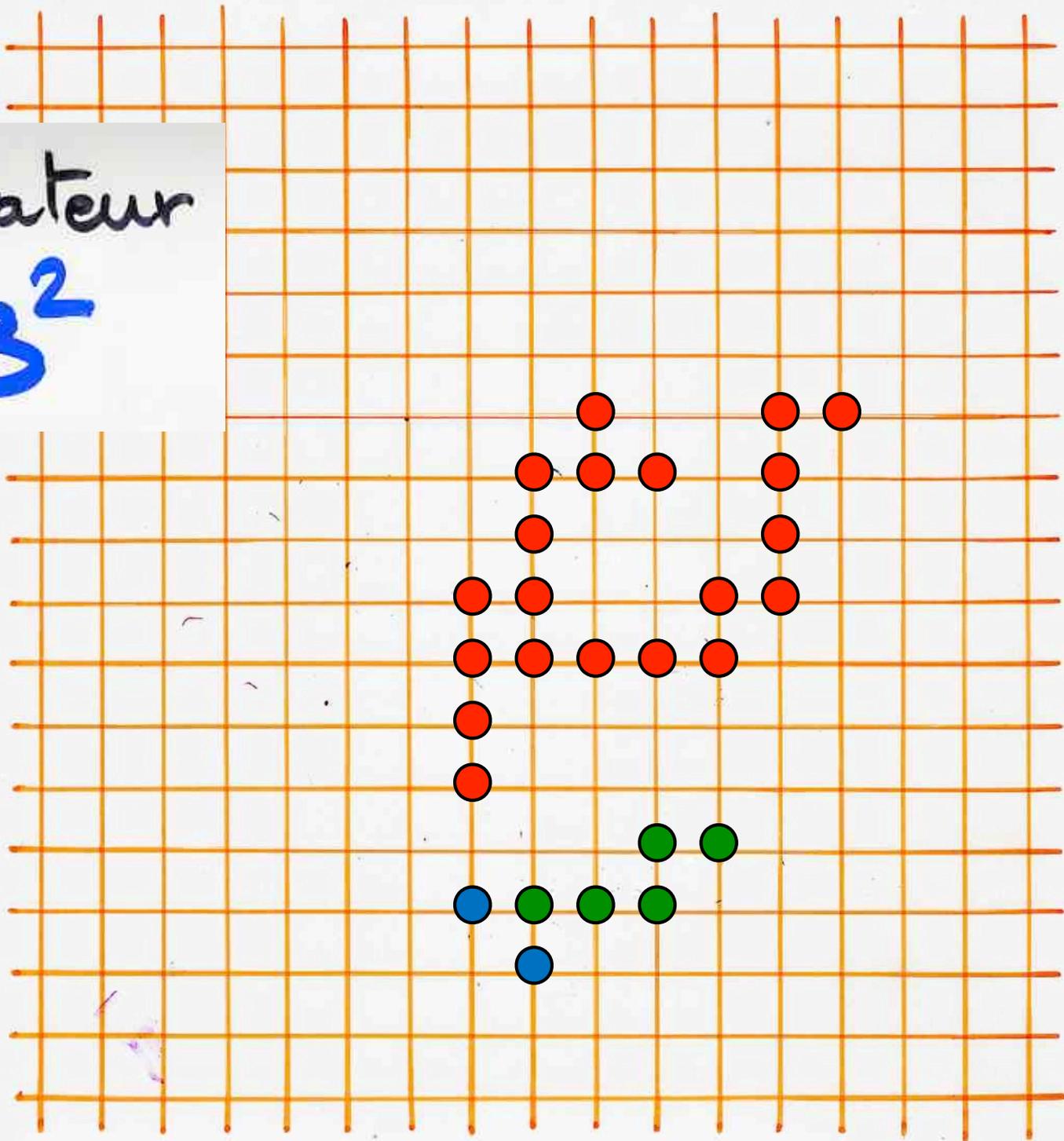
opérateur  
 $B^2$



opérateur  
 $B^2$



opérateur  
 $B^2$



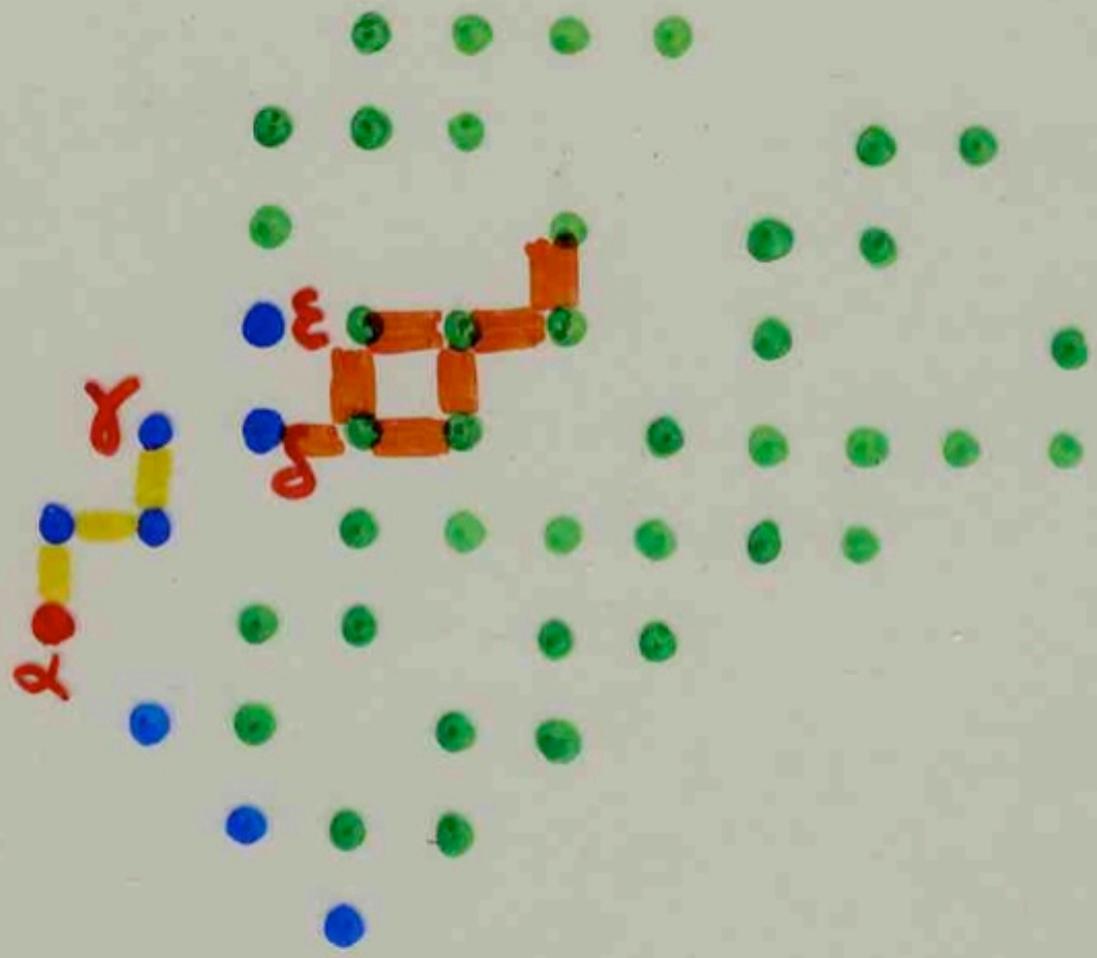


opérateur  
B

?

C'est très compliqué ....



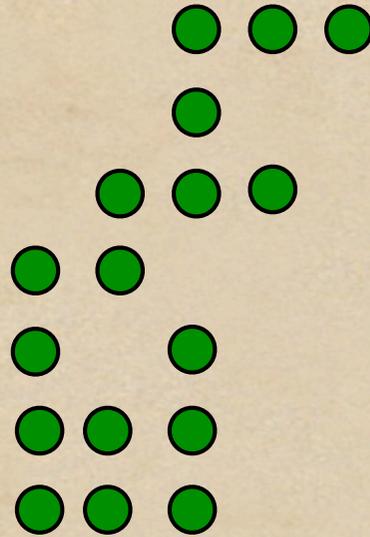






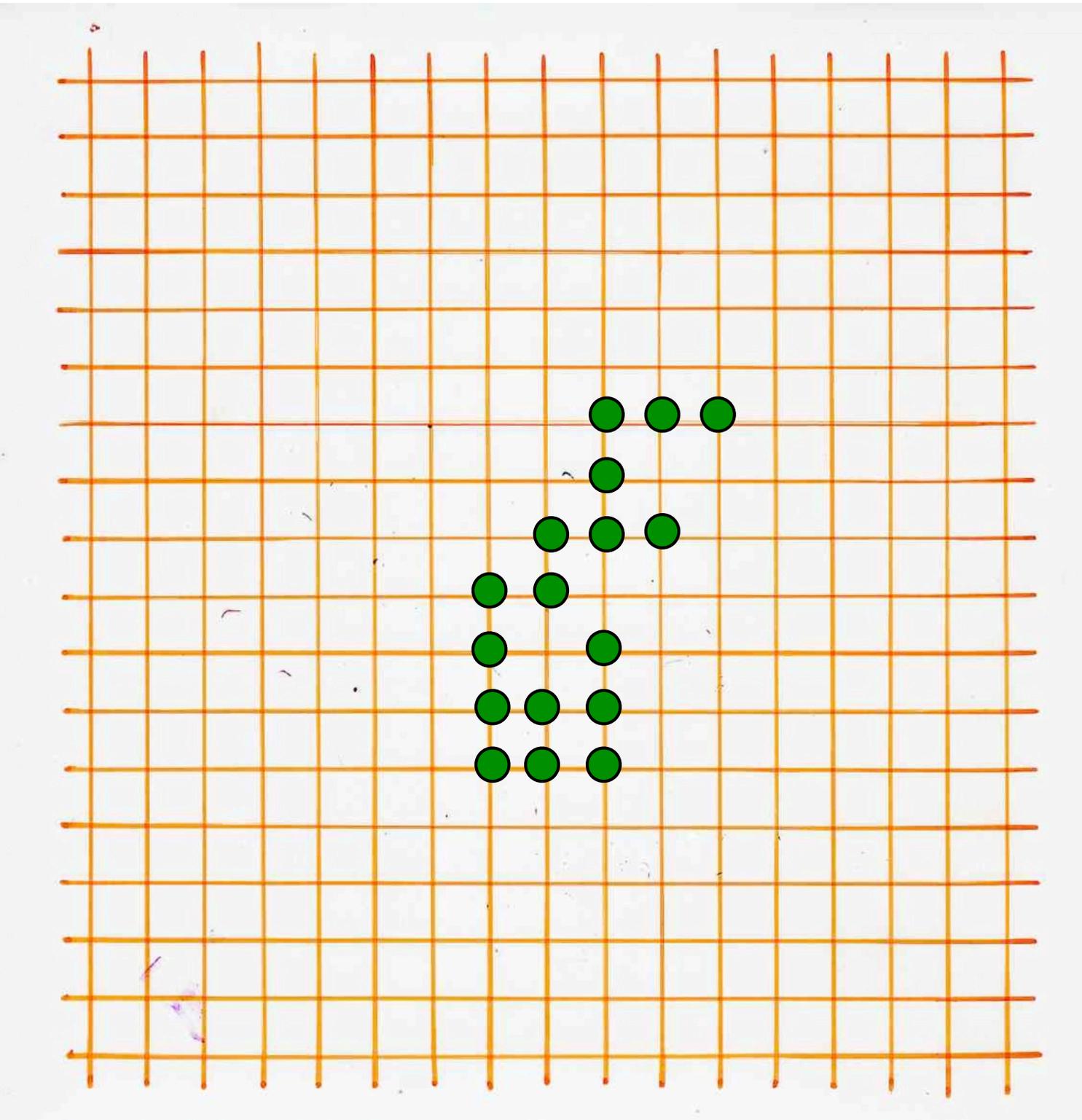
la bijection

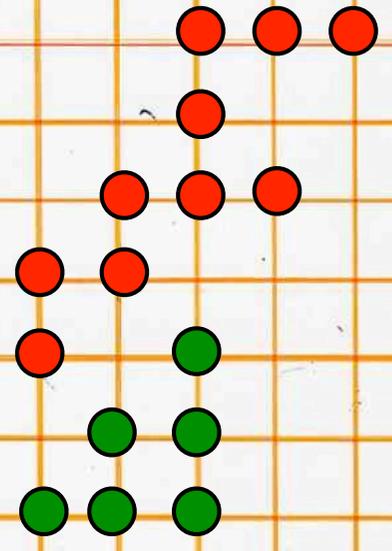
$3^n$

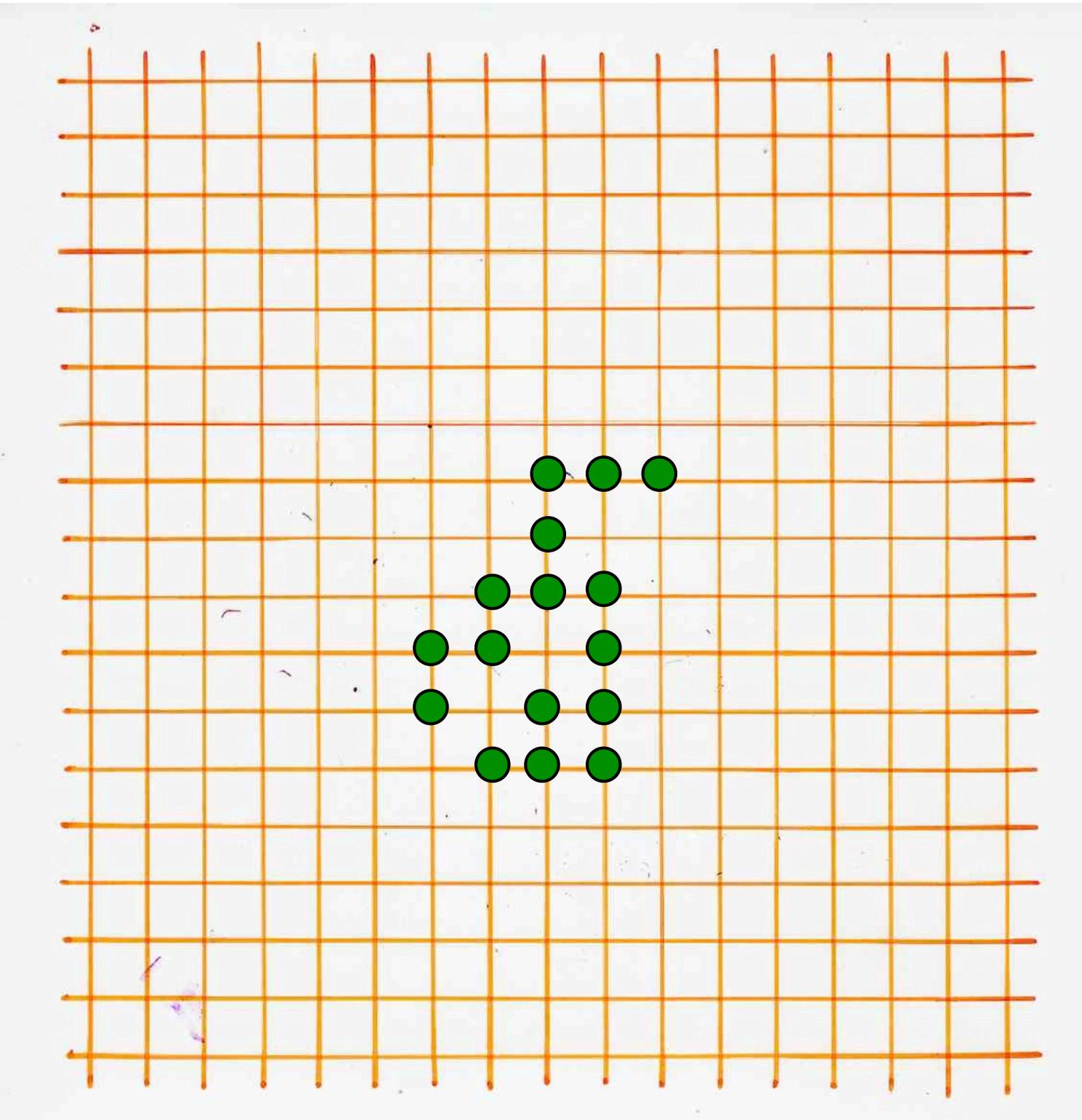


A A C C C B C A B A C B B B B C

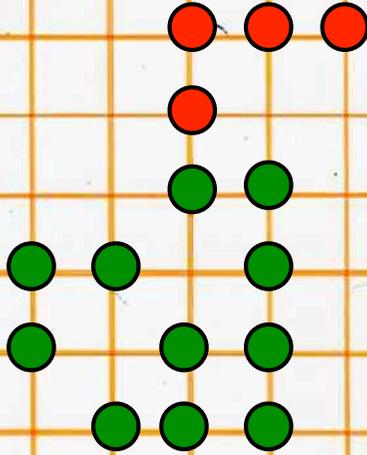
animation au violon





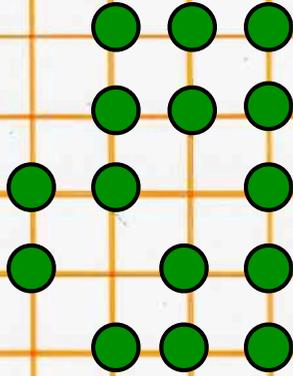


B



B C

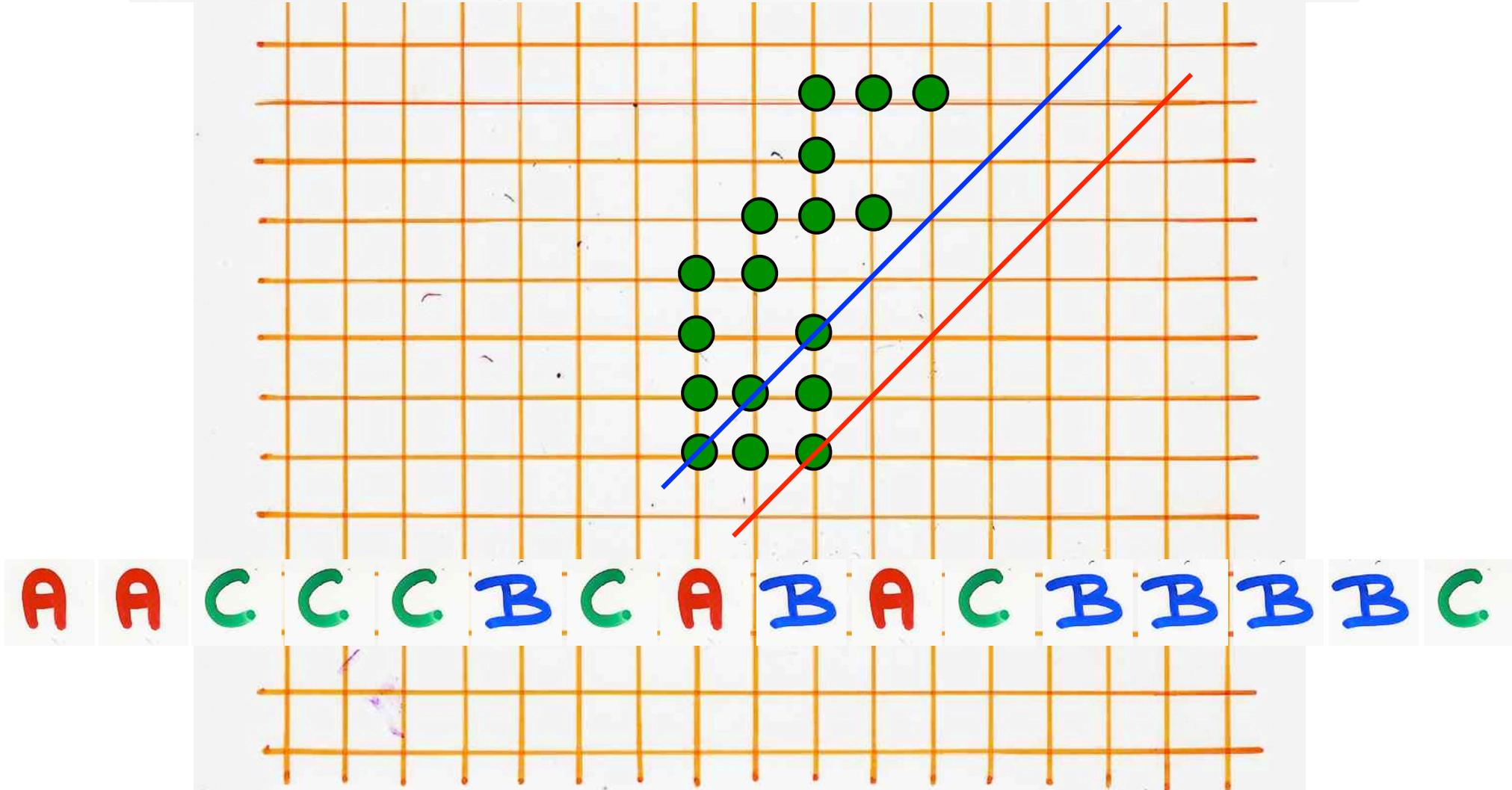
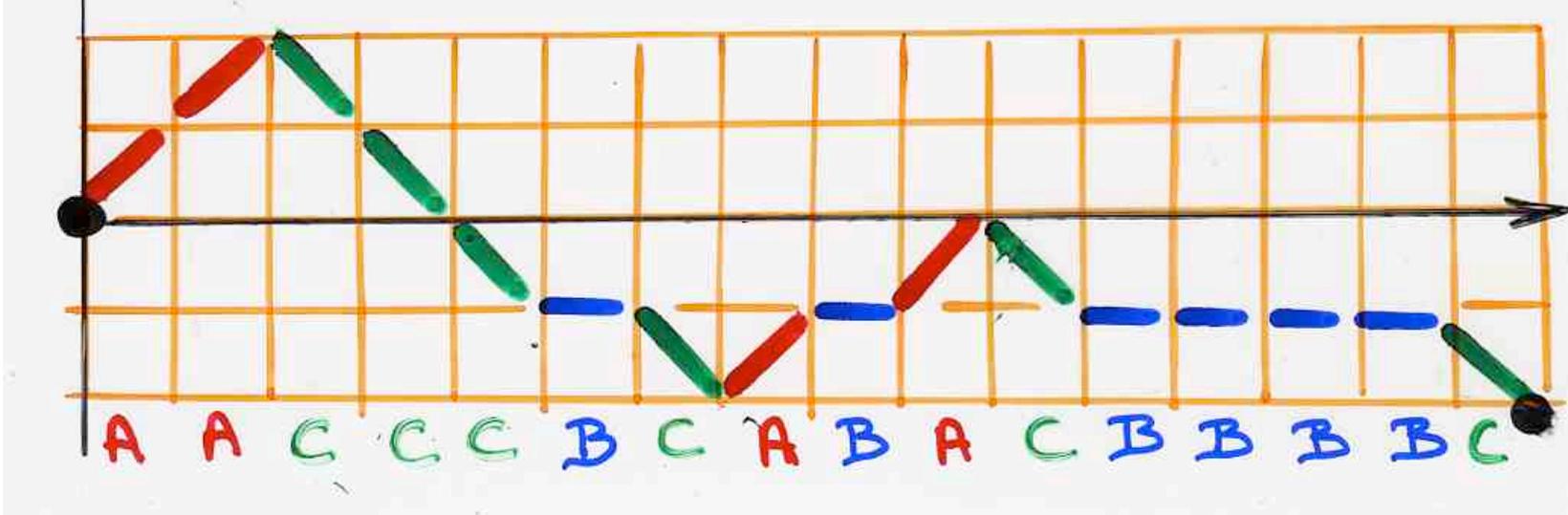
B



B C

A A C C C B C A B A C B B B B C





formule pour le nombre  
d'animaux dirigés de taille  $n$

$$a_n = \sum_{i \geq 0} \binom{n}{i} \binom{i}{i/2}$$

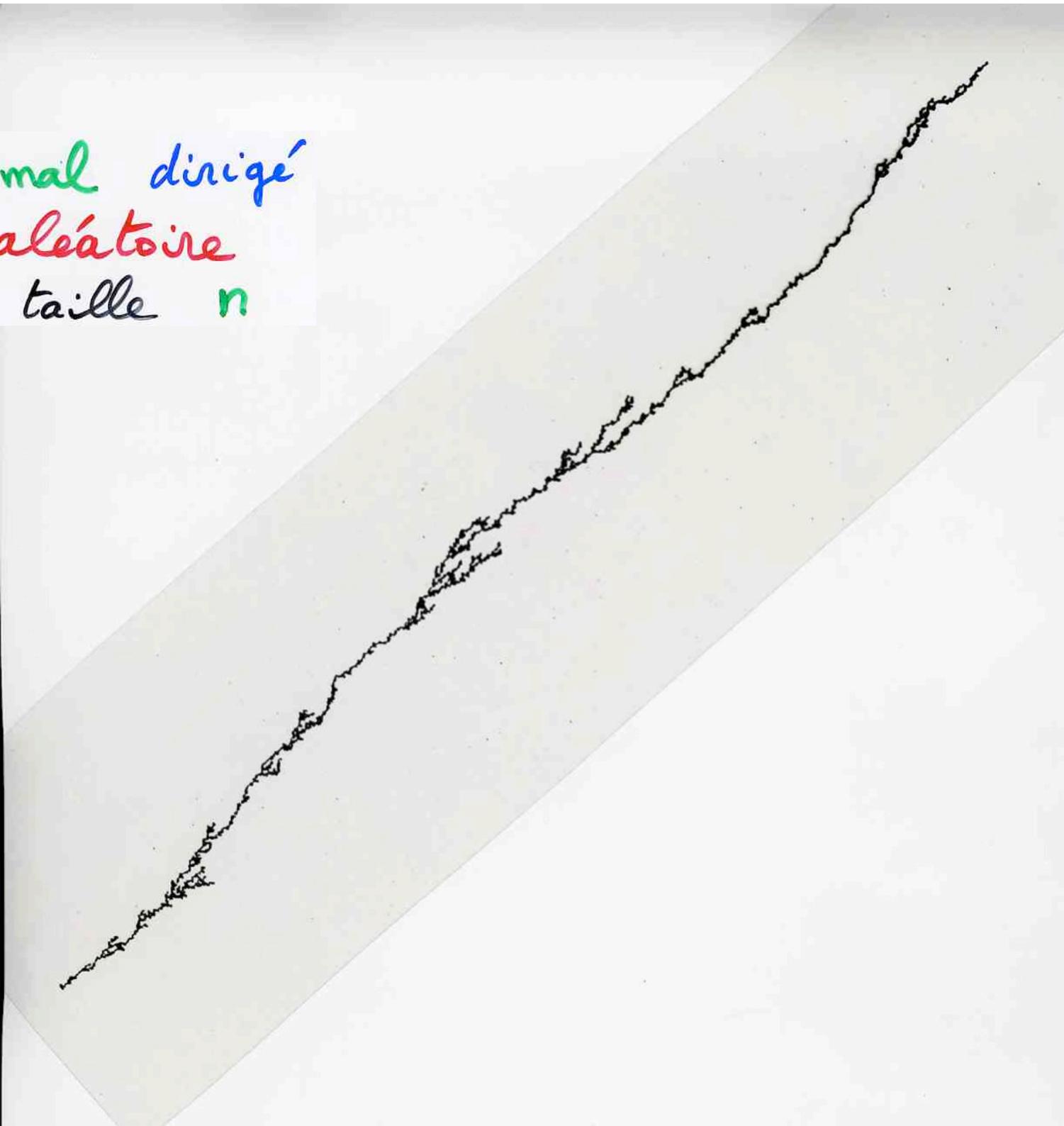


D.Dhar

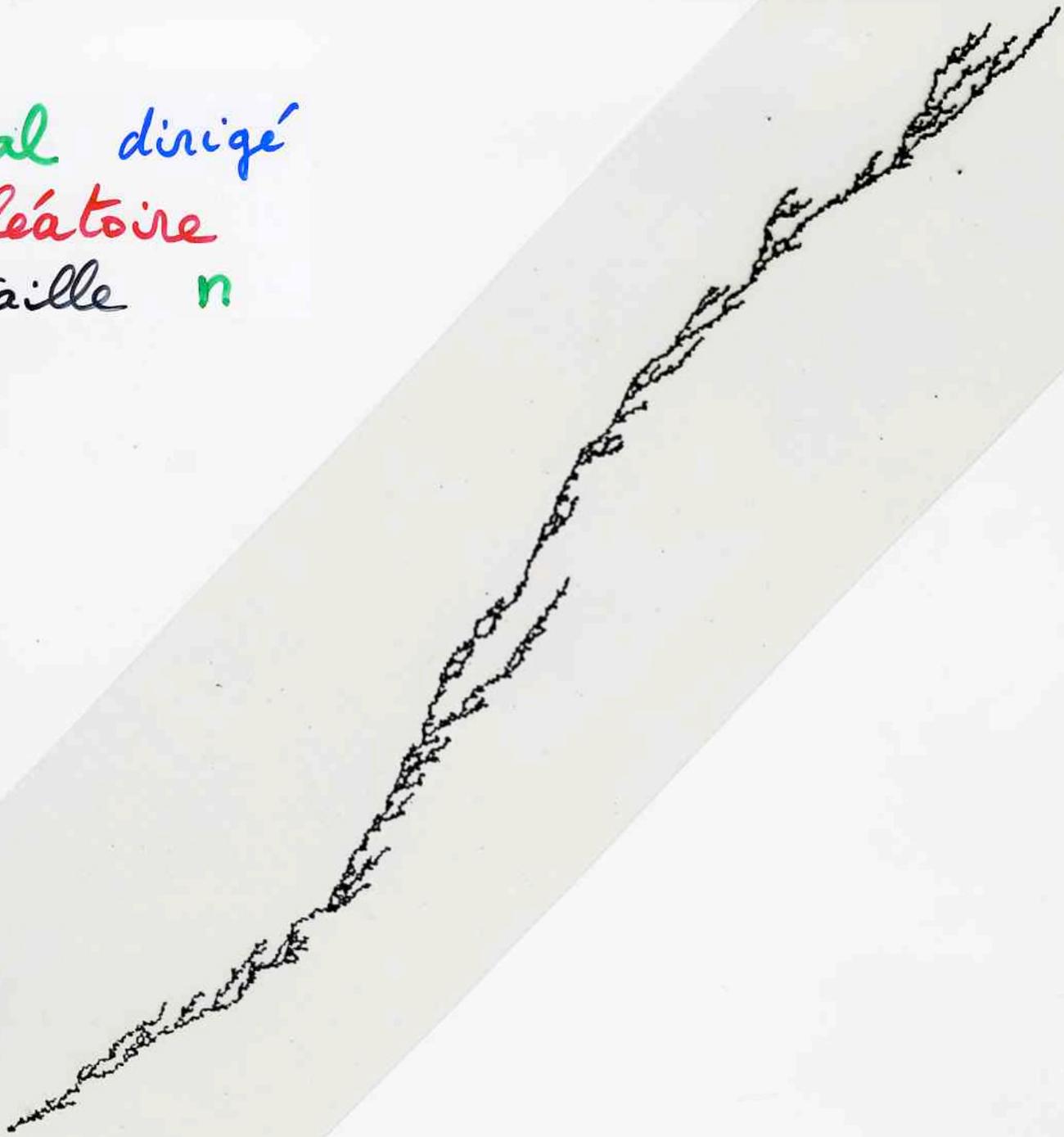
modèle de gaz  
de "particules dures"  
R. Baxter

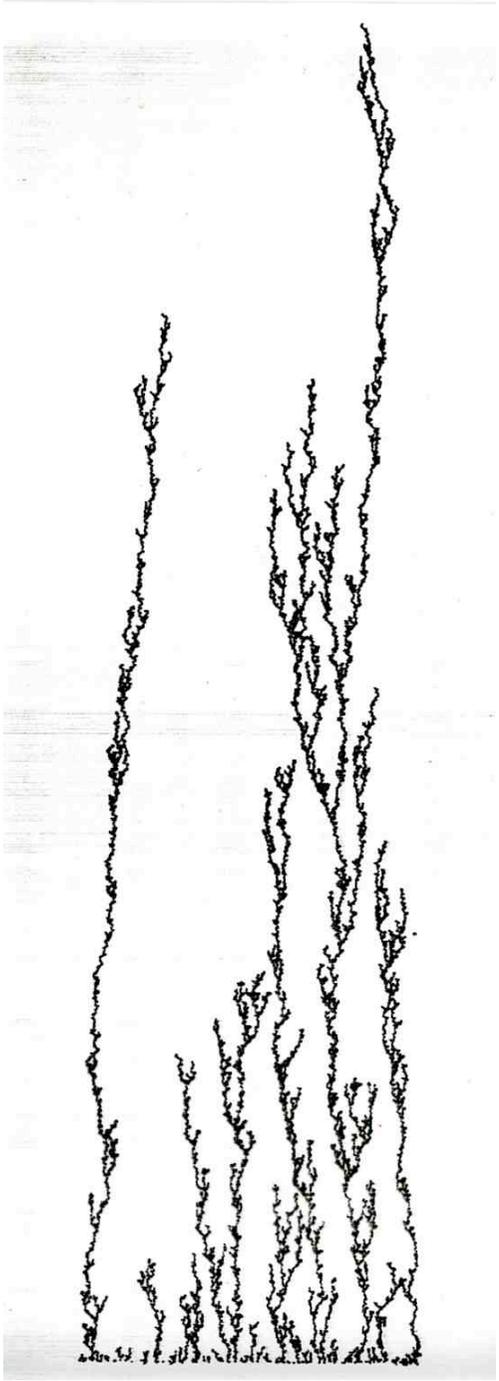
animaux dirigés aléatoires

animal dirigé  
aléatoire  
de taille  $n$



animal dirigé  
aléatoire  
de taille  $n$





1. **Introduction**  
The purpose of this study is to investigate the effects of a new educational program on student learning outcomes. The program is designed to enhance critical thinking and problem-solving skills through a series of interactive activities and projects.

2. **Methodology**  
The study employed a quasi-experimental design. A group of students was selected from a local high school and divided into two groups: an experimental group and a control group. The experimental group participated in the new educational program, while the control group followed the traditional curriculum. Data was collected through pre-tests, post-tests, and student self-reports.

3. **Results**  
The results of the study indicate that the experimental group showed significantly higher scores on the post-test compared to the control group. This suggests that the new educational program had a positive impact on student learning outcomes. Additionally, students in the experimental group reported higher levels of engagement and motivation throughout the program.

4. **Conclusion**  
Based on the findings, it can be concluded that the new educational program is effective in improving student learning outcomes. The program's focus on interactive activities and projects appears to be a key factor in its success. Further research is needed to explore the long-term effects of the program and to identify ways to integrate these practices into the traditional curriculum.

ALEA

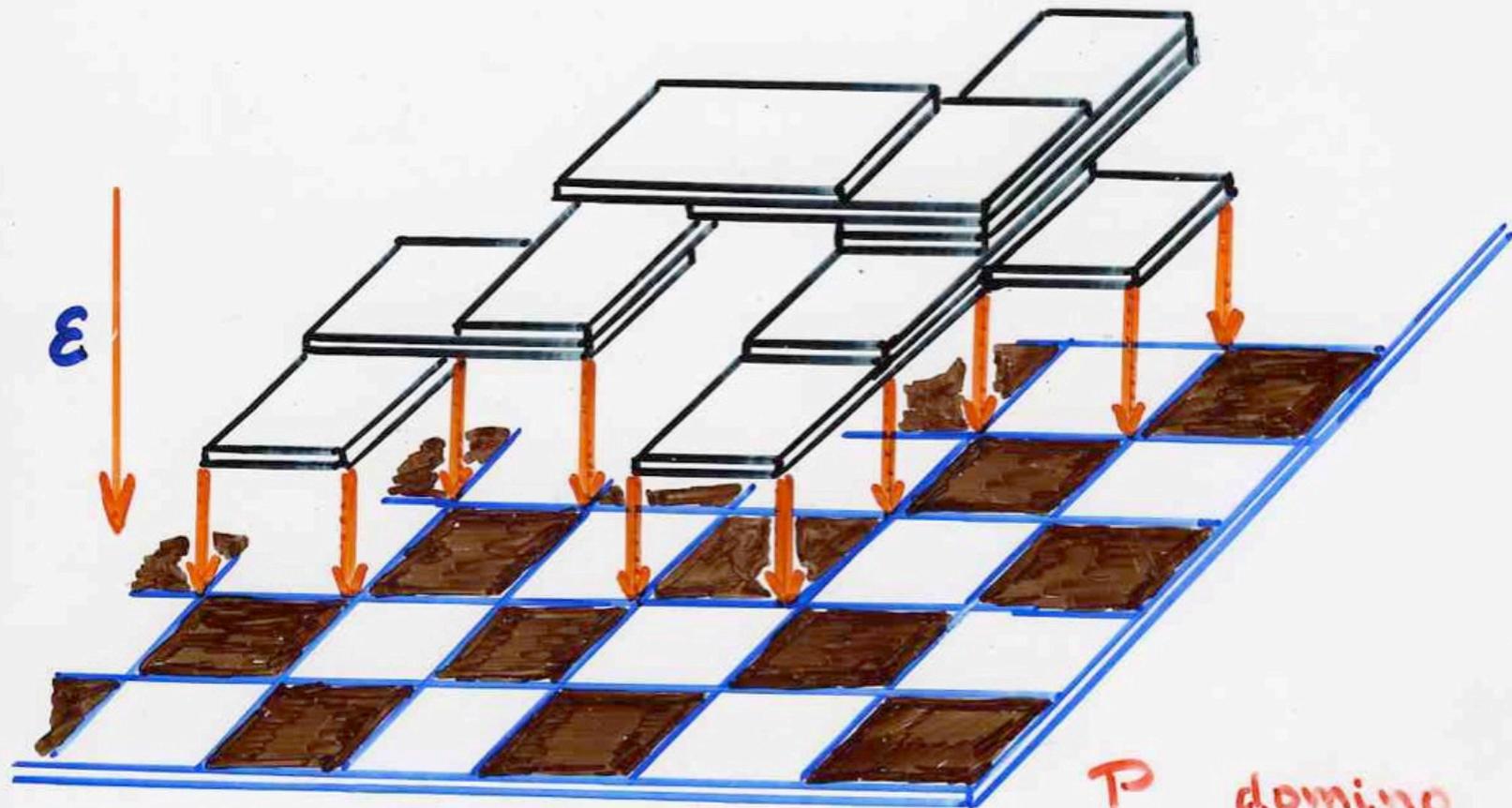
probabilités  
combinatoire  
physique statistique  
analyse d'algorithmes



combinatoire analytique

P.Flajolet

empilements de pièces

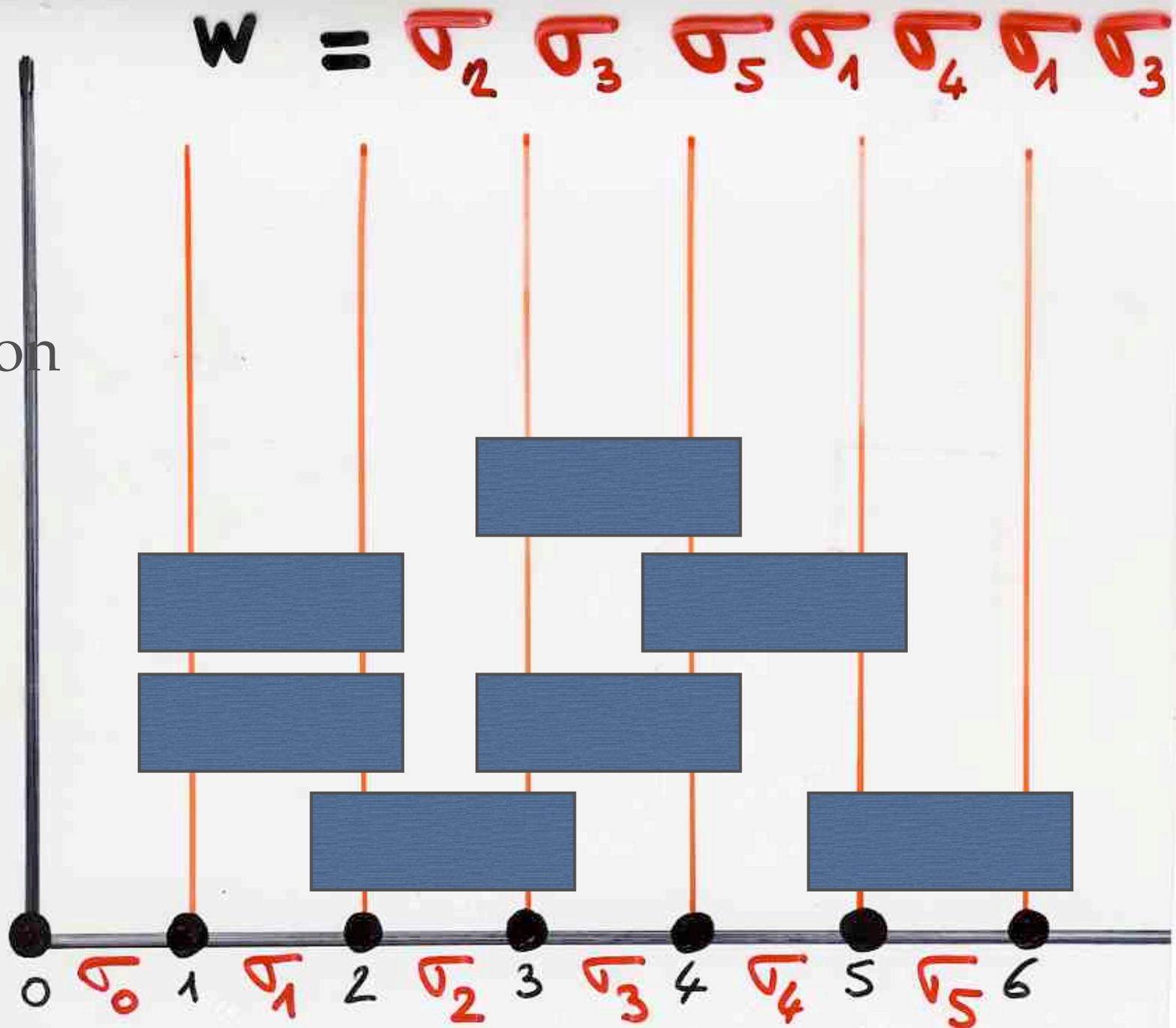


$$B = \mathbb{R} \times \mathbb{R}$$

$P$  domino

$\pi = Id$

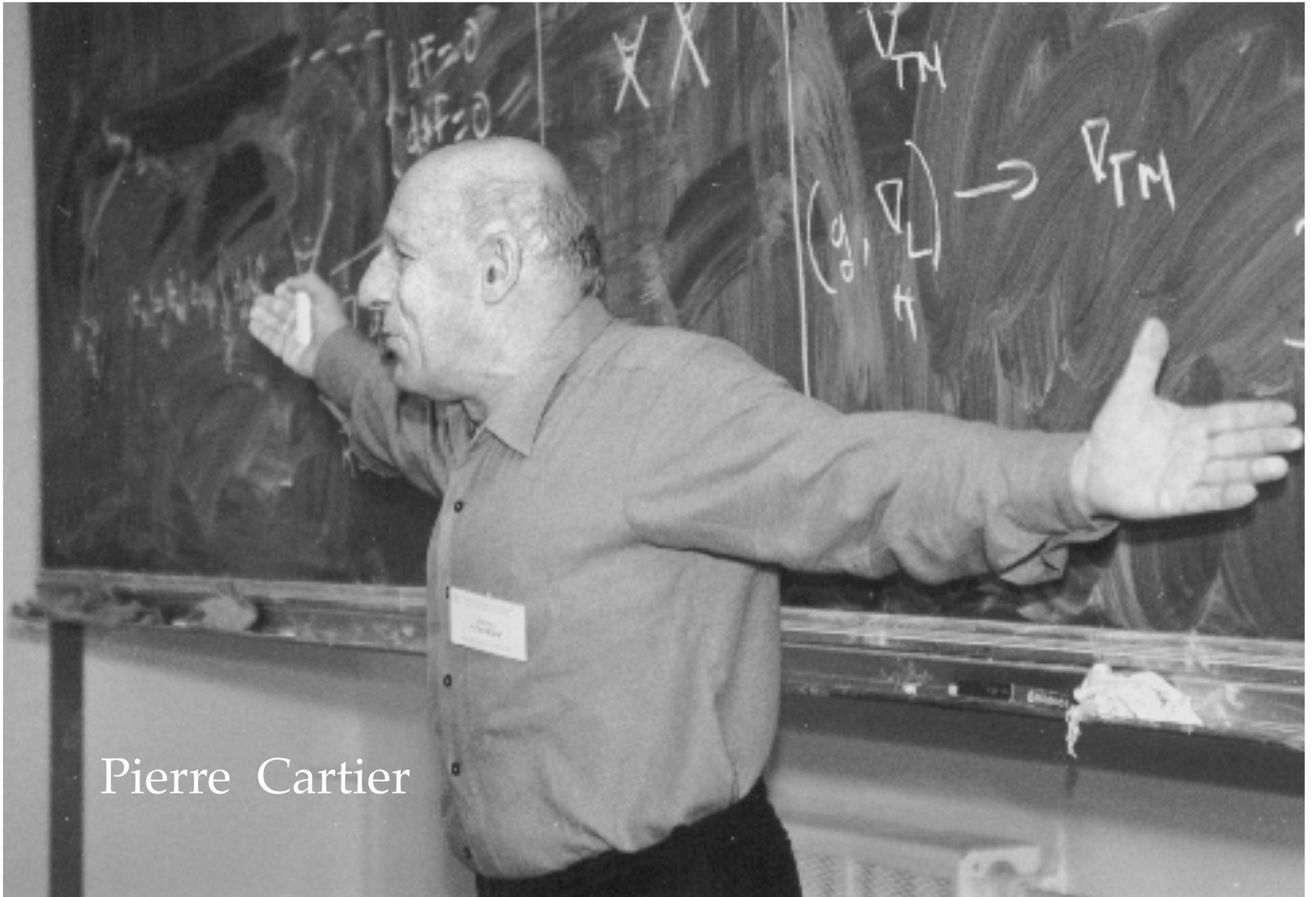
animation  
video



empilements  
de  
Pièces

monoïdes  
de  
commutation

Cartier et Foata



Pierre Cartier

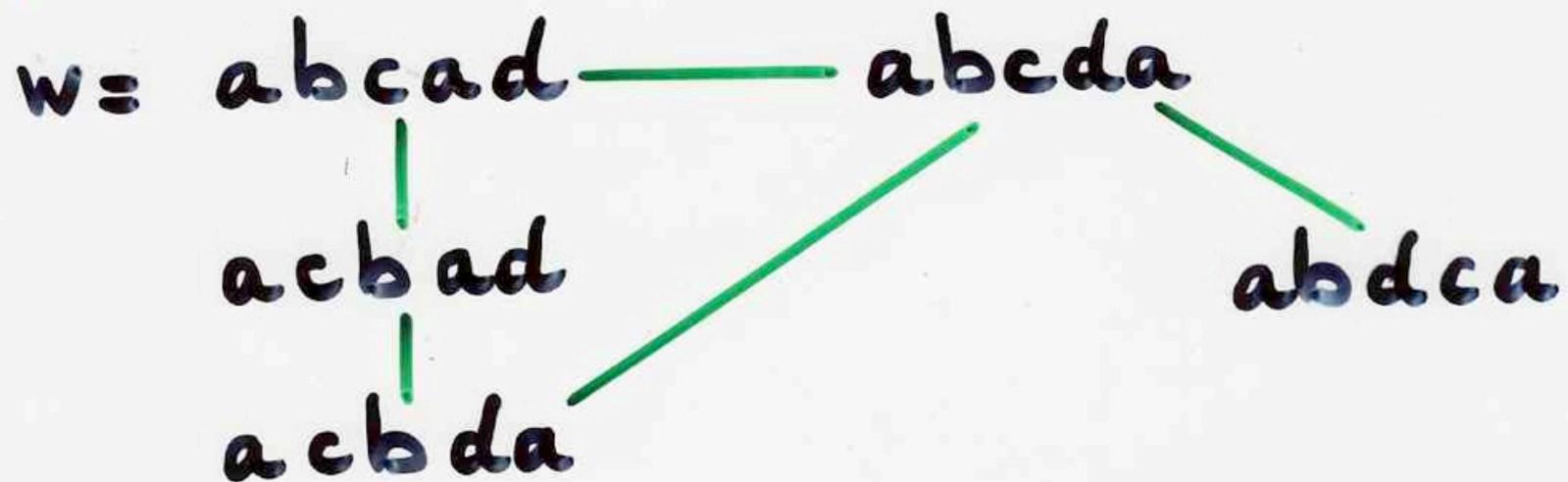


Dominique  
Foata

ex:  $A = \{a, b, c, d\}$

$C \begin{cases} ad = da \\ bc = cb \\ cd = dc \end{cases}$

equivalence class



heaps of dimers

$(i, i+1)$

on  $\{0, 1, \dots, n-1\}$

generators

$\{\sigma_0, \sigma_1, \dots, \sigma_{n-1}\}$

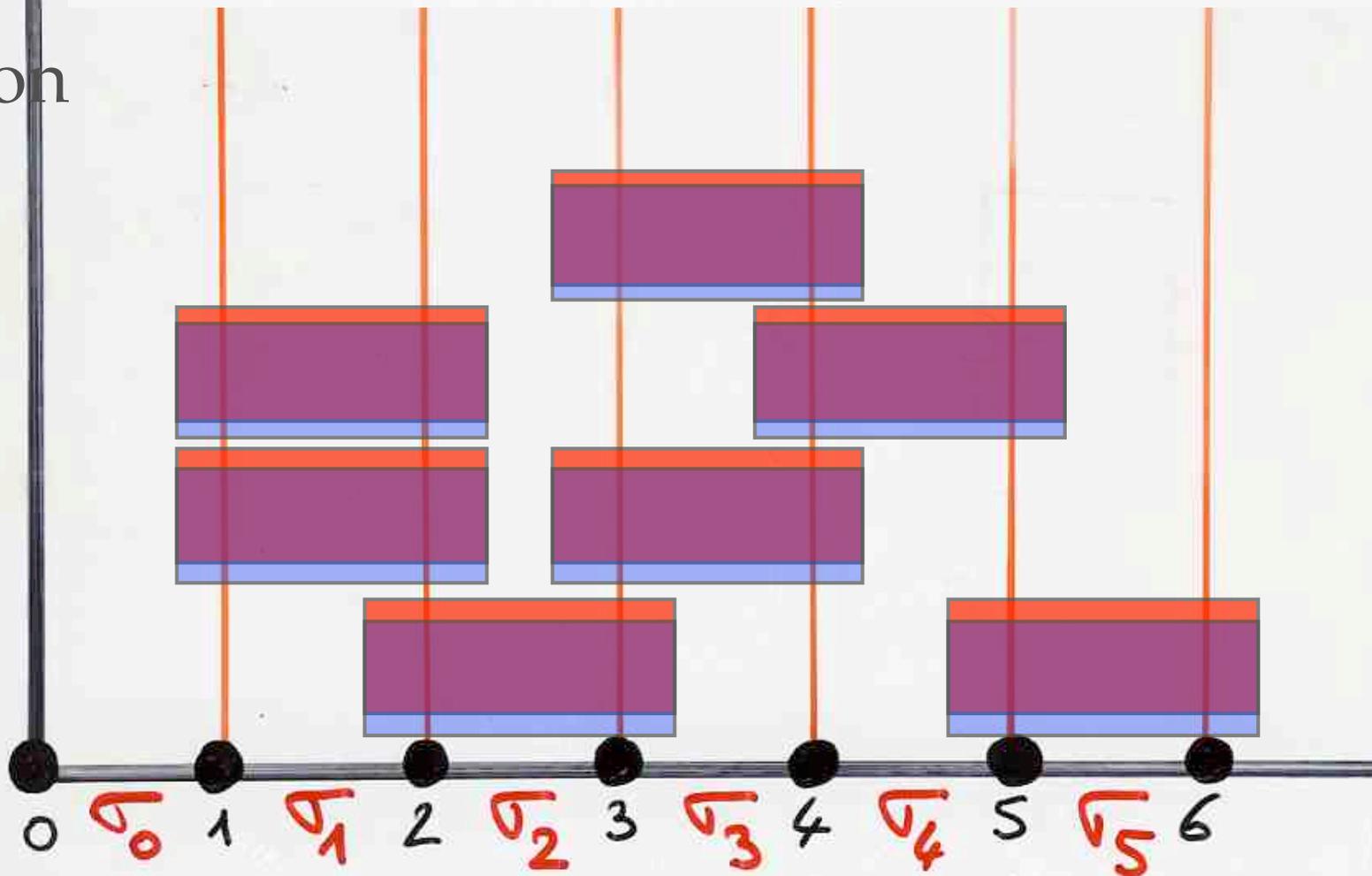
$$\sigma_i \sigma_j = \sigma_j \sigma_i$$

iff  $|i-j| \geq 2$

$$W = \sigma_2 \sigma_3 \sigma_5 \sigma_1 \sigma_4 \sigma_1 \sigma_3$$

$$W = \sigma_5 \sigma_2 \sigma_1 \sigma_1 \sigma_3 \sigma_4 \sigma_3$$

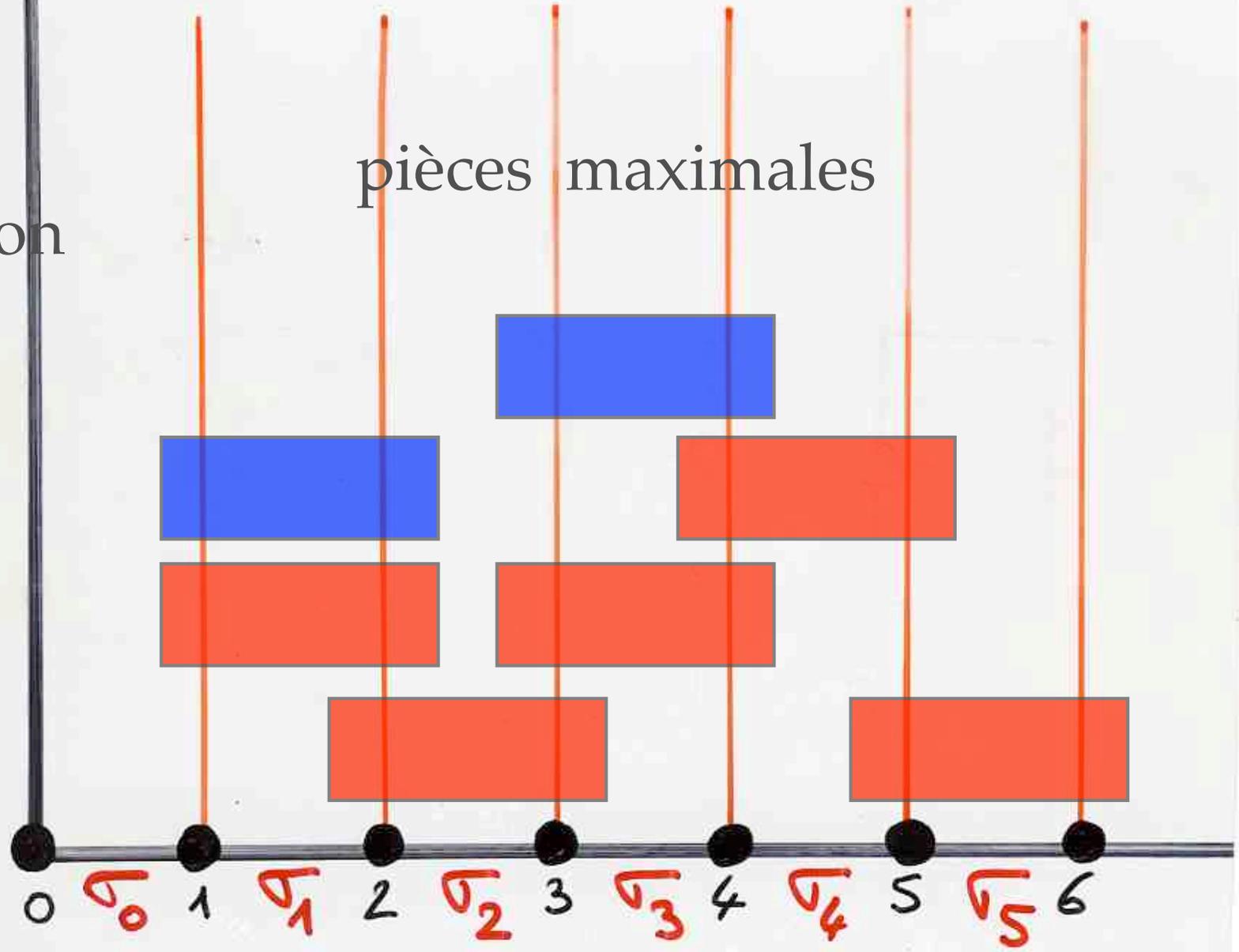
animation  
video



$$w = \sigma_2 \sigma_3 \sigma_5 \sigma_1 \sigma_4 \sigma_1 \sigma_3$$

pièces maximales

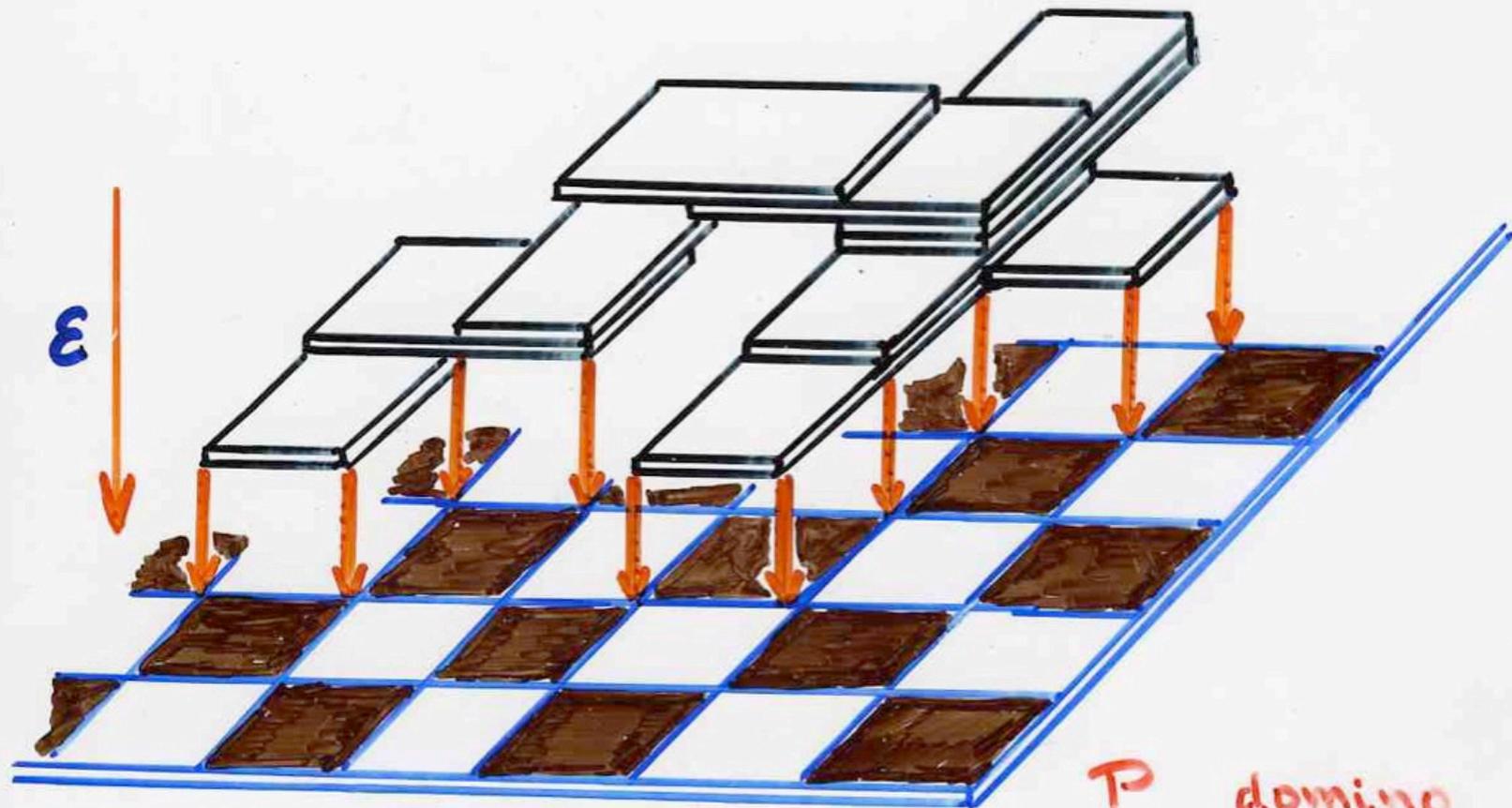
animation  
video



Pyramide

empilement  
ayant une seule  
pièce maximale



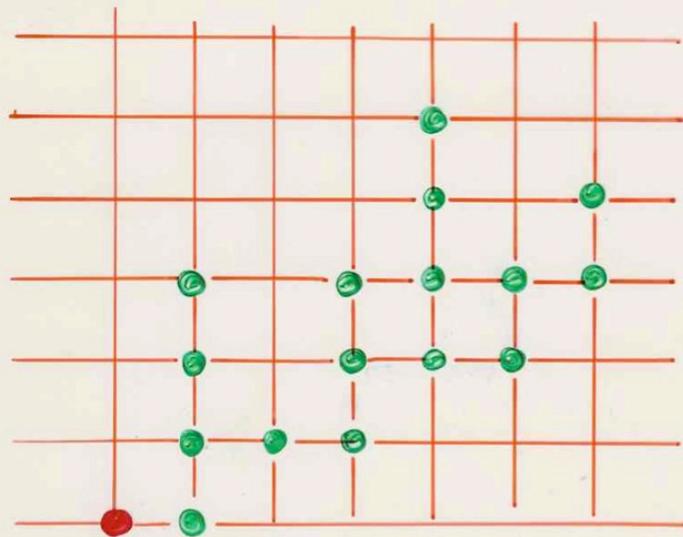


$$B = \mathbb{R} \times \mathbb{R}$$

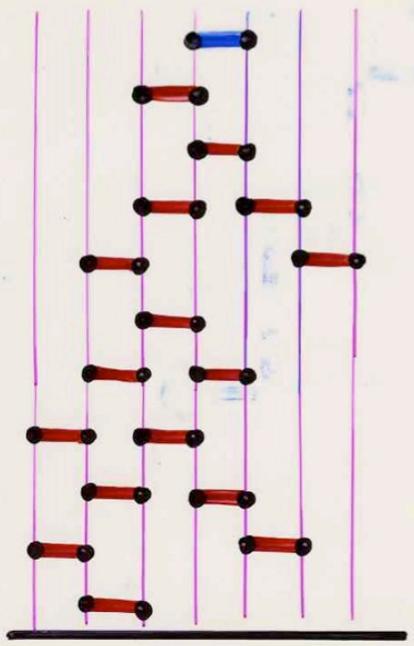
$P$  domino

$\pi = Id$

des animaux dirigés  
aux empilements de pièces

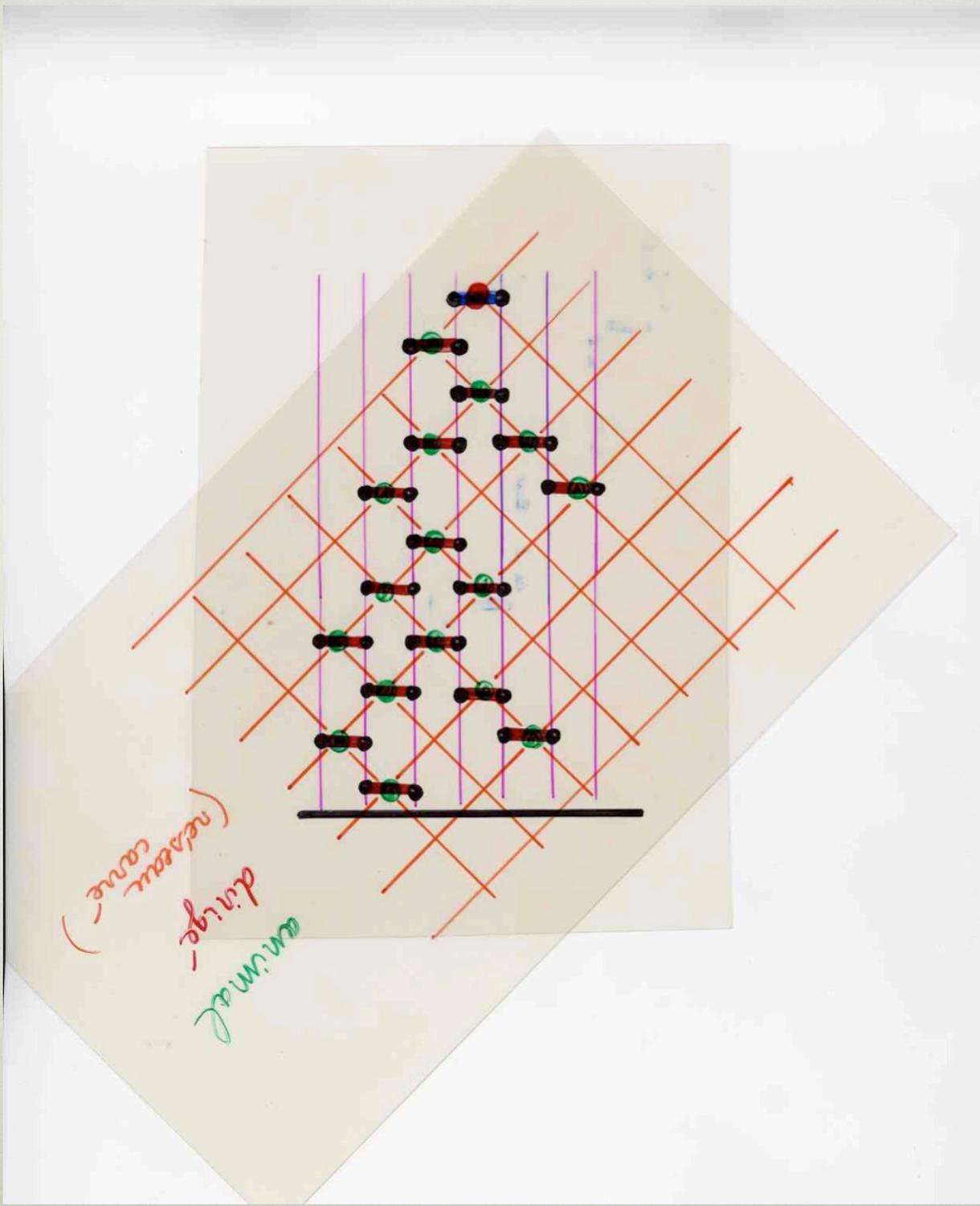


animal  
dirigé  
(réseau  
carré)



strict  
heap





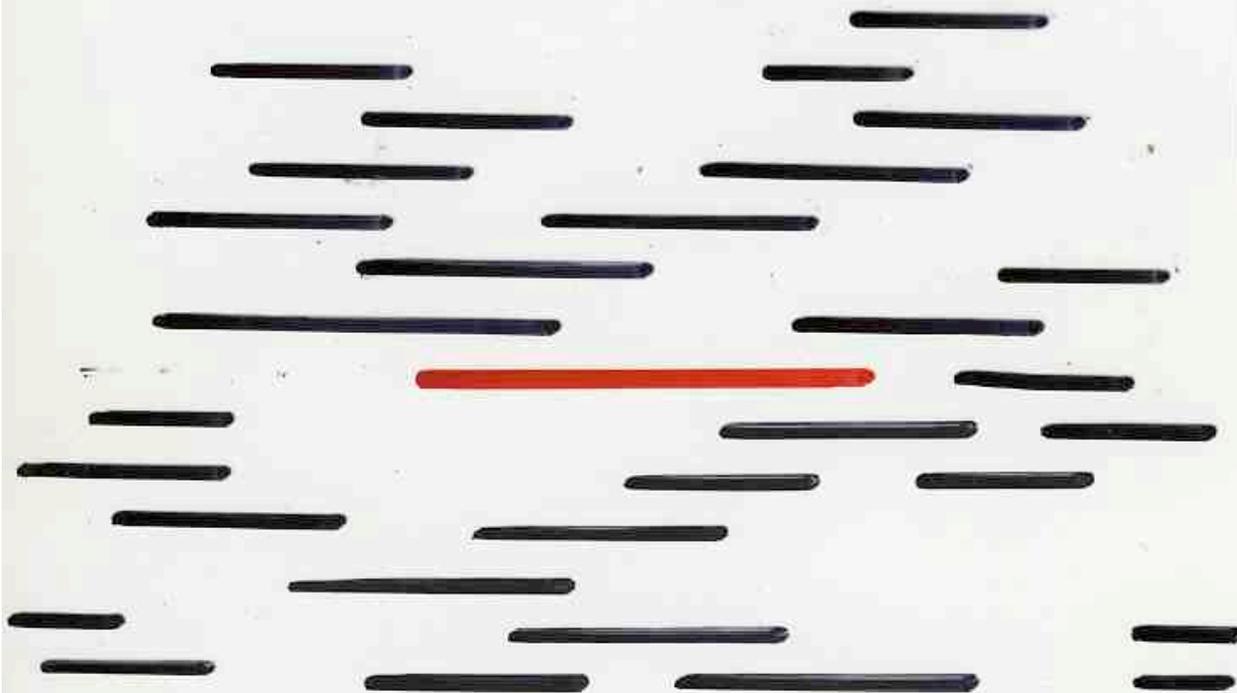
des équations "visuelles"

pour les animaux dirigés

Opérateur

"Poussez"

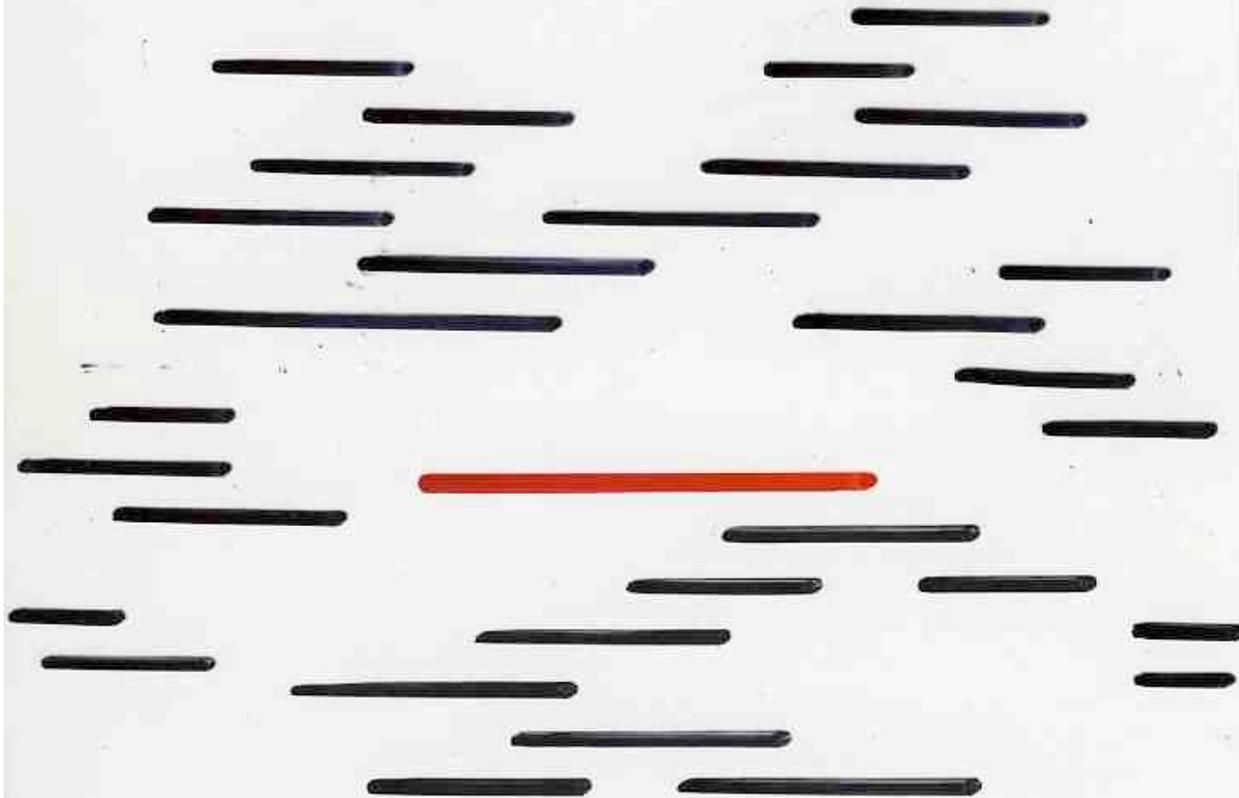
...



Opérateur

"Poussez"

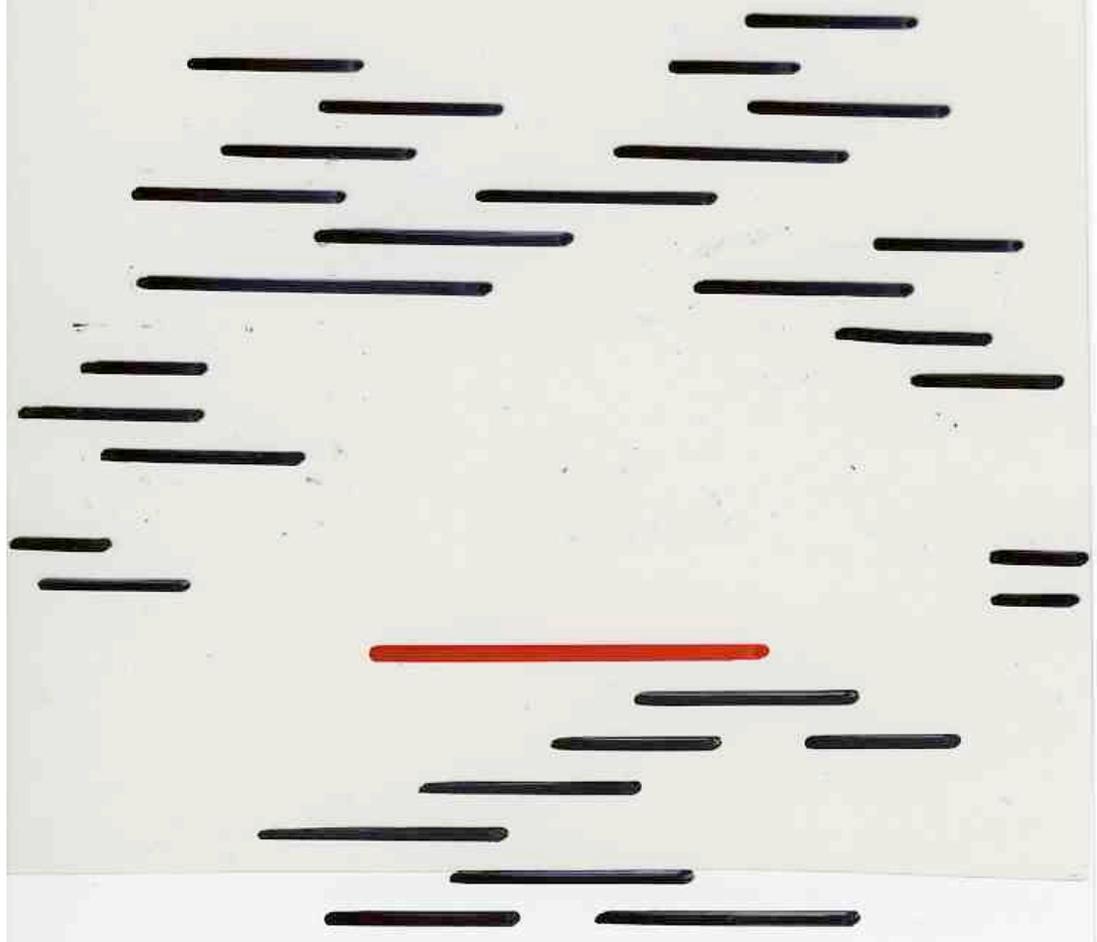
...



Opérateur

"Poussez"

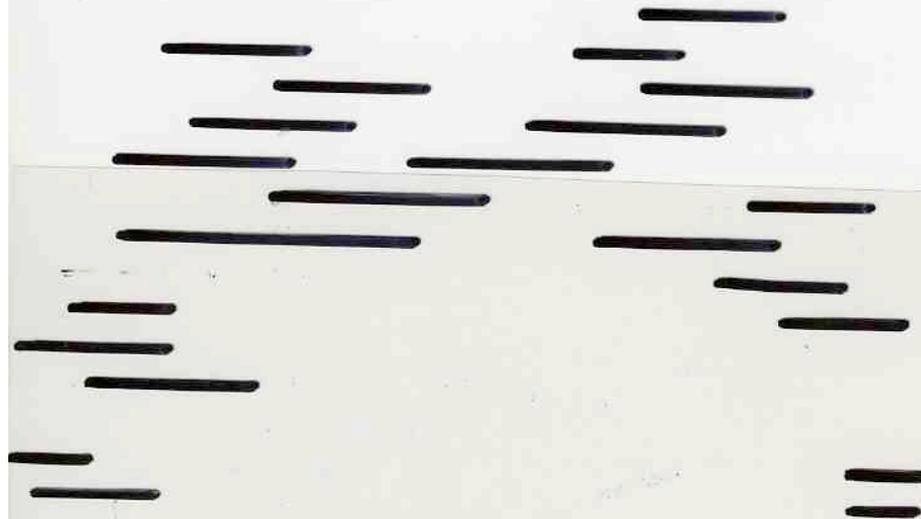
...



Opérateur

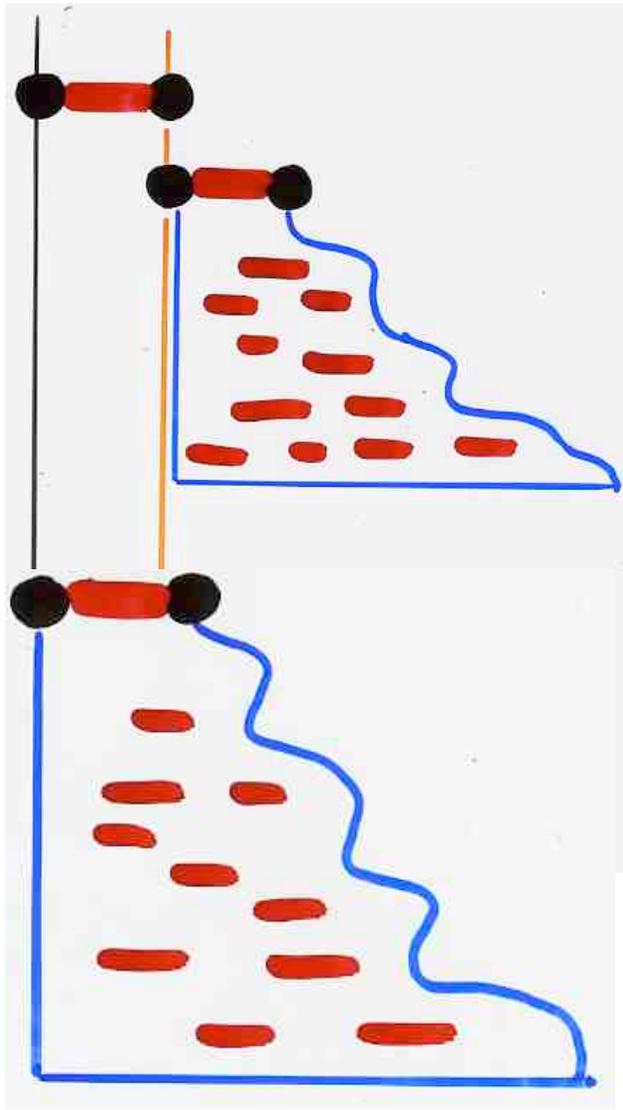
"Poussez"

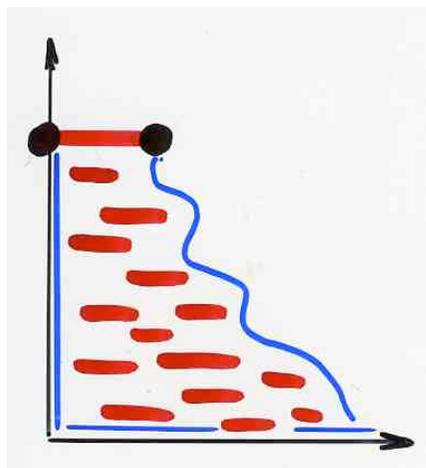
...





+

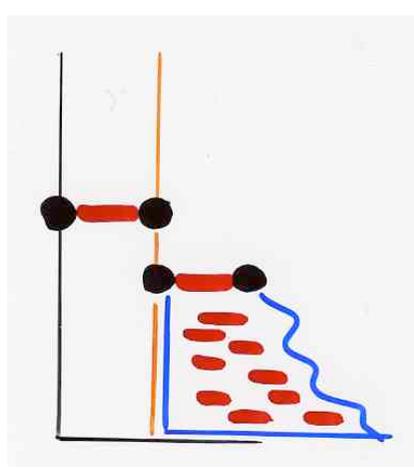




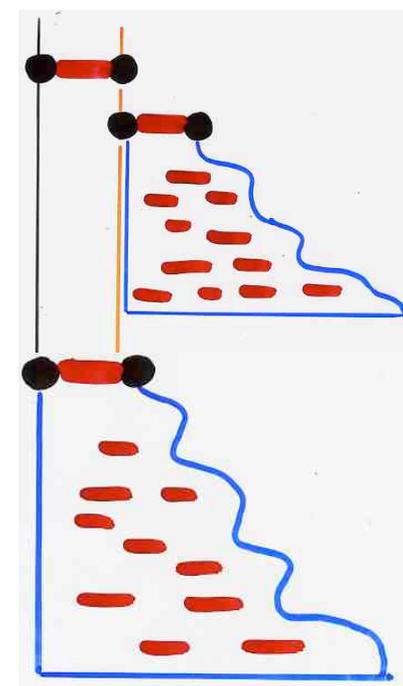
=



+



+



H

=

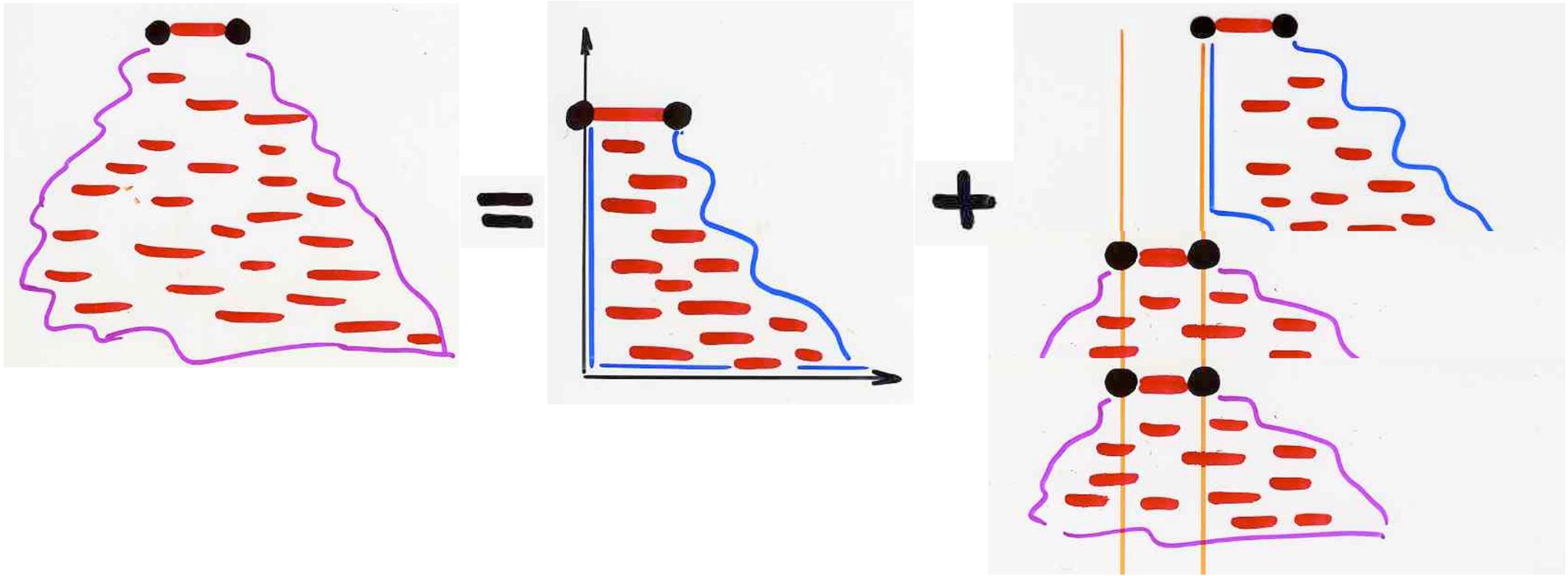
z

+

z H

+

z H<sup>2</sup>



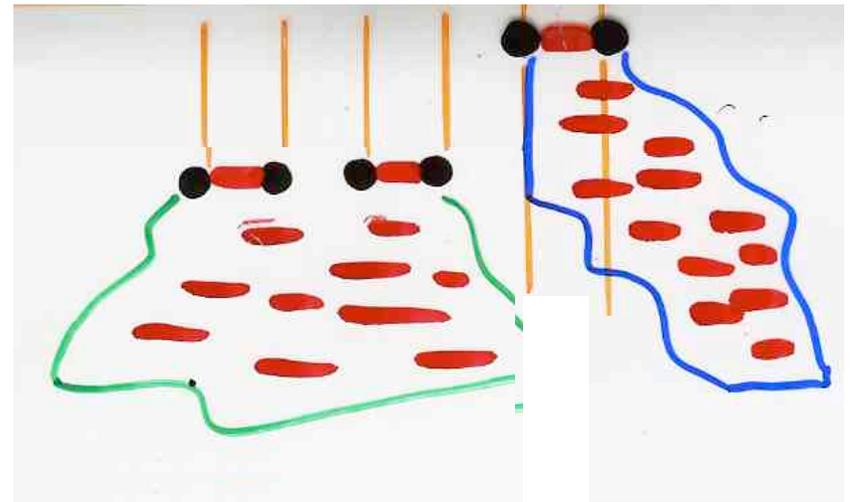
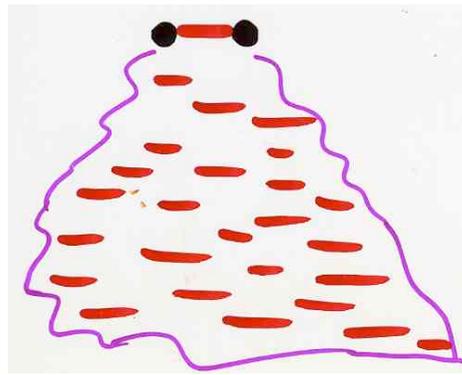
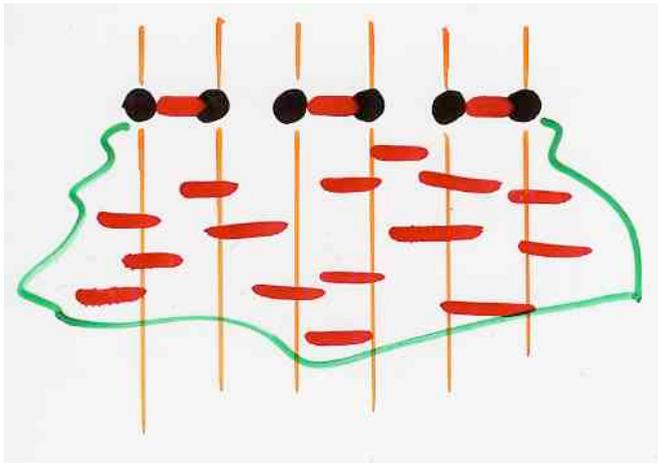
P

=

H

+

PH



X

=

P

+

X P

dessiner des calculs

...

calculer des dessins

le retour des figures  
en mathématiques

# Diagrammes de Feynman

$$\sigma^{\gamma}(\Upsilon) = \text{---} \circ \text{---} \circ \text{---} \circ \text{---}$$

$$\sigma^{\gamma}(\Upsilon) = \text{---} \circ \text{---} \text{---} \text{---} + \text{---} \circ \text{---}$$

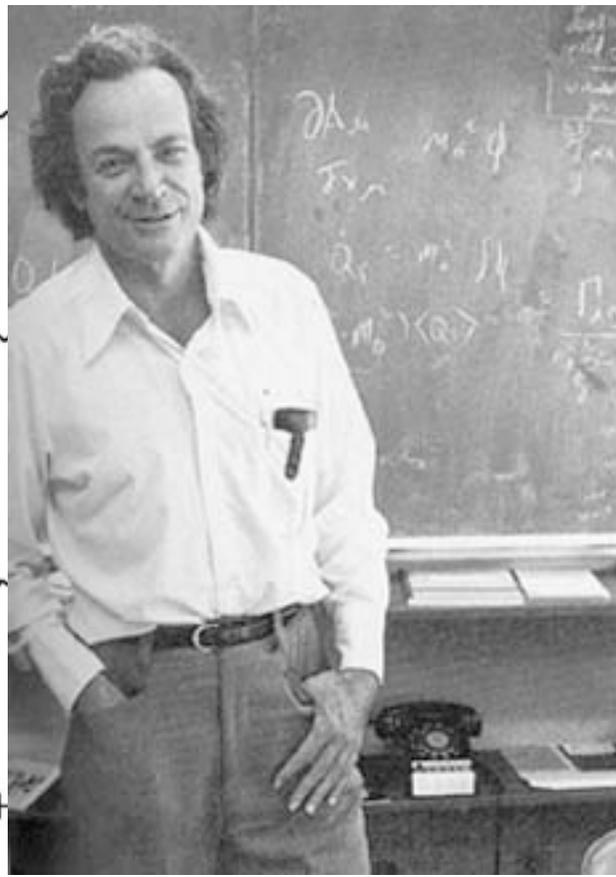
$$\sigma^{\gamma}(\Upsilon) = \text{---} \text{---} \text{---} \circ + \text{---} \text{---} \text{---} \circ$$

$$\sigma^{\gamma}(\Upsilon) = \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$$

$$\sigma^{\gamma}(\Upsilon) = \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$$

$$\text{---} \text{---} \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$$



$$H = z + zH + zH^2$$

$$P = H + PH$$

$$X = P + XP$$

$$X = \frac{z}{1-3z}$$

J. Betrema  
J.G. Penaud

$$= z + 3z^2 + 3^2z^3 + \dots + 3^n z^{n+1} + \dots$$



D. Zeilberger

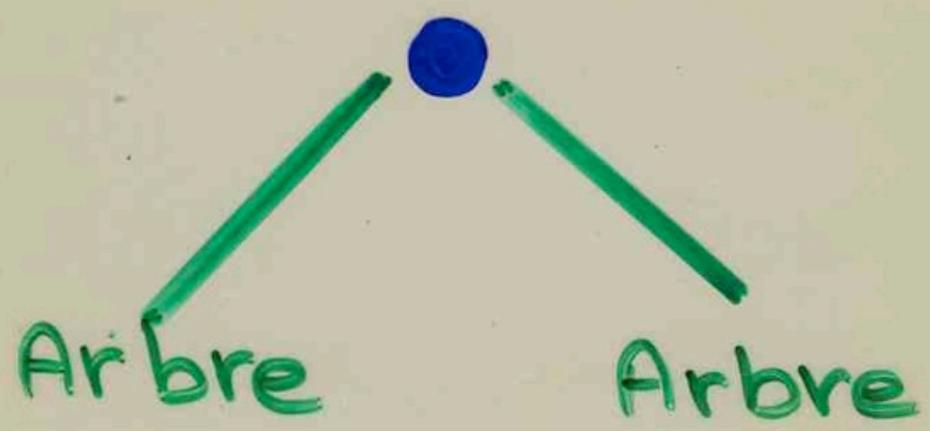
livre

$$A=B$$

$$A = 1 + tA^2$$

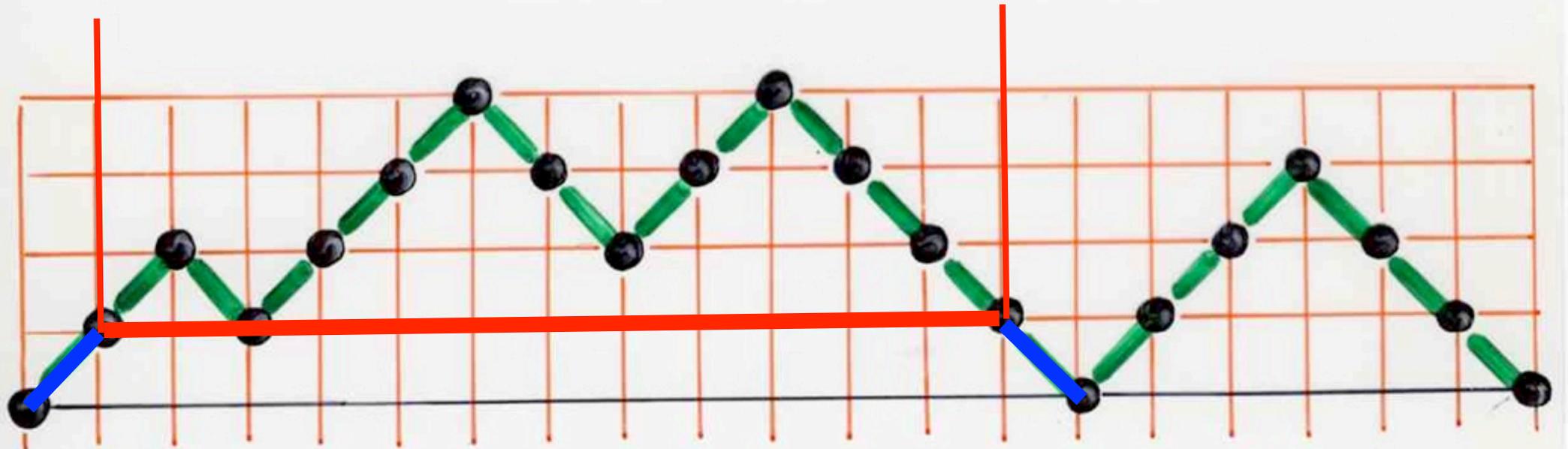
Arbre

=

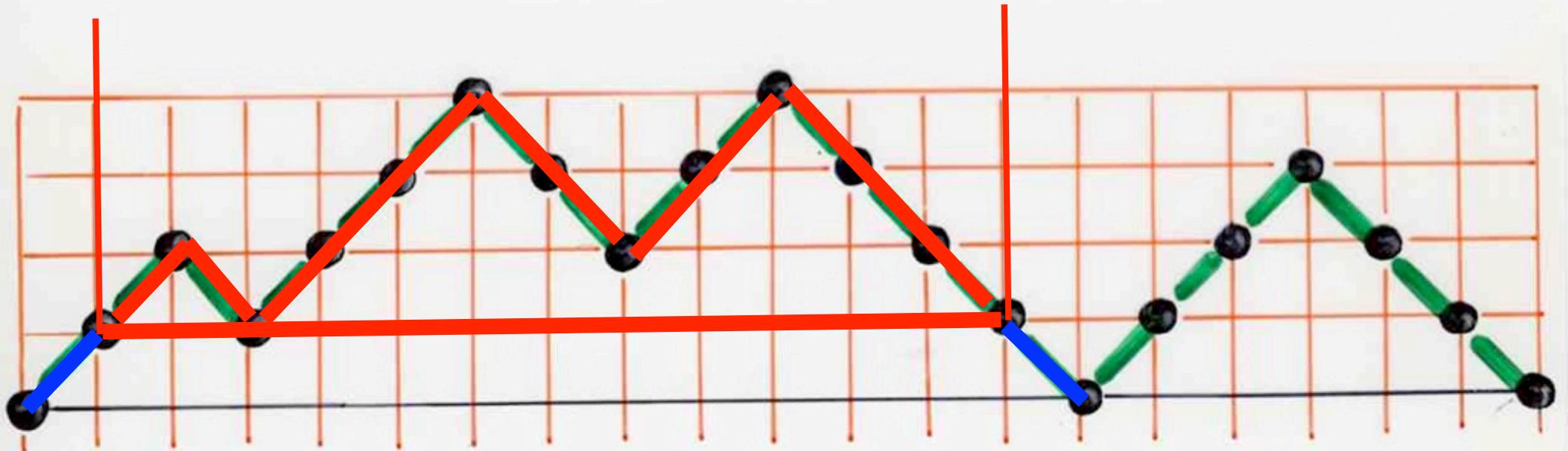




# chemins de Dyck



# chemins de Dyck



$$D = 1 + t D^2$$

$$H = Z + ZH + ZH^2$$

$$A = 1 + tA^2$$

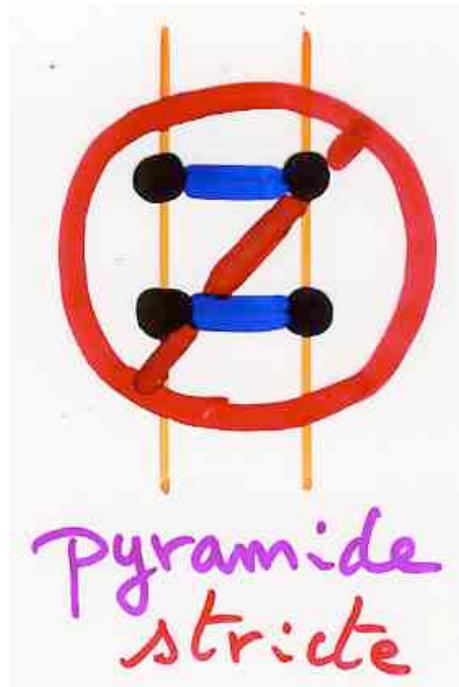
arbres binaires

$$D = 1 + tD^2$$

chemins de Dyck

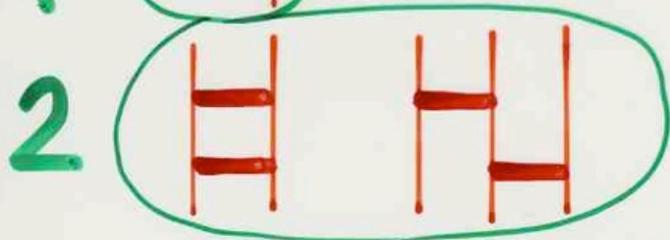
H

P



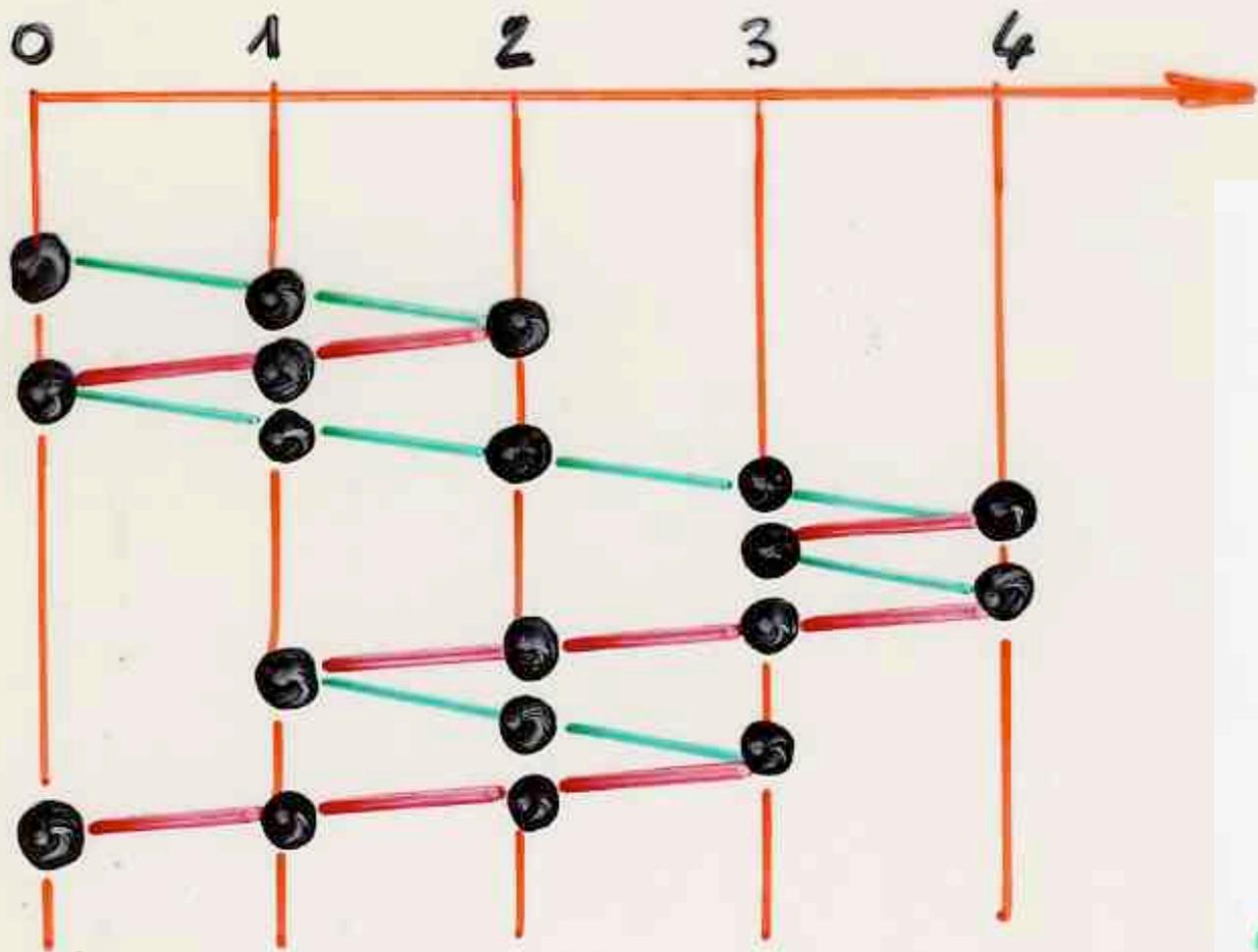
pyramide stricte

demi-pyramides  
sans restriction



$$A = 1 + tA^2$$

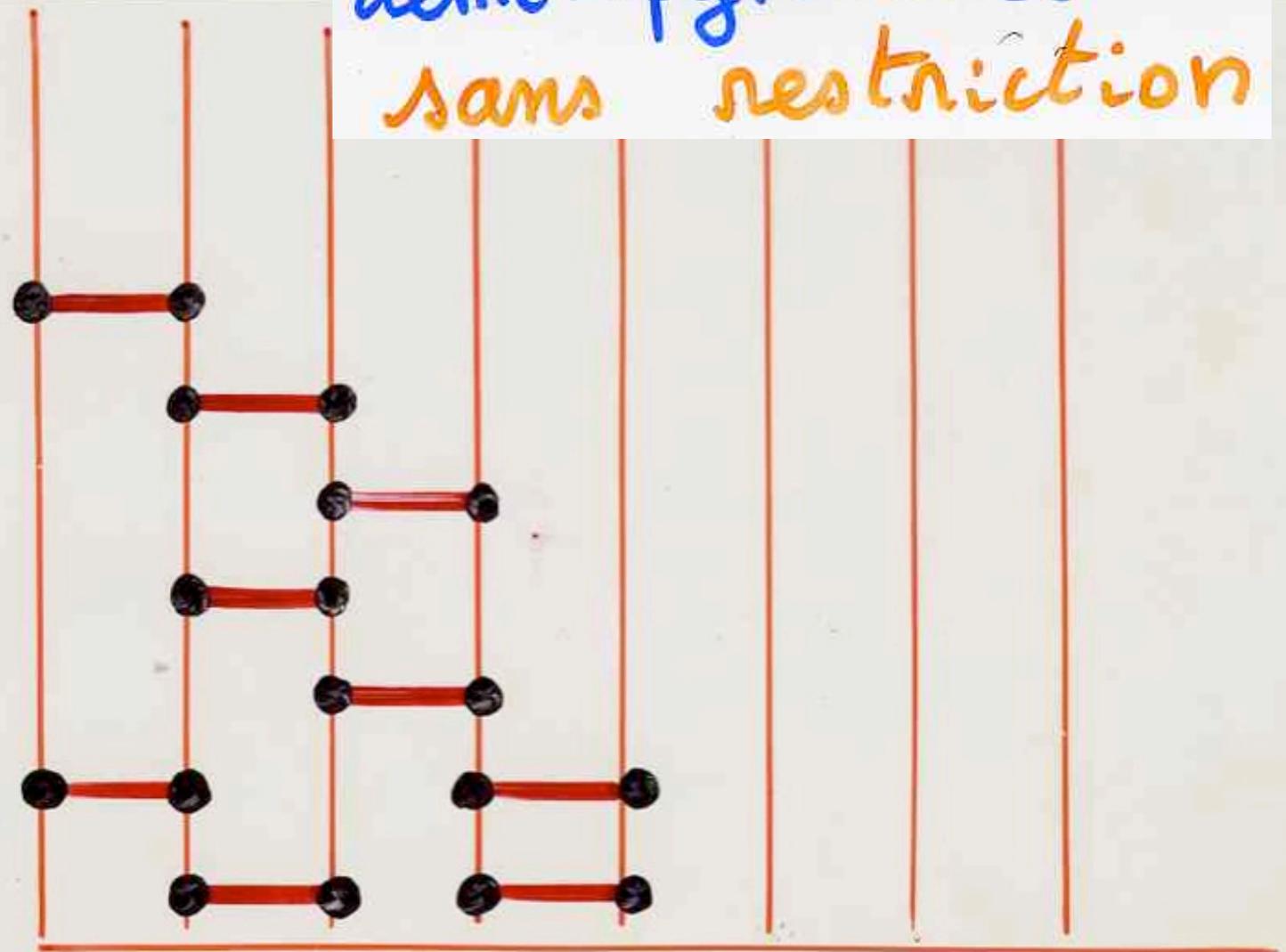
nombre  
de  
Catalan  
 $C_n$



chemins de Dyck

$$D = 1 + t D^2$$

demi-pyramides  
sans restriction



métamorphose:

des chemins de Dyck  
aux empilements de dominos

animation au violon

voir la vidéo sur le site: [www.xavierviennot.org](http://www.xavierviennot.org)

